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Slope Across the Curriculum: Principles and Standards for School Mathematics and Common Core State Standards

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This article provides an initial comparison of the Principles and Standards for School Mathematics and the Common Core State Standards for Mathematics by examining the fundamental notion of slope. Each set of standards is analyzed using eleven previously identified conceptualizations of slope. Both sets of standards emphasize Functional Property, Real World Situation, and Linear Constant conceptualizations of slope and describe a similar instructional sequence during the middle grades. However, the elementary and high school standards include differences that reflect alternative approaches to covering key prerequisite notions related to slope and in extending ideas of slope to non-linear functions. Both documents are examined in light of their respective purposes, with careful attention to potentially unintended consequences on the treatment of slope across the curriculum. The findings warrant careful consideration of other key mathematical topics to understand the curricular implications of adopting a new set of standards.

The National Council of Teachers of Mathematics (NCTM) spurred the standards-based reform of mathematics education by publishing the *Curriculum and Evaluation Standards for School Mathematics* in 1989 and then the

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Principles and Standards for School Mathematics (*PSSM*) in 2000. The writers of the *PSSM* aimed to "set forth a comprehensive and coherent set of learning goals for mathematics for all students from prekindergarten through grade 12 that [would] orient curricular, teaching, and assessment efforts" (NCTM, 2000a, p. 1).

PSSM's goals are organized according to five content standards, including Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. *PSSM* provides both an overview of the content of these standards in prekindergarten through grade 12 as well as a more detailed outline of the content in each standard for four distinct grade bands: Prekindergarten through grade 2, grades 3–5, grades 6–8, and grades 9–12. In addition to the five content standards, *PSSM* describes five process standards that describe how students should acquire and apply their content knowledge (NCTM, 2000b). The goal of *PSSM* was not to provide an exhaustive list of all the objectives to be taught, but instead to provide a vision for mathematics education across the prekindergarten through grade 12 curriculum. Although *PSSM* was spurred by efforts to provide guidance for national curricular development and instructional decisions, great variability continues to be reported between individual states' mathematics standards (Porter, Polikoff, & Smithson, 2009; Reys, Chval, Dingman, McNaught, Regis, & Togashi, 2007).

The Common Core State Standards Initiative is one of the most recent developments in the United States' preK-12 mathematics education reform movement. The National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) led the development and release of *Common Core State Standards for Mathematics* (National Governors Association for Best Practices & Council of Chief State School Officers, 2010). The *Common Core State Standards for Mathematics* (*CCSSM*) seek to provide national standards that are "focused and coherent" while aiming for "clarity and specificity" (NGA Center & CCSSO, 2010, p. 3). In contrast to *PSSM*, *CCSSM* describes standards individually for each grade level from kindergarten to grade 8 (rather than using grade bands) and for six categories of high school mathematics (Number and Quantity, Algebra, Modeling, Functions, Geometry, Statistics and Probability). Similar to *PSSM*'s process standards, *CCSSM* describes standards for mathematical practice as the ways in which students should be encouraged to engage in mathematics. Adopted by 44 states nationwide (Achieve, 2012), *CCSSM* has the potential to influence the mathematics curriculum across the nation. As with any curricular reform, we believe it is important to study differences between the emphasis of previous and newly adopted documents to understand how instruction may ultimately affect students' mathematical understanding.

Porter, McMaken, Hwang, and Yang (2011) investigated the alignment of the various states' standards, the *PSSM*, and the *CCSSM*. In their study, Porter and colleagues used the Surveys of Enacted Curriculum (Porter, 2002) to study alignment using both content and level of cognitive demand as a lens for the degree of agreement. The authors found a great amount of variability between the states' standards in terms of both content and focus, making it difficult to generalize comparisons from *CCSSM* to the state standards as a whole. Although the state-to-*CCSSM* alignments varied, they were generally described as having low to moderate alignment. In general, *CCSSM* demonstrated a slight shift to higher cognitive demand, placing more emphasis on conceptual understanding and less on skills and procedures when compared with state standards (Porter, McMaken, Hwang, & Yang, 2011). They found similar results for *PSSM*, which aligned to *CCSSM* better than some states but not as well as others, aligning as well as the "average" states (Porter et al., 2011). Alignment did improve slightly when Porter et al. aggregated the standards across grade levels to eliminate the influence of sequencing and timing. The findings suggest that adoption of *CCSSM* may involve significant adaptations to the current curriculum and that the adaptations will vary greatly from state to state. A careful analysis of the treatment of each of the foundational mathematical concepts under the previous and new standards is critical for a fluid transition to *CCSSM*.

