Change and Stability in Research in Mathematics Education

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Anyone who surveys research activities in mathematics education around the world cannot help but see a field that is in a state of flux. On the one hand, the public's and the profession's dissatisfaction with school mathematics instruction has led to, among other things, calls for research that can guide and inform practice. The clear implication is that the existing research has not been doing that. On the other hand, researchers are increasingly questioning the epistemological and methodological foundations of their work. They too are dissatisfied with what has been accomplished, but they are unsure of what else can be done. The pressure for research that would have a more practical value has risen just at a moment when many researchers are doubting that research will ever be able to prescribe what to do and are becoming content simply to understand more of what happens when children learn and teachers teach. At the same time, other researchers argue that both education and society must be transformed in radical ways and that research must be the agent of that transformation.

These shifting currents are not unique to our field, but because research in mathematics education has been so eclectic, because it has reached out not only to behavioral psychology but also to cognitive science, linguistics, developmental psychology, anthropology, philosophy, sociology, artificial intelligence, history, and mathematics itself, the crosscurrents have been especially strong. Many behavioral scientists are reappraising the foundations of their fields and the research methods they have been using. At the same time, all scientists are being called upon to contribute to national economic competitiveness.

Educational research has long been justified in terms of its affinity to science and technology; it is now being justified more openly in terms of the economic advantages it might confer (McDonald, 1985). In the United States, there has lately been an intense preoccupation by educators and educational researchers with the accomplishments of Asian countries, especially Japan. Among other differences, Asian students are seen as using their time in school more productively than American students do, and the test performance of Asian students is much higher. The conclusion is that Asian education must be more efficient than American education. And one response has been to turn to research to find ways to make education more efficient.

It is interesting that, as Peter Fensham (1986) observes, the United States has not looked, in its efforts to improve science education, to countries such as Kenya, Nepal, the Netherlands, Sri Lanka, Tanzania, and Thailand. Each of those countries has made impressive gains in improving the teaching of science, but the United States seems not to be interested in that. Instead, it looks to Japan, not because of what the Japanese are doing in science education but because of what they have done economically and because of their high scores on international achievement tests.

Of course, educational research has long been promoted as an applied science, and built into that view is the goal of maximizing the efficiency of school practice. The ends of education are specified, and the role of research is to find effective, efficient means. In this conception, the teacher's role easily slips into that of mere technician. As educational research then yields more and more self-instructional materials, teachers are deskilled. That is what has happened in more than one country over the last half century. The teacher has become a consumer--someone who needs no technical knowledge, only the ability to make discerning choices among competing products and then to use those products appropriately. Although the declines in teachers' status and autonomy that have occurred in technologically advanced countries arise from many sources, surely one of them stems from the view that education itself is largely technical and that research contributes to its efficiency.

The conflict between the increased pressures to make education more efficient and the rejection of a technical view of education is being felt by researchers in mathematics education in many countries. This conflict is part of a deeper clash of unresolved positions that affects our work. The purpose of this paper is to sketch some of the currents of change in who we are and what we do and to show that beneath the surface turmoil lie some enduring issues that shape our research efforts and that may limit the possibilities for substantive change in our field if they are not resolved.

Changes in the research community

Over the past decade or so, the most obvious change in the research community in mathematics education, aside from its continued numerical growth, is in its increasingly international and crossdisciplinary character. The number of conferences, presessions, seminars, and publications devoted to research in mathematics education has mushroomed as collaborations have formed across the borders of countries and of disciplines. Researchers in mathematics education have always borrowed freely from anyone whose ideas they liked. In the last few years, however, more people whose work has been borrowed have begun to do collaborative work with some of the borrowers.

Many researchers, of course, still work in relative isolation. They participate in the community dialogue only vicariously, by reading books and journals and perhaps occasionally attending a meeting where research is reported. The establishment of various centers to undertake research in mathematics education and the growing use of information technology to disseminate research have helped to reduce some of that isolation and have contributed to the formation of "invisible colleges" of researchers who share ideas freely through the mails and through meetings at which collaborative work is done. These invisible colleges help to promote trends in research and give some cohesiveness to a field comprised of individuals holding diverse conceptions of the role and nature of research.

