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Microworlds of Children's Construction of Rational Numbers of Arithmetic

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Children's Construction of Rational Numbers of Arithmetic (Fractions Project), a project sponsored by the National Science Foundation, is designed to determine how children construct rational numbers and fractions. The Fractions Project is a successor to a project on investigating multiplication schemes used by children (Steffe, 1992). It hopes to extend models developed by Steffe in his multiplication project to include children's construction of rational number schemes.

The project will study children's construction of rational numbers for three years starting in the third grade. By using computer microworlds as research tools, the project will be able to gather useful data on the schemes used by children to handle rational numbers and gain insight into children's development of rational numbers in arithmetic tasks. Thirty-minute teaching episodes provide the context for the collection of data. A teaching episode typically consists of two children and a project member in the role of the teacher. The microworlds act as the medium in which tasks are proposed and solutions are found. The tasks themselves serve a dual purpose. By examining how the children solve the tasks posed to them, the current state of their mathematical schemes will be investigated. The tasks also serve as a way to help the children construct operations that we feel are necessary to the construction of rational numbers.

This paper focuses on the computer microworlds developed for the project. We begin by providing theoretical background on the project and discussing how this theoretical background is realized in the microworlds. We provide an overview of each microworld and discuss how we expect the children to use the microworlds as part of their construction of rational numbers. Finally, we give some indications of the future use of the microworlds both on the project and afterwards.

Theoretical Background

The theoretical background for the project has been provided by the models developed by Steffe (1992), most notably his work with composite units. A summary of the ideas presented is given here.

Since mathematics is constructed in the mind, each child has his or her own interpretation of a mathematical world. By children's mathematics, we refer to children's schemes for working with numbers, their schemes which have been developed for the solution of tasks. These schemes make up the basis of their mathematics. It is the premise of the Fractions Project that children can construct their own mathematics; in fact, they do it all the time. Often, this mathematics may not correspond to the teacher's view of what the children's mathematics should be.

Children construct schemes to solve problems and perform tasks. In one situation, a child may be given a goal the path to which contains obstacles. The child must then develop a scheme to overcome the obstacles. In another situation, with a similar obstacle, the child activates the scheme again. When a child uses a scheme and realizes that it is ineffective, a perturbation is set up. The scheme is re-examined by the child, an act of *re-presentation* that can eventually cause the child to interiorize the scheme. The re-presentation action can result in a greater abstraction of the scheme in an attempt to generalize it and increase its domain of application. This is the basic outline of how children construct their own mathematics (Steffe, 1992).

The goal of the teacher in the Fractions Project, therefore, is not to give his own construction of mathematics to the children, but rather to encourage the children to develop their own mathematics. The teacher would do this by providing tasks using the microworlds which encourage each child to develop new schemes and re-present previously developed ones; thus providing the children with raw material from which to construct their own mathematics. Repetition of a task with different parameters and different obstacles encourages re-presentation of the task and can cause the child to interiorize the scheme thereby making the scheme more abstract and powerful. The purpose of the microworlds is to provide frameworks for these tasks and tools for the solution of these tasks. By examining the tools the children used to solve problems, it may be possible to understand something of the operations occurring in their minds and thus develop models of the mental operations children use to construct rational numbers. Therefore, it is necessary to have the microworlds mirror as closely as possible the mental operations that we hypothesize the children possess. This mirroring process is a feedback loop in which the operations that the children express a desire to use are put into the microworlds, others are altered so that they are more in line with what the children expect them to do, and still others are eliminated. This process is based on the comments of the children, but also reflects how the theory of children's construction of rational numbers changes based on the data gathered during the teaching episodes.

The multiplication theory that forms the basis of this project proposes that children possess mental representations of number sequences. Exactly which number sequence a child possesses tends to determine both the type of problem which the child can solve and the method used for the solution. According to Steffe (1992), number sequences are constructed from previous sequences through mental operations of disembedding, uniting, and re-presentation.

A child develops a verbal number sequence by placing his or her number words in one-to-one correspondence with a collection of objects. The counting acts that result from this correspondence establish records of counting in the mind of the child. When a child progresses to the point that a number word is associated with these records of counting to the extent that the word can replace future counting acts, the child is said to possess an *initial number sequence*. Thus, a child without an initial number sequence may take a collection of seven objects and count them "one, two, three, four, five, six, seven." He does not disembed the objects from his environment and treat them as a united collection; rather, he continues to treat them as individual objects. On the other hand, a child with an initial number sequence can disembed the collection from his environment and recognize that the word "seven" stands for a collection of objects which, if counted, would have a one-to-one correspondence with "one, two, . . . , seven." The child has abstracted the counting acts that he performed initially and can now operate with the number seven without having to perform those counting acts (Steffe, 1992).