Why Slope?

Slope is a particularly important topic for an investigation of curricular coverage. Slope is a foundational mathematical topic that appears throughout the elementary and secondary mathematics curriculum from beginning algebra to calculus (NCTM, 2000a, 2000b; NGA Center & CCSSO, 2010). In addition to coverage across the curriculum, there are many conceptualizations of slope emphasized. Building on Stump's initial work (1999, 2001a, 2001b), Moore-Russo and colleagues (Moore-Russo, Conner, & Rugg, 2011; Mudaly & Moore-Russo, 2011) have identified eleven unique conceptualizations of slope. Each of the eleven conceptualizations, listed in Table 1, has been recognized among secondary or post-secondary students and instructors (Moore-Russo, et al., 2011; Mudaly & Moore-Russo, 2011; Nagle & Moore-Russo, 2013; Nagle, Moore-Russo, Viglietti, & Martin, 2013; Stanton & Moore-Russo, 2012). The multitude of ways to conceptualize slope suggests that different sets of standards might promote the development of varying ways that individuals come to understand slope. As a result of its prominence and diversity across the curriculum, slope is particularly susceptible to changes in curricular reform. In this study, the researchers compare *PSSM* and *CCSSM* by focusing on the concept of slope, considering: (a) which of the eleven conceptualizations are emphasized and (b) the sequencing of instruction across the curriculum. The following research questions guided the study:

1. What conceptualizations of slope are emphasized across the curriculum by *CCSSM* and *PSSM*?
2. What conceptualizations of slope are emphasized at each grade band by *CCSSM* and *PSSM*?
3. What overall notion of slope is supported by *CCSSM* and *PSSM*?

Methods

The research team used the "all-code-all" approach to code the *PSSM* and *CCSSM* documents using the eleven conceptualizations of slope adopted by Stanton and Moore-

Table 1
Slope Conceptualizations as adopted from Stanton and Moore-Russo (2012).

Category	Slope as …
Geometric Ratio (G)	Rise over run of a graph of a line; ratio of vertical displacement to horizontal displacement of a line's graph (often seen as graph of a line with right triangle highlighting both the horizontal and vertical displacement)
Algebraic Ratio (A)	Change in y over x; representation of ratio with algebraic expressions (often seen as either $\Delta y/\Delta x$ or $(y_2 - y_1)/(x_2 - x_1)$)
Physical Property (P)	Property of a line often described using expressions like grade, incline, pitch, steepness, slant, tilt, and "how high a line goes up"
Functional Property (F)	(Constant) rate of change between variables or quantities (e.g., when *x* increases by 2, *y* increases by 3) found in various representations of functions; sometimes seen in situations involving related rates or constants of proportionality (where the unit rate is the slope)
Parametric Coefficient (PC)	The variable *m* (stated either as "*m*" or as its numeric value) found as a coefficient in $y = mx + b$ and $(y_2 - y_1) = m(x_2 - x_1)$
Trigonometric Conception (T)	Property related to the angle a line makes with a horizontal line (usually the positive *x*-axis); tangent of a line's angle of incline/decline; direction component of a vector
Calculus Conception (C)	Measure related to derivative either specifically as the slope of a secant or tangent line to a curve or as relating to the instantaneous rate of change for any (even a nonlinear) function
Real World Situation (R)	Static, physical situation (e.g., wheelchair ramp) or dynamic, functional situation (e.g., distance vs. time)
Determining Property (D)	Property that determines if lines are parallel or perpendicular; property with which a line can be determined (if a point on the line is also given)
Behavior Indicator (B)	Property that indicates the increasing, decreasing, or horizontal trends of a line or the property that indicates amount (or severity) of a line's increase or decrease; if nonzero, indicates that a line has an intersection with the *x*-axis
Linear Constant (L)	"Straight" or "flat" absence of curvature of a line that is not impacted by translation; property unique to "straight" figures (can be referenced as what makes a line "straight" or the "straightness" of a line); mention that any two points on a line may be used to determine slope