Despite these developments, it has long been true that apart from the continuing large fraction of doctoral students, most of the people doing research in mathematics education have full-time jobs doing something other that research (Sowder, 1989). Research may well be expected of them, but they are usually given little or no time in which to do it. The majority of these researchers are in colleges or universities, usually as teacher educators. Although they may occasionally be awarded a grant to do a research study, they rarely have the luxury of working on research full time. The part-time nature of our field does not distinguish it from our sister fields in educational research, but it is different from other sciences. And the people in the schools who might collaborate with researchers from the universities typically have even less time to devote to research. It is not so surprising, then, that we seem so fascinated with what others outside mathematics education are doing and so prone to imitate them. We often lack the time and energy to work out our own paths.

The generally small scale of educational research activities has often been noted. "In the developing countries little money has ever been expended on research activities, and even in those countries thought of as 'big spenders' the actual percentage of the educational budget allotted to research and development has always been miniscule" (Howson & Wilson, 1986, p. 83). Research in mathematics education has shared in and suffered from this paucity of resources. Making research in mathematics education a full-time activity for more people would require substantial investment. Although some countries are attempting to increase their education budgets, educational research is ordinarily not given a high priority in those attempts.

The research community in mathematics education is developing a more stable, cohesive character as it begins to define its own problems, as it grows across institutional and national boundaries, and as its members begin to seek solutions in collaboration with others across and outside the community. It still has some way to go, however, in overcoming the isolation and fragmentation of much of its membership.

Changes in research

Much of the apparent movement in our field comes from the themes we address in our research. There are themes in our research today that were not well represented 20 or even 10 years ago. They include sex differences and ethnic differences in mathematics learning, teachers' thoughts and beliefs, and mathematics outside the school context. There are also themes that were present 10 or 20 years ago but that seem to have increased in prominence. They include error analyses, analyses of teaching, the use of computer technology in instruction, and the learning of rational numbers and algebra. We have themes that endure, such as problem solving and spatial reasoning, and themes that have declined in interest, such as Piagetian studies and the search for aptitude-treatment interactions.

A common way of viewing research studies in mathematics education has been in terms of the mathematical content with which they deal. One thinks of counting, geometry, and probability as not only topics in the school curriculum but also sites of intensive research activity. Much research to date can be characterized according to mathematical topic, and some topics have been nuclei for the formation of invisible colleges of researchers. Research-reporting sessions at conferences and chapters in books reviewing research are often organized along content lines.

It is becoming more and more difficult, however, to use mathematical topic as the primary rubric for discussing research. The questions that researchers are addressing more seriously--questions of assessment, equity, affect, technology, language, metacognition, teachers' beliefs, teacher education, to name a few--cut across content boundaries. Although the school curriculum remains central in much research, attention seems to be turning from what is taught and learned to where, when, and how teaching and learning occur. One of two major themes identified in a recent series of conferences to set a research agenda (Sowder, 1989) was instruction, and "environments for learning" was a major part of that theme. In general, there appears to have been a tidal shift in research from the content of mathematics teaching and learning to the context in which they take place.

Parallel to that shift has been another in the view taken of knowledge and its formation. The other major theme from the research agenda conferences

(Sowder, 1989) was the nature of mathematical knowledge. The school curriculum is increasingly being seen as less a collection of topics than a set of experiences. Learning is viewed as active construction rather than passive absorption, teaching as facilitation rather than transmission. The knowledge that both teacher and student bring to the educational encounter is being examined more closely so as to clarify how that knowledge shapes teaching and learning. The misconceptions that persist after instruction become crucial and revelatory instead of merely unfortunate. Helping children gain a sense of the empirical side of mathematics becomes a central goal of instruction that also needs to be examined in research.

A third shift has occurred in the view taken of research itself and consequently in some of the methods used in doing research. Research in mathematics education has long followed the empirical-analytic tradition of the natural sciences. In that tradition, the basis for knowledge is presumed to be confined to what can be observed, and observation permits those phenomena to be decomposed into their constituent parts (Popkewitz, 1984, p. 36). The goal is to uncover law-like regularities that permit one to explain, predict, or control phenomena. The world is seen as a system of interacting variables to be controlled experimentally if possible so that cause-and-effect relations can be discerned.

The dominant research methodology in mathematics education has relied on the statistical model that was originally developed for agricultural research (Sowder, 1989). In that model, operationally defined variables are manipulated through design, randomization processes, or statistical means to permit the study of the effects of various treatments. In mathematics education research, such treatments have typically been different ways of teaching the same item of mathematical content. Techniques of statistical inference are used to test research hypotheses that relate treatments to effects.