A *tacitly nested number sequence* is a result of a re-presentation act by a child with an initial number sequence. A child with a tacitly nested number sequence has two internal number sequences as opposed to the child with an initial number sequence who has only one. A child who has constructed a tacitly nested number sequence is able to operate on his number sequence as well as operate with his number sequence. The child with a tacitly nested number sequence is able to disembed a part of his number sequence to find out how many items are between the third and seventh items in a collection by counting "four, that's one; five, that's two; six, that's three; seven, that's four." The presence of two internal number sequences is shown by this act of double counting (Steffe, 1992).

An *explicitly nested number sequence* is also a result of a re-presentation act, this time on a tacitly nested number sequence. While a child with a tacitly nested number sequence can disembed part of his number sequence in order to double count, he cannot use this disembedded part independently as input for further operations: The child with an explicitly nested number sequence, on the other hand, is able to do just that. In an example given by Steffe (1992), a teacher gave a child possessing an explicitly nested number sequence twelve blocks. The teacher also kept a certain number of blocks unknown to the child. The child was told that there were a total of nineteen blocks. How many blocks does the teacher have? In order to solve this problem, the child first decomposed nineteen into ten and nine. This is within the capabilities of a child with a tacitly nested number sequence. This child, however, went further. By using the results of her disembedding operation, she decomposed nine into seven and two, then united the two with the ten to make her twelve blocks. This left seven blocks, which she knew must be how many blocks the teacher had. A child with a tacitly nested number sequence could not have used the results of the decomposition of nineteen into ten and nine as inputs into the decomposition scheme once again and then put everything back together as this child did.

By manipulating the disembedded pieces of a number sequence as independent entities, a child with an explicitly nested number sequence has the operations necessary for multiplication and division. Within the Fractions Project, these operations are considered to be extremely important for the construction of rational numbers of arithmetic. In an attempt to help the children in the project develop an explicitly nested number sequence, the operations of disembedding and uniting exist in each microworld as visual representations of mental operations. In particular, analogs to an initial, tacitly nested, and explicitly nested number sequences exist in the microworld called Toys.

The Microworlds

Three microworlds have been developed for the Fractions Project: Toys, Sticks, and Candybars. These microworlds are designed to be used in a spiral sequence. First, the children are introduced to discrete quantities in Toys, then, one-dimensional continuous quantities in Sticks, and finally, two-dimensional continuous quantities in Candybars. After the children have used Candybars, they return to Toys to confront those problems which gave them difficulty the first time around.

An important goal in developing the microworlds was to make the technology non-threatening so that operating the machine does not get in the way of formulating mathematical schemes to solve problems. We have tried to make the operations within the microworlds natural so that they may be accomplished with a minimum of effort. We are constantly updating and revising the microworlds in an attempt to make the technology easy to use.

One important aspect of the use of the computer microworlds is the play aspect. When confronted with the microworlds, none of the children seem to be intimidated. They enjoy exploring the microworlds. Since the microworlds are graphically oriented, the children tend to use them as drawing programs. This exploration is very important in allowing the children to accommodate themselves to the limitations of the microworlds. It was up to the teachers of the project to turn this play into mathematical activity by posing tasks.

Toys

The Toys microworld is perhaps the closest to standard manipulatives, consisting of discrete objects that can be moved around, covered, and grouped into units. Most of the activities of the children take place in a large window divided into three main regions: a Playground, a Toybox, and a button control region. Toys are the fundamental objects in the microworld. Figure 1 is a reproduction of a screen from the Toys microworld. In the window of the microworld, there is a Toybox that provides a never-ending supply of the five basic types of toys, regular polygons. Clicking the mouse cursor on a toy highlights it in black, letting the child pick that particular toy to work with. By clicking anywhere in the Playground, children can make as many copies of a toy that is highlighted as they wish. Clicking outside of the Playground returns the highlighted toy to its original color and stops the copying to use the numerosity of the individual strings to find the total number of toys in the resulting string. A string of four toys may be joined with a string of five toys to produce a string of nine toys. By stringing the four toys together and then stringing the five toys together to form a four-string and a five-string, respectively, the child focuses on the strings rather than on the individual toys and treats them as units. Strings are formed by using the String button and then broken back into individual toys by using the Unstring button.