Russo (2012). Both members of the research team independently reviewed the Slope Conceptualization codes and the *PSSM* and *CCSSM* documents; then, they collaboratively coded the documents discussing the coding as they proceeded. In instances of an initial disagreement on coding, the researchers discussed the excerpt until a consensus was reached. In addition to the actual standards, introductory paragraphs, overviews, and sample problems were coded for references to the concept of slope. *PSSM*'s Process Standards and *CCSSM*'s Standards for Mathematical Practice were also included in the analysis. A single sentence was coded for each conceptualization evidenced. In a few instances multiple sentences described the same idea and so they were coded as one unit. See Table 2 for examples of codes assigned.

Table 2
Excerpt from PSSM and CCSSM with Corresponding Coding

Slope Reference	Source	Coding
Example of plant growth in which students use verbal, tabular, and graphical representations to describe the rate of growth over time with a focus on comparing situations with constant versus variable rate of change and recognizing graphical representations of each; "… precursor to later, more focused attention on what the slope of a line represents, that is, what the steepness of the line shows about the rate of change."	*PSSM* Algebra Standard (Grades 3-5) p. 163	F, P, L
"By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y-2)/(x-1)=3$."	*CCSSM* Standard for Mathematical Practice (Grades K-12) p. 8	A
"Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y=mx$ for a line through the origin and the equation $y = mx+b$ for a line intercepting the vertical axis at b."	*CCSSM* Expressions and Equations Standard (Grade 8) 8.EE.6, p. 54	G, L, PC

When a standard described a sample problem, the researchers attempted to solve the problem to determine which conceptualizations of slope were required. When multiple conceptualizations were required to solve a single problem, the problem was coded for all such conceptualizations. However, in a few instances, multiple approaches to a single problem were possible (but not required), and one approach led to a particular conceptualization while another did not. For instance, standard 8.EE.8C in *CCSSM* describes using two pairs of points to determine whether lines passing through the pairs of points intersect. This problem could imply the use of an *Algebraic Ratio* (y_2-y_1/x_2-x_1) to find the slope of the line passing through the two points and then *Determining Property* to recognize that lines are parallel precisely if their slopes are equal. However, this problem could also be solved by plotting the two pairs of points on a coordinate system, using a ruler to sketch the line passing through each pair of points, and visually inspecting whether the lines will meet. Because it was unclear which approach was anticipated and, therefore, which conceptualizations would be utilized to solve the problem, no code was assigned to this standard. A small number of such ambiguous cases occurred, and each was handled in the manner described above.

Results

The number of references made throughout the standards documents determined the frequency of each conceptualization of slope. The total number of slope references across the elementary and secondary curriculum was similar for *PSSM* (57) and *CCSSM* (53), and the most dominant conceptualizations of slope (i.e., *Functional Property*, *Linear Constant*, and *Real World Situation*) are also consistent across the *PSSM* and *CCSSM* documents. Figure 1 illustrates the number of references to each conceptualization of slope under *PSSM* and *CCSSM* across the preK-12 curriculum. Because the total numbers of slope references were not equal, Figure 2 illustrates the percentage of the intended slope instruction aligned to each of the eleven conceptualizations under *PSSM* and *CCSSM*. Although *Functional Property* was the most common conceptualization for both documents, it was evidenced twice as often in *CCSSM* compared to *PSSM*. Conversely, *PSSM* emphasized *Behavior Indicator* and *Physical Property* conceptualizations of slope and included evidence of both the *Trigonometric* and *Calculus* Conceptions of slope; neither *Trigonometric* nor *Calculus* Conceptions were found in *CCSSM*. Despite the differences cited, the overall emphasis of slope is quite similar between the two sets of standards.