The results of efforts to apply the statistical model have been rather generally disappointing to researchers in mathematics education, and as a consequence, some of them are abandoning the empirical-analytic tradition. Because researchers have undertaken experimental studies in the absence of exploratory work and without strong theoretical rationales, much of the hypothesis testing has been ad hoc and tentative. The small scale of most of the studies has meant low statistical power, which coupled with the use of weak and inappropriate instruments has meant an inability to detect effects. Many, probably most, studies of treatments in mathematics education have yielded either no differences between groups or differences that could not be interpreted in relation to a body of related knowledge.

As the empirical-analytic tradition has begun to lose favor in mathematics education as well as in education generally, researchers have sought other alternatives. Increasingly attractive, especially to researchers in North America, is the view of research as interpretive understanding (Eisenhart, 1988). The empirical-analytic researcher attempts to stand apart from the educational encounter so as to make an objective assessment of what is happening. The interpretive researcher, in contrast, comes into the classroom and attempts to enter that encounter so as to capture and share the participants' understanding of what they are teaching and learning. Methodology changes too as researchers, in the words of Carr and Kemmis (1986), "replace the scientific notions of explanation, prediction and control, with the interpretive notions of understanding, meaning and action" (p. 83).

Interpretive research attempts to illuminate educational activities by describing them in ways that would make sense to the participants. But it goes further than that. It also attempts to discover "the rules of the game," that is, the social perceptions, implicit agreements, and tacit principles that permit people to work together in certain settings at tasks they jointly construe as aimed at education. The purpose of this research is not to uncover general laws about human behavior but rather to supply "local knowledge" (Geertz, 1983) about social action within a context.

In contrast, some researchers, especially in Australia and New Zealand but also Europe, have taken what they call a critical approach (Bates, 1979; Bernstein, 1976; Carr & Kemmis, 1986; Popkewitz, 1984). The empirical-analytic researcher stands outside the arena of practice, aiming at general laws that will transcend time, place, and circumstance. The interpretive researcher moves into the arena of practice but still maintains a neutral, nonjudgmental stance. The researcher who takes a critical approach enters the arena of practice with an eye toward changing it in the direction of greater freedom and autonomy for the participants. Normative issues--standards and values--become neither excluded from research nor interesting objects of study but central concerns of the researcher.

The critical theorists maintain that one cannot understand any system--a mathematics classroom, say, or a school--apart from the other aspects of society that have given it form and structure. It is not enough to come inside the classroom and observe the educational encounter. One needs also to guide practice directly. That requires a much closer collaboration between teacher and researcher than has been common in the past.

So far, both the interpretive and the critical approaches to educational research have been more talked about than employed by researchers in mathematics education. It is true that one now encounters more case studies and protocol analyses in the literature, and many researchers seem to be abandoning the statistical testing of hypotheses, or indeed any testing of hypotheses. Qualitative studies do seem to be on the rise. But relatively few researchers in mathematics education have gone out to live among the natives in the classroom. And even fewer have organized groups of teachers to study and reform their own work. More precisely, we have not yet seen the work of many such researchers.

Persistent issues in research

Below the surface of these various changes are some durable ideas. Levels for classifying the products of mathematics learning, for example, seem to be perennial; what changes is the scheme, from Bloom, to Skemp, to van Hiele, to the SOLO taxonomy. The underlying question is really one of assessment: How do we capture what students have learned?

Much of the work with Logo and other computer languages seems to be aimed at seeing whether, with their aid, children can develop a deeper understanding of mathematical ideas that have long been recognized as important. Even though technology changes, the role that technology should play in teaching mathematics remains unresolved.

Although recent models for analyzing how children respond to problems in addition and subtraction seem to have arisen from work in the computer simulation of cognition, they also reflect mathematics educators' enduring concern with the ways in which, when you change the task you set for children, they change their responses. What is new are some of the tasks and the ways they are analyzed. It seems that we return again and again, perhaps with new insight and sophistication and certainly with new techniques, to the same concerns that have bothered mathematics educators for years.