The next level of unitization is the chain, consisting of strings joined together into a rectangular array. An example of a four-by-three chain may be seen in Figure 1. Chains are created with the Chain button and broken back into individual strings with the Unchain button. The necessity of higher levels of unitization is to promote higher orders of abstractions of a child's number sequence. Some tasks involving chains follow

1. A chain with four strings of six toys each is produced and hidden under a cover. Eighteen toys are then added under the cover. How many toys are under the cover? How many strings of six toys could be produced with the toys under the cover? If the eighteen toys were added to the chain, how many strings would be in the chain?

2. A chain of seven strings of four toys each is produced and hidden under a cover. Under a second cover, a chain of five strings of four toys is produced and hidden. If the two chains were joined together, how many strings would be in the resulting chain? How many toys would be in the resulting chain?

In task 1, asking how many toys are under the cover process leaving the user with a number of toys to work with in the Playground. This creation process is generally the first thing done by the children with the microworld.

The second type of object in the Toys microworld is a linear object called a string. A string is a collection of individual toys, grouped together into a unit. A four-toy string may be seen in Figure 1. The string reflects the uniting process used to develop an initial number sequence. By encouraging children to think of a string as one object that may be represented by a number, rather than as individual things that need to be recounted, it is hoped that an initial number sequence of a child may be developed or expanded. The string may be united with other strings to form a larger string, allowing the child to use the numerosity of the individual strings to find the total number of toys in the resulting string. A string of four toys may be joined with a string of five toys to produce a string of nine toys. By stringing the four toys together and then stringing the five toys together to form a four-string and a five-string, respectively, the child focuses on the strings rather than on the individual toys and treats them as units. Strings are formed by using the String button and then broken back into individual toys by using the Unstring button.

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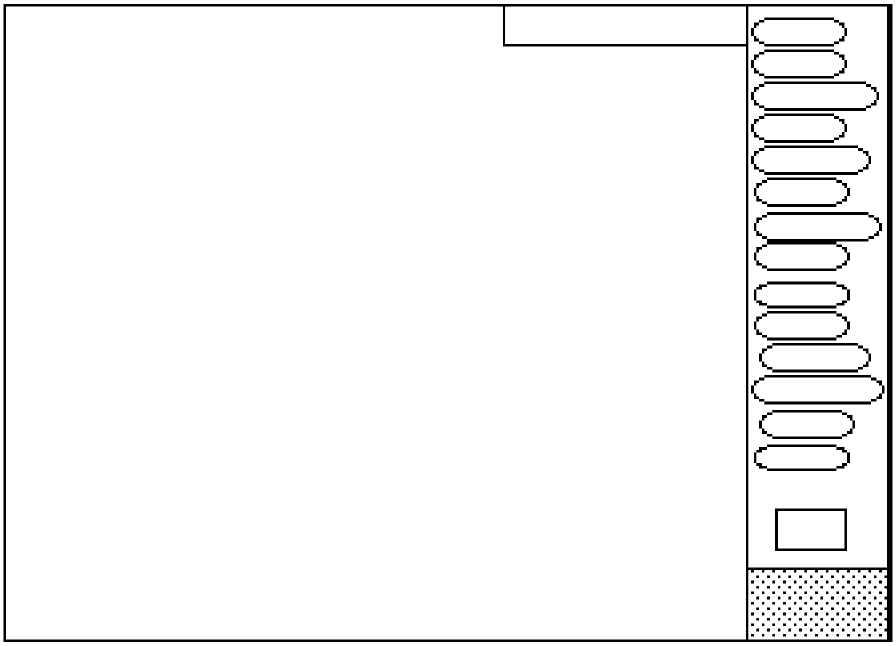

Figure 1: A reproduction of a screen from the Toys microworld.

requires the child to calculate how many toys are in the chain. The child is required to disembed the individual toys from the chain and must coordinate two viewpoints, the toys as elements of the chain and also as twenty-four individual elements to be combined with the eighteen loose toys. Asking how many strings could be produced with the toys under the cover could encourage uniting operations on the eighteen loose toys to form three strings and disembedding operations on the four strings in the chain.