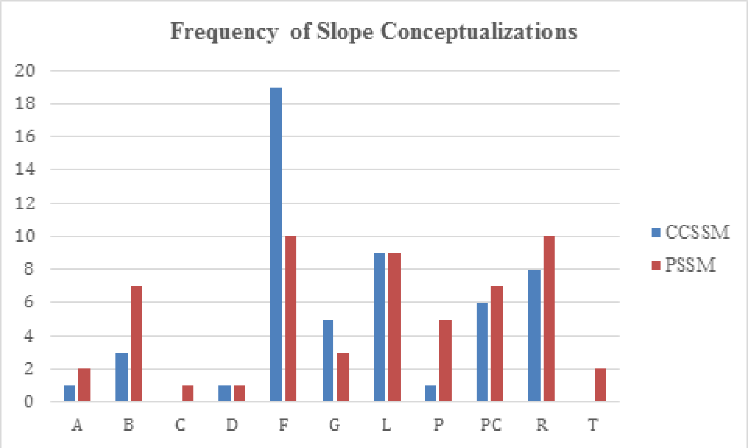

Figure 1. Frequency of conceptualizations in *PSSM* and *CCSSM*.

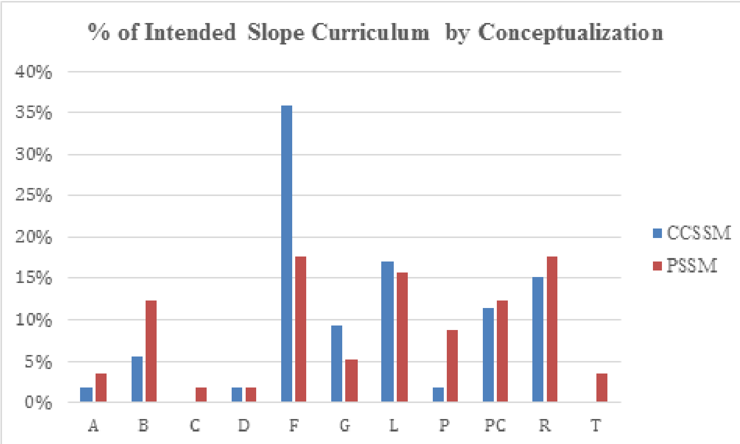

Figure 2. Distribution of conceptualizations in *PSSM* and *CCSSM*.

Grade Level Analysis

To understand the sequencing of slope instruction, we compared conceptualizations by grade bands where slope was referenced (3–5, 6–8, 9–12) to compare the emphasis at various stages of the curriculum. Figure 3 displays an illustrated summary of conceptualizations in the three grade bands. Discussion of the findings for each grade band follows.

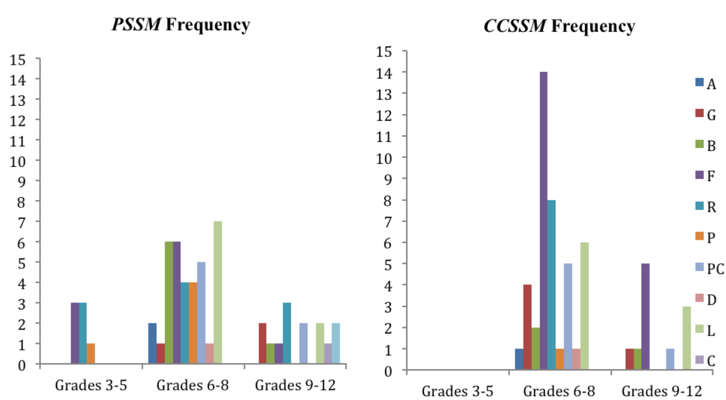

Figure 3. Conceptualizations of slope emphasized across the various grade bands in *PSSM* and *CCSSM*.

Grades 3-5. The overall similarity of slope across the curriculum does not translate to similar treatment of the topic in grades 3-5. Although *PSSM* supported the development of *Functional Property*, *Real World Situation*, and *Physical Property* conceptualizations of slope during grades 3-5, no references to slope were made in these grade level standards for *CCSSM*. *PSSM*'s emphasis in grades 3-5 is illustrated by the plant growth example described in Table 2. In particular, *PSSM* stresses the development of analyzing change in grades 3-5, including describing the relationship between simultaneously changing quantities to understand rate of change. The rate of change between two covarying quantities is at the heart of the *Functional Property* conceptualization of slope. It is noteworthy that although *CCSSM* places emphasis on describing change through analyzing patterns and relationships in grades 3-5, covariational reasoning of simultaneously changing quantities is not explicitly mentioned. As defined by Carlson, Jacobs, Coe, Larsen, and Hsu (2002), covariational reasoning refers to "…coordinating two varying quantities while attending to the ways in which they change with relation to each other" (p. 354). Instead of this focus on coordinating the change between two variables (as related to *Functional Property*), the *CCSSM* focus is on describing change in a single quantity. At first, this seems like a subtle difference. However, attention to coordinating change between two covarying quantities has been closely linked to reasoning about slope, as described in more detail below.