One hears the argument that researchers in mathematics education now have a much more relativistic view of research and that they are abandoning the hard-science ideal they have hitherto pursued in favor of a softer, humane approach. Although they do seem to be redefining to some degree what it means to do science, much of the change in their thinking may be more apparent than real. I enjoy reading various attacks on behaviorist research in mathematics education because they are usually such delightful caricatures. In my experience, very few researchers in our field ever bought the complete behaviorist line. Many ran experiments when they could and tried to put some operationalism into their instruments, but they did not act much like true believers. They insisted on talking about mental constructs, none of them wasted much time checking validity, and the best of them questioned the appropriateness of elaborate statistical models even as they used them. Consequently, I will not be surprised if the crowds around the people who are arguing that we need to topple those behaviorist idols soon thin out, as researchers wander away to see what else is new.

Researchers face the difficult task of examining various conceptions of research from within whatever framework they have constructed for themselves. We develop our own epistemologies, and as they develop, they throw up barriers to our understanding (Bachelard, 1983, p.13). The very act of knowing dulls our sensitivity to phenomena outside the way we know. Our elaborate schemes of understanding entangle us and prevent us from moving in new directions. Our theories themselves pose natural obstacles to the development of new theories. Without sustained collective effort, we cannot rise above our parochial concerns.

If our research is to meet both our standards and the needs of society, it must take many forms and adopt various perspectives. To accomplish that will require a greater tolerance and encouragement of diversity than has been evident in recent years. Three features of research need to be enhanced.

The first is *community*. As Donald Campbell (1985) says, "Science advances through competitive, disputatious communities of scientists, who find it important to keep each other honest" (p. 20). Even as researchers break into separate groups to pursue particular lines of inquiry, they need to remain part of that larger community that agrees on "the social norms of the shared inquiry" (Campbell, 1986, p. 119). As the critical theorists require, we must see that teachers are made full colleagues in the community. And we must continue to include those scholars who look at the educational enterprise from a sociological, anthropological, historical, or philosophical perspective. We must agree to disagree, to check our own work, and to check one another's work, as together we seek not a final version of the truth but the best versions we can collectively construct.

A second feature of research that needs enhancement is our attention to *context*. Some views of research attempt to decontextualize inquiry, others embed it in a pedagogical context, still others embed it in a social content. Any inquiry occurs in multiple contexts. What we should strive to do is to *recontextualize* our research activity, to relate it to as many as possible of the contexts in which it occurs. That means we need to make the familiar strange, treating as problematic various features of the situation under inquiry that we take for granted because we know them so well or cannot imagine them otherwise.

The third feature to be enhanced is our *courage*. We tend to underestimate the simple courage it takes to submit our ideas to the test that research demands. Progress in our field requires that we know that our theory is faulty and actively seek to refute it at the same time that we believe in it enough to try it out. As George Polya used to say about solving problems in mathematics, we need courage--both the courage to guess and the courage to doubt our guess. Some researchers take the easy way out; they seek only support for their preconceptions, never subjecting them to acid tests of disconfirmation. One of the great strengths of the scientific ideal is that we come together figuratively in journals and literally in scientific meetings to question in public the validity of each other's work. That scientific spirit needs to prevail over the negatively political spirit of sheltering one another from the rough and tumble of debate.

Margaret Eisenhart (1988) has put her finger on a primary reason the substance of our research may have changed so little despite much surface change. As she says, "Research questions in mathematics education tend to be derivatives of the general question: How can mathematics teaching and learning be improved?" (p. 100). It is that desire to improve, and not simply to study or make sense of, the circumstances in which mathematics education occurs that keeps us coming back to the old issues and that gives form and substance to whatever is seen as a new issue. We are in this business because we want mathematics to be taught better than it is and learned better than it is. We are drawn to undertake studies in which instruction is manipulated because we want it to improve. We work directly with students and teachers so as to help them accomplish not only their goals but ours. Intellectual curiosity drives our work, to be sure, but it is curiosity in the service of amelioration.

A continuing barrier to change is the failure of researchers and teachers in our field to participate together in the research enterprise. Everyone seems to operate on the premise that the practical implications of research are what matters, that the researchers are supposed to provide them, and that the teachers will receive and use them. There is nothing wrong with wanting to improve the teaching and learning of mathematics. On the contrary, as I have noted, that desire seems to be the engine that drives the research in our field. But there does seem to be something wrong with having one group decide what to do and the other do it.

Researchers in mathematics education should not interpret their mission as helping teachers when in fact both need help and both need to help. If, amid the stability of issues and the changes in themes and methods, our research is to be improved, it will need to transcend the limitations of any single vision. And that will require sustained collaborative effort across not only the boundaries of nations and academic disciplines but also across the gulf that separates most researchers in mathematics education from their colleagues who teach mathematics.

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