In task 2, we aim for an awareness on the part of the child that the numerical operations performed on individual toys can also be performed on strings. Strings are abstracted from collections of toys to mental objects. These mental objects may then be operated upon by the child.

The use of covers allows some of the toys to be hidden. Covers are important to encourage a child to perform a re-presentation. Arrangements of toys can be made and hidden, then questions may be asked about the arrangements, causing the child to re-present the hidden toys to himself. For example, a chain of four strings with five toys in each might be covered. The child would be asked how many toys are under the cover. The child would be forced to rely on his memory of the toys under the cover; that is, the child would have to re-present the internalized sensory data to himself. Initially, the child may try to remember every toy under the cover and count by ones using his initial number sequence to find the number of toys. It is easy, however, to overload such a scheme and thus cause the child to find another scheme for finding the number of toys under the cover. Perhaps the child may eventually find that he only needs to know the number of strings and the number of toys in a string to solve the problem. The child can still count by ones by keeping track of the number of strings counted and the number of toys in each string counted so that he knows when to stop. This is on the way to the next level of number sequences, a tacitly nested number sequence, thus our goal in proposing the task to help the child construct the next level has been achieved.

Other operations available in the microworld are the ability to make copies of objects using Copy, the ability to Cover and Uncover objects, and the ability to disembed a part of a whole united object through the use of the Pull parts button. Peek is used to examine what is underneath a cover. Line is used to organize the playground by forming strings into lines. Image creates a picture of desired objects and is generally used for comparison between two objects. Fill allows the filling of individual toys with selected colors for highlighting purposes. It also changes the color of covers and allows distinguishing between covers by color.

Menu items in the microworld, which replace the standard items on the computer's menu bar, allow the child to find out information about what is in the playground, such as the number of toys in the playground or any area of the playground. Also, menus are included that allow the child to find the number of strings and chains in the playground. Menus are also available to save the state of the microworld at any time, print out the screen, and configure the buttons and menus to determine what buttons are available to the child at any time.

The tasks posed through the medium of this microworld are many and varied. Since this microworld is based on standard counter-type manipulatives, certainly any task which can be done with standard manipulatives can be posed in this medium as well. In addition, multiple copies can be made so that numbers in the hundreds can be used. An important feature of the microworld, considering the importance of unitizing and disembedding in the model of children's number sequences, is the visible representation of the unitizing and disembedding operations. The creation of strings and chains provides such a visible representation of unitizing; while the cutting and pulling apart of strings and chains provides the representation of disembedding.

In the Fractions Project, it is felt that such visible representations are crucial. Those of us involved in the project wish to allow children to construct the operations necessary to develop multiplication and rational number schemes by presenting them with tasks that create opportunities for such construction. The microworld provides the child with visible representations of operations they can use in developing strategies to solve tasks. A child confronted with a chain of six strings of seven toys each and a chain of four strings of seven toys each might actually Join the chains together to find out the number of toys. Having joined them together, the child might realize that the resulting chain of ten strings of seven toys each corresponds to the multiplication problem 10 x 7 and count by tens to obtain 70. Although initially the use of such operations might be almost trial and error, further use of the operations in the microworld provides the opportunity for interiorization of the operations and finally results in the incorporation of the operations into the child's strategies. Thus, after solving several similar tasks involving joining chains together, the child in the above example might develop a strategy to solve the tasks based on mentally joining the chains together and seeing what multiplication problem results. The operations will become active and usable by the child in the development of new strategies that are the basis of mathematical thinking.

This microworld provides an opportunity for children to develop their number sequences and begin pre-fraction reasoning involving multiplication and division tasks. We give two examples of division tasks in this microworld that involve operations felt to be necessary to the development of rational numbers:

1. Given a collection of twenty-eight toys, how many strings of four toys each can be made from the collection?

2. Given a collection of twenty-eight toys, how many toys would be in each string if we wanted seven strings?

The first task is a "segmenting" task; to solve it, all that is necessary is to segment the group of twenty-eight toys into groups of four. The second is a more difficult partitioning task. All of the children to whom this task was presented had a great deal of difficulty solving it, but the members of the project feel that the schemes necessary for its solution are exactly those schemes necessary to the development of rational numbers.