The difference in the treatment of relationships and change in grades 3-5 may influence students' understanding of slope. Lobato, Ellis, and Muñoz (2002) described students' focusing phenomenon, highlighting distinctions between a focus on change in a single quantity versus coordinated change in covarying quantities. *CCSSM*'s coverage of patterns and relationships focuses on uncoordinated sequences of values and differences of quantities, such as using a constant additive rule to generate terms of a sequence, whereas *PSSM* describes using patterns to relate covarying quantities (e.g., time and height in the plant growth example). According to Lobato, et al. (2002), *CCSSM*'s approach may lead to students interpreting slope as a difference (e.g., y_2-y_1) rather than as a quotient dependent on two covarying quantities (e.g., y_2-y_1/x_2-x_1). Covariational reasoning has received much attention as an important foundation for the concept of slope in particular (Lobato, et al., 2002; Lobato & Siebert, 2002) and precalculus in general (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Carlson, Oehrtman, & Engelke, 2010). Johnson (2012, 2013) distinguishes between two types of covariational reasoning: *simultaneous independent* (comparing changes in two quantities) and *change dependent* (coordinating change in two quantities). *Change dependent* reasoning helps students understand variation in the intensity of change (e.g., increasing increases), a critical foundation for the concept of concavity in calculus (Johnson, 2012, 2013). The above findings highlight the importance of understanding how curricular reform impacts students' development of covariational reasoning with regard to slope.

Grades 6-8. Under both *PSSM* and *CCSSM*, slope receives the most attention during the middle grades. Both standards documents address nine of the eleven conceptualizations (all except the *Trigonometric* and *Calculus* Conceptions) during grades 6-8, indicating a very broad coverage of the concept (see Figure 3). The increased attention to *Functional Property* and *Real World Situation* by *CCSSM* may be explained partially by the differences in the suggestions for the grades 3-5 curriculum. Recall that although both *Functional Property* and *Real World Situation* were included in grades 3-5 under *PSSM*, the middle grades mark students' first exposure to these ideas under *CCSSM* standards. Thus, some of the increased emphasis in the *CCSSM* middle school standards may be a result of their absence in the elementary school standards. This idea is supported by analyzing the focus of 6th grade standards under *CCSSM*, which appear to be in line with the grades 3-5 focus under *PSSM*. Figure 4 illustrates *CCSSM*'s grade 6 focus on *Functional Property* and *Real World Situation* to build covariational reasoning. This is also evident in the grades 6-7 ratios and proportions section of the *Progressions for the Common Core State Standards in Mathematics* (Common Core Standards Writing Team, 2011) document. The *Progressions* document includes a focus on coordinating changes to build an understanding of ratios and proportional relationships. One example of this has students explore the additive and multiplicative changes of two quantities in a juice concentrate mixture (5 cups grape to 2 cups peach) to build understanding of the 5 cups grape to 2 cups peach ratio and extend to understanding the 5/2 cups grape per 1 cup peach rate (Common Core Standards Writing Team, 2011). With prior attention to *Functional Property* and *Real World Situations* in grades 3-5, *PSSM* emphasizes graphical interpretations of slope via *Behavior Indicator* and *Physical Property* conceptualizations in the middle grades.