Sticks

The Sticks microworld provides a computer environment in which students, with a click of the mouse, construct line segments of varying lengths and then investigate relationships between these segments. Sticks has the same format as Toys in that buttons are similar in appearance; most buttons perform the same operations. Thus by being proficient in the Toys program, students should be able to use Sticks with little difficulty.

Sticks mimics natural acts that students could perform with string or sticks. Such actions as cutting with a knife or scissors, marking with a pencil to divide a segment into parts, and joining together either by gluing or tying are simulated by the computer. The main benefit of using the computer environment is that these operations can be conducted with as many sticks, actually line segments, as can be drawn and with greater speed and accuracy. Students can thus be expected to do more investigations than with actual sticks, quickly clearing mistakes from the screen when and if they occur.

Figure 2 shows a screen from the Sticks microworld. The screen of this microworld in which the children's activities take place consists of three regions: buttons, standard sticks, and a workspace where sticks can be drawn. The button region is similar to the one in Toys in that when a particular button is turned on the action can be carried out in the workspace. In the standard stick region at the lower left of Figure 2, there are seven sticks, the last of which has a length of one unit. The lengths of the sticks above the unit stick are in a predetermined ratio to the unit stick, namely 1/2, 1/3, 1/4, 1/5, 1/6, and 1/8. In Figure 2, a stick has been drawn at the top of the screen using the Draw button. Two copies of this stick were made using the Copy button. One of the copies was duplicated by using Repeat; the duplicates were joined to the selected segment to produce a line segment whose length is four times that of the selected stick. The second copy was divided into four equal parts using the Parts button. This microworld shows a visual representation of the partitioning operation that is necessary if children are to understand fractional relationships. A child can perform the partitioning manually using the Mark button, or by specifying the number of parts, the child can let the computer partition the segment. By comparing the results of the two ways of partitioning, a child can obtain immediate feedback on the results of his estimation.

The Copy feature of Sticks is an essential action in all of the microworlds. By allowing the students to construct exact replicas of objects, actions can be conducted on the copies leaving the original figure intact. This is a very important feature because students working on a scheme sometimes forget not only the problem posed but also the aspects of the original figure.

One of the reasons that the standard sticks were developed was to provide ways to represent equivalent fractions. Children can copy the unit stick and one or all of the other sticks into the work region. Then using the Repeat button, they can compare how many times each stick needs to be repeated until they are of equal length, for example, representing 2/3 equals 4/6. This scheme would also allow a child to compare the lengths of the sticks in a ratio. Through the use of the ratio, a child can begin to see one of the original sticks as a fraction of the other. For example, two sticks might be repeated until they are of the same length. By inspection, the child sees that three repetitions of one stick gives the same length as four repetitions of the other, giving the ratio of 3:4. From this a child may be able to develop the concept of 3/4 and realize that the shorter stick is 3/4 the length of the longer one.

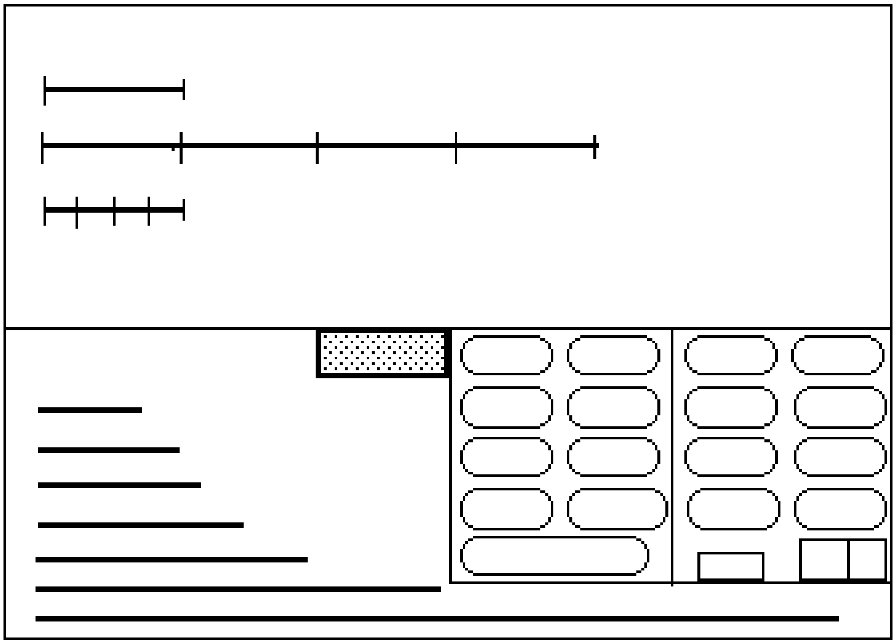

Figure 2: A reproduction of a screen from the Sticks microworld.