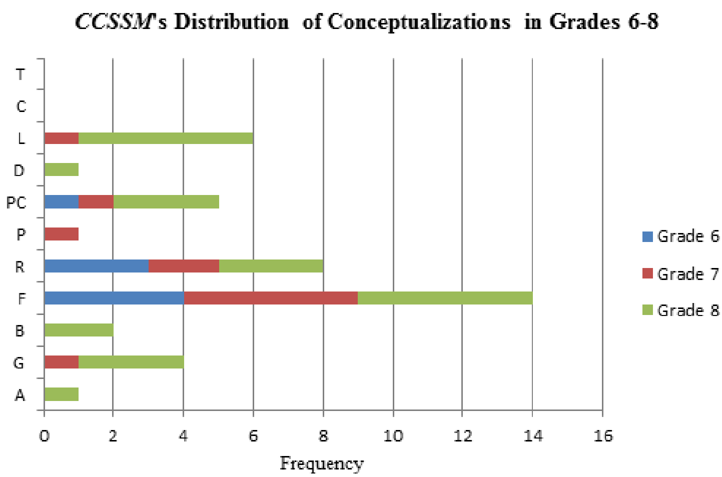

Figure 4. Distribution of slope conceptualizations for the middle grades under *CCSSM*.

Despite variations stemming from the treatment of slope in the primary grades, the descriptions of developing notions of slope are similar throughout grades 6-8. Because *PSSM* does not provide detail for individual grades, NCTM's *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (2006) provided additional insight into the focus of slope instruction at each grade level. *Curriculum Focal Points* (NCTM, 2006) describes three central mathematical topics to be addressed at each grade level preK-8, which were used to clarify the grade-level emphases during the middle grades. In both *Curriculum Focal Points* and *CCSSM*, grade 6 instruction focuses on building proportional thinking to prepare students for directly proportional relationships (of the form $y = kx$) in grade 7 and linear relationships (of the form $y = mx + b$) in grade 8. Thus, where the core of instruction on slope is concerned, there is a general consensus between *CCSSM* and *PSSM* standards regarding the focus and sequencing of instruction.

Grades 9-12. Figure 3 illustrates some meaningful differences in the treatment of slope in grades 9-12 under *PSSM* and *CCSSM*. Overall, *CCSSM* describes a more focused treatment of slope in high school, providing evidence of five conceptualizations compared with *PSSM*'s eight (including all five found in *CCSSM*). Multiple occurrences of *Linear Constant* in both documents highlight the extension of slope to make judgments about the linearity of relationships in high school. However, the means by which this extends to nonlinear situations differs. *CCSSM*'s heavy focus on *Functional Property* in the middle grades carries into the high school grades as well by comparing the constant rate of change of linearly related quantities with the variable rate of change of nonlinear (e.g., quadratic and exponential) relationships. Under *PSSM*, the high school focus is on using multiple representations to interpret *Real World Situations* rather than on understanding covariational characteristics of classes of functions through Functional Property.

Overall, *CCSSM* supports a more concentrated presentation of slope in grades 9-12 and across the curriculum. *PSSM* promotes a somewhat broader interpretation of slope in grades 9-12 and more gradually builds conceptualizations across the curriculum, as illustrated in Figure 5.

Discussion

This study provides a detailed analysis of the treatment of slope as described by two sets of standards for mathematics education. Although the treatment of slope appears to be similar across the preK-12 *PSSM* and *CCSSM* standards, a more detailed analysis across grade bands highlights important differences that will influence instruction and ultimately, how students come to understand and make use of slope. This

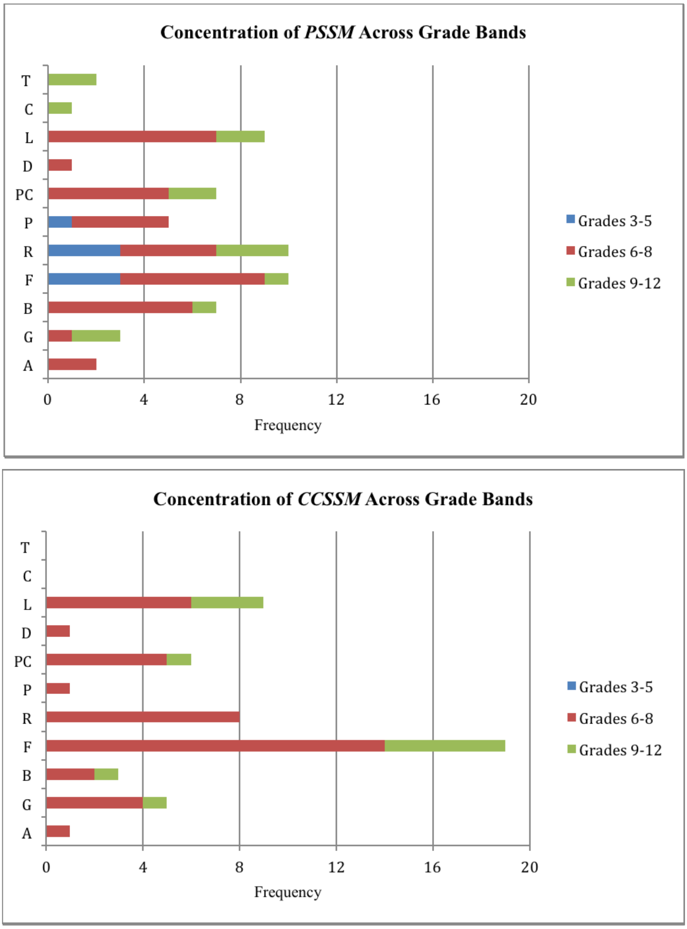

Figure 5. Distribution of the slope conceptualizations across *PSSM* and *CCSSM*.

discussion section first outlines those differences and then interprets those differences in light of the goals of the *PSSM* and *CCSSM* documents.

Comparing the Standards

The first important difference revealed in the treatment of slope under the two documents is the timing of introductory instruction on preliminary notions of rate of change and covariational reasoning. Based on past research highlighting the importance of students' ability to coordinate covarying quantities on their understanding of slope (Carlson et al., 2002; Carlson, et al., 2010; Lobato et al., 2002; Lobato & Siebert, 2002), this difference should not be dismissed. Whereas *PSSM* builds a foundation for covariational reasoning through real world examples in grades 3-5, *CCSSM* describes a focus on such ideas during grades 6-8. Because instruction in grades 6-8 is otherwise very similar, the importance of this shift to a later focus on covariational reasoning could easily be overlooked. Under NCTM's *PSSM* and *Curriculum Focal Points*, a teacher in grade 6 would build proportional reasoning from students' prior experiences with basic covariational reasoning. *CCSSM* does not change the grade 6 focus on proportional relationships; however, these students will simultaneously be building their images of change between two covarying quantities because this did not appear earlier in the curriculum. Overlooking this subtle, but important, difference may result in students who misinterpret slope as a difference rather than as a ratio (Lobato et al., 2002) and who struggle to advance from average to instantaneous rate of change (Carlson et al., 2002; Orton, 1984).

Important differences were also found in high school standards. Although students learn about the concept of slope and linear relationships in grade 8, the high school curriculum will continue to influence how students come to understand and use slope. Based on the emphasis of standards, we conjecture *PSSM*'s focus may lead to students who can use various representations to interpret real world situations involving linear and nonlinear functions, while the *CCSSM* approach may promote students with a more conceptual sense of various functions and the relationship between inputs and outputs. How middle grades instruction on slope is extended to make sense of nonlinear functions could impact students' overall conception of slope.

The results suggest a middle grades teacher might assume there appears to be little curricular impact in the changes described by *CCSSM*. However, how students come to understand and make use of slope builds from their many experiences of slope across the curriculum (Tall & Vinner, 1981) and is influenced by the development of prerequisite notions prior to formal study of slope as well as by the focus of advanced instruction that builds upon and extends the concept. This highlights the risk of building a false sense of correspondence between curricular documents when a comprehensive analysis of the topic across grades preK-12 is not considered.

The Goals of *PSSM* and *CCSSM*

The differences between the presentation of slope under *PSSM* and *CCSSM* may be partially understood by considering the intentions of these two documents. In particular, *PSSM* set out to describe an overarching, comprehensive set of learning goals appropriate for prekindergarten through grade 12 mathematics. This is evidenced in the organization of *PSSM* around standards that span the curriculum with additional details regarding how each standard may be addressed for specific grade bands. By contrast, *CCSSM* aims to provide a more concentrated description of the specific learning goals for each grade level or high school topic. Although more general domains are included for each grade level, specific grade level standards provide a more detailed description of the content students should learn on a grade-by-grade basis. In providing these details, *CCSSM* seeks to address the issue of a mathematics curriculum that provides only trivial coverage of a large number of topics (Center for K-12 Assessment & Performance Management at ETS, 2013; NGA Center & CCSSO, 2010).