Another reason for including the standard sticks was to represent addition of fractions visually. When a stick of 1/2 unit is combined with one of 1/4 unit, the resulting length can be compared with a unit length to find that 1/2 + 1/4 = 3/4. Other examples that can be done with children are

1. A student is asked to draw a segment and then draw one that is 1/2 or 1/3 the length of the original. Through the use of either the Copy or Image buttons, student can see if the segment he creates is correct and if not, develop schemes to reach the correct solution.

2. Segments can be drawn, divided into *n* equal parts, then filled with different colors so that 1/2, 1/3, or 1/4 of the original segment is filled. By using the Copy or Image button, children can "prove" that their construction of 1/2, 1/3, or 1/4 of the original segment is correct.

3. A segment can be drawn, copied twice, with the first copy divided into halves and the second into quarter sections. Children can then compare these partitions to express equivalent fractions.

Sticks has a continuous nature in contrast with the discrete nature of Toys. It is in Sticks that the children begin using fraction language on a regular basis because they are able to perceive that measurement is occurring. It is here that the children begin to develop fractions through segmenting and partitioning of line segments. It is hoped that tasks such as those given above will allow children to construct concepts of fractions.

Candybars

The Candybars microworld is a two-dimensional extension of the Sticks microworld. Many of the operations in Sticks appear again in this microworld in a two-dimensional context. The emphasis of this microworld is on disembedding, unitizing, and re-presentation; thus many of the same operations that were found in Toys are also found here.

A candybar is a rectangle that is created with the Make button by dragging the mouse in a similar manner that covers are constructed in both Toys and Sticks. As many candybars as desired can be constructed as long as they fit on the computer screen. Once the children create candybars, various actions can be performed on the bars. Figure 3 is a reproduction of a screen from the Candybars microworld. The Candybars microworld has two regions: a button region where actions are selected and a workspace similar to both Toys and Sticks where bars are made and operations are carried out on the bars. In Figure 3, a candybar has been created and copied. The copy was partitioned into twelve pieces using Parts, then copied again and the new duplicate was broken into twelve smaller candybars using Break.

Through the use of the Color and Fill buttons, children can make candybars of seven different colors. Similar buttons exist in Toys and Sticks to color the objects in those microworlds. Each candybar can be moved around the screen just by clicking on the bar, holding the mouse button down, and dragging the bar to the desired location. Other buttons that have the same functions as in both Toys and Sticks are the Copy, Cover, Uncover, and Join buttons. The Rotate button rotates a candybar 90 degrees thereby giving the children another perspective of the candybar.

Unique to Candybars are multi-function buttons. One of these buttons allows the children to select between various modes: Marks, Cuts, Parts, and Shade. In Figure 3, this button is currently in the Marks mode. These buttons perform the operations that their names imply. Marks lets the children partition a candybar into pieces with the click of the mouse. Children can use their estimation skills in determining what 1/2, 1/3, and so on looks like by placing a mark, actually a line segment that is drawn by the computer across the bar, on the candybar. Since this is a two-dimensional microworld, children have the option of making either horizontal or vertical marks with the use of another multi-function button. The Cuts button lets the children cut any size piece of the bar in the same way that they mark the bar. Again, cuts can be made horizontally or vertically. The Shade button lets the children fill a bar either horizontally or vertically with the color currently on the Color button. If a child were to shade 1/2 of the bar vertically and then shade 1/2 of the bar horizontally, the computer would show the intersection of these operations; thus the child could speculate about what part of the total bar is the intersection. The Parts button works in tandem with a counter that lets the child decide into how many parts a candybar should be divided. Again, the partitioning can be both horizontal or vertical. After selecting the number of parts and the direction, the child clicks on the candybar and the computer draws lines across the bar that divide it into exact portions. The Break button, breaks the candybar into the number of parts that were marked on the bar. These pieces then become smaller candybars that can have the same operations performed on them as the original bar. An example of the usefulness of these operations occurs when a bar is partitioned into 4 parts, broken, and then one of the pieces is again partitioned into 4 parts. Children can be asked how one of the parts of a piece compares to the original candybar. It is through the interiorization of operations such as these that the child's concept of 1/16 may be constructed. Other examples of tasks that can be done with children follow:

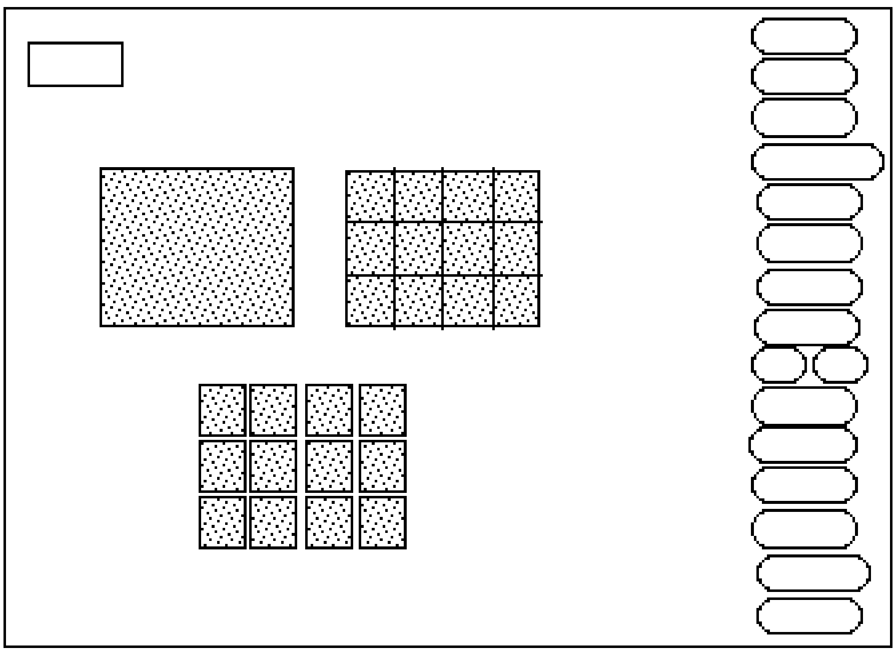

Figure 3: A reproduction of a screen from the Candybars microworld.

2. Children could be presented with a candybar and asked to share it fairly between twelve people. Initially, the children might mark the bar into twelve pieces of the same size using the Parts button and then use the Break button to separate the bar. Ultimately, it would be desirable for children to activate their number schemes developed in Toys to realize that the bar can be divided into, say, three parts vertically and four parts horizontally, also to obtain twelve pieces of the same size. Such activation of their number schemes would show that they are performing the operations of abstraction that we desire between microworlds.

Summary

In using these microworlds in the Fractions Project, we begin with Toys. Its discrete units allows children to make use of their counting skills in developing multiplication and division schemes. Later, the children move into Sticks, which introduces them to continuous quantities. Candybars gives them access to two-dimensional continuous quantities. By providing the children in the project with opportunities in Sticks and Candybars to activate their number schemes developed in Toys, we hope to help them construct rational numbers. In Sticks and Candybars, rational numbers are seen in the context of fractions of a whole. By allowing the children to use their number schemes with these fractions, we hope to eventually encourage them to abstract their concepts of fractions out of the context of the microworlds and into the realm of rational numbers. At this time, we are only beginning the use of the microworlds in the development of fractions. Since the children involved in the project are in various stages in the development of their number sequences, some are already using Candybars for fraction work while others have entered only recently Sticks for pre-fraction work. Discussion among the members of the project often centers on the development of tasks for encouraging the children's development of fractions through the use of the microworlds.

These microworlds are not static; they are constantly being updated and revised. Future papers will include the updates of these microworlds, give problems that can be posed in the context of these microworlds, and discuss future microworlds that are now in the design stages.

Reference

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"If we work upon marble, it will perish; if we work upon brass, time will efface it . . . but if we work on immortal minds . . . we engrave on those tablets something which will brighten all eternity."

- *Daniel Webster*

"The subject which involves all other subjects, and therefore the subject in which education should culminate, is the Theory and Practice of Education."

- *Herbert Spenser*

"Again, the sciences that deal with first principles are the most exact of all, for those that employ few principles like arithmetic, are more exact than those that employ more, like geometry."

- *Aristotle*