Based on the data analysis, it appears that for the topic of slope, *CCSSM* has succeeded in providing a more focused development of the concept than was previously described by *PSSM*. While *PSSM*'s coverage of slope is more widely spread across grade levels and conceptualizations, *CCSSM*'s coverage is more heavily concentrated on a subset of conceptualizations. Although nine conceptualizations are introduced in the middle grades, only five of these are elaborated in the high school grades, with a particularly heavy high school emphasis on slope as a *Functional Property* and *Linear Constant*. By contrast, *PSSM* addresses all 11 conceptualizations, including eight at the high school grade levels, and gives nearly equal attention to each of the eight conceptualizations in the high school curriculum.

Although the findings for *CCSSM* may not be surprising in light of the intentions for its creation, we must consider both the intended and potentially unintended consequences of turning to *CCSSM* as a curricular guide. The study suggests that adoption of *CCSSM* may lead to more focused instruction of the mathematical concept of slope in terms of *Functional Property*, *Linear Constant*, *Real World Situation*, *Parametric Coefficient*, and *Geometric Ratio* conceptualizations. However, if careful attention is not paid to the subtle differences in the curricular treatment of slope, unintended gaps in student knowledge might result. It is important for those who write curricula to examine how shifts in the content taught in earlier grade bands will impact the development of concepts at the next level. It is also important for teachers to look beyond the grade level state standards they are responsible for teaching to understand how the standards fit together and the overall understanding emphasized. This is one of the affordances of *PSSM*, which helps to situate the content by describing the key developments in the various content strands across all grade bands (NCTM, 2001). If state-adopted standards follow the lead of *CCSSM*, students may not have elementary school experience with slope as a *Physical Property*, *Real World Situation*, and *Functional Property*, as recommended by *PSSM*. If teachers are not aware of these shifts, they could lead to gaps in student knowledge that would undermine efforts to build rich and detailed understanding of slope.

Conclusions and Future Work

The results of the *PSSM* and *CCSSM* document analyses for slope warrant similar analyses for other important mathematical concepts. As demonstrated in this study, a crosscheck of standards at the point of instructional focus is not sufficient to understand the total impact of adopting a new set of curricular standards. Changes to instruction of prerequisite notions may lead to gaps in prior knowledge, while a shift in focus at more advanced stages of instruction may lead to unexpected student interpretations or understanding of mathematical ideas. Thus, a detailed analysis of the treatment of mathematical concepts across the curriculum is required to fully understand the extent of curricular reform.

In this study, the researchers provided an analysis of the intended curriculum set forward by the *PSSM* and *CCSSM* but do not offer any information about the enacted or realized curriculum. It is important to note that neither *PSSM* nor *CCSSM* provides instructions for how to implement the standards, stating what content should be included in the curriculum but not addressing how this content should be taught. Beckmann (2011) acknowledges that teaching is the key to the success of *CCSSM* and calls for teachers across elementary, secondary, and post-secondary levels to build a community focused on improving mathematics education. In light of past research highlighting preservice teachers' difficulties teaching slope as a *Functional Property* (Stump, 2001a) and students' infrequent use of *Functional Property* conceptualizations (Nagle et al., 2013), the enactment by teachers and realization by students of the intended *CCSSM* slope curriculum (which has a heavy *Functional Property* emphasis) deserves further attention. Because textbooks may influence classroom instruction, research should examine the conceptualizations of slope emphasized in commonly adopted texts. Future work should focus on examining whether the *Functional Property* focus of *CCSSM* in grades 9-12 has any impact on the ways that secondary students come to understand and work with slope, with particular attention to their ability to interpret rate of change in real world contexts.

Finally, *CCSSM* describes building covariational reasoning together with proportional reasoning in grade 6, a diversion from *PSSM*'s approach of building a foundation of covariational reasoning in grades 3-5 on which to understand proportional relationships in grade 6. Future work should investigate the merit of these approaches to determine whether proportional reasoning is dependent on first understanding the nature of covarying quantities (as in *PSSM*) or whether investigating proportional relationships provides a context to examine the covariation of two quantities (as in *CCSSM*).

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