# On Differences in Pupils' Conceptions about Mathematics Teaching 

Erkki Pehkonen

If constructivism is taken as a basis for teaching and learning mathematics (e.g. Davis, Maher, \& Noddings, 1990), teachers' and pupils' mathematical beliefs have a key role for researchers who are trying to understand their mathematical behavior (e.g. Noddings, 1990). Research has revealed that knowing the right facts, i.e. algorithms and procedures, does not necessarily guarantee success in solving mathematical problems. Other factors including decisions made by the solvers and the strategies they use, as well as their emotional states at the time they are solving mathematical tasks, have a major effect on the performance of solvers (Schoenfeld, 1985). "Purely cognitive" behavior is rare. Belief systems shape cognition, even though some people may not be consciously aware of their beliefs.

Many studies on pupils' beliefs have been undertaken over the last decade. Underhill (1988) has compiled a review of research results on pupils' beliefs. The reviews of the role of affect in mathematics education conducted by McLeod $(1989,1992)$ provide additional information on the research results in this field. Butthe international comparison of pupils' (and teachers') mathematical conceptions seems to be still a rather unexplored field. Only in the Second International Mathematics Study (Kifer \& Robitaille, 1989) are pupils' responses to some questions on the affective domain dealt with in a background questionnaire.


Figure 1: Factors affecting pupils' mathematical behavior.

Although beliefs are popular as a topic of study, the theoretical concept of belief has not yet been dealt with thoroughly. The main difficulty has been to distinguish between beliefs and knowledge, a difficulty which has not yet been clarified (e.g. Abelson, 1979; Thompson, 1992). As a consequence of vague definition, many researchers use their own definition of belief. In this context, we understand beliefs as the individual's subjective knowledge of a certain object for which there is not necessarily any tenable ground in objective considerations. Then it follows that conceptions are conscious beliefs, i.e. conceptions form a subset of beliefs. The notion of a belief system is a metaphor used for describing how one's beliefs are organized (Green, 1971).

## The Meaning of Mathematical Beliefs for Pupils' Learning

The central meaning of beliefs for the successful learning of mathematics has been noticed by many mathematics educators. For example, Baroody and Ginsburg (1990) state that beliefs can have a powerful impact on how children learn and use mathematics. Both Schoenfeld (1985) and Silver (1985) have pointed out that pupils' beliefs regarding mathematics may form an obstacle for s o l ving nonroutine problems and for ef-

Erkki Pehkonen is a senior lecturer in the field of mathematics education at the University of Helsinki in Finland. He is interested in problem solving with a focus on motivating middle grades students and understanding students' and teachers' conceptions about mathematics teaching.
fective mathematics learning. Borasi (1990) stresses that pupils who have rigid and negative beliefs regarding mathematics and its learning easily become passive learners who emphasize remembering more than understanding in learning.

In her dissertation, Martha Frank (1985) introduced a schematic picture of some factors affecting pupils' problem solving behavior. Since most of the factors are acting via pupils' belief systems, we have here organized the components in the scheme in another way (Figure 1).

Pupils' mathematical beliefs form a filter which deals with almost all their thoughts and actions concerning mathematics. Pupils' prior experiences in mathematics affect their beliefs-usually unconsciously. Their mathematical knowledge also operates through the filter of their belief systems. But pupils' motivation and needs as mathematics students are not connected only with their individual mathematical beliefs. There are also societal mathematical myths, e.g. boys are better than girls in mathematics, which also influence pupils' mathematical behaviors via their belief systems (for more myths see Paulos, 1992).

## Data Collection

The starting point of the survey was the background
studies of the earlier research project Open Problems in Mathematics (e.g. Pehkonen \& Zimmermann, 1990) which was sponsored by the Academy of Finland. In the background studies, pupils' conceptions of mathematics teaching as well as their experiences and wishes regarding mathematics teaching were clarified with the aid of a questionnaire and interviews. From these background studies, a large independent research project International Comparison of Pupils' Mathematical Conceptions has emerged in which the pilot study was composed of a comparison between four European countries (cf. Pehkonen, 1993). Here we have added the U.S. data to the results of the pilot study.

## Research Problems

The purpose of this survey is to clarify, in middle schools in Finland, Estonia, Hungary, Sweden and the U. S., pupils' conceptions of mathematics teaching as well as their experiences and wishes regarding mathematics teaching. The main question of this study involves the comparison of pupils' conceptions: Are there essential

|  | S | E | H | S | US |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 doing calculations mentally | 1.8 | 1.8 | 1.9 | 1.5 | 2.4 |
| 3 doing computations with paper and pencil | 2.0 | 3.1 | 2.4 | 2.2 | 2.5 |
| 6 drawing figures | 2.3 | 2.8 | 1.8 | 2.6 | 2.3 |
| 9 doing word problems | 2.2 | 2.9 | 1.9 | 2.0 | 2.3 |
| 14 use of calculator | 2.0 | 1.5 | 2.4 | 2.2 | 1.9 |
| 17 different topics, such as calculation , of percentages geometry, algebra, will be taught and learned separately | 2.8 | 2.8 | 2.9 | 2.6 | 2.9 |
| 19 studying mathematics has practical benefits | 1.5 | 1.5 | 2.0 | 1.8 | 2.0 |
| 22 calculations of areas and volumes | 2.1 | 2.5 | 1.9 | 2.0 | 2.4 |
| 28 the constructing of different concrete objects and working with them | 2.9 | 3.1 | 2.7 | 3.1 | 2.2 | differences in pupils' conceptions of mathematics teaching in different countries?

The objectives of the research can be extracted from this question and categorized into three target fields: The aim of the research is to clarify a) pupils' conceptions of mathematics teaching, b) pupils' subjective experiences and wishes regarding mathematics teaching, c) whether there are any differences in these points in different countries.

## Questionnaire

In the questionnaire used, there are 32 structured statements about mathematics teaching, and the pupils were asked to rate their views of the statements on a Likert scale (see Appendix). At the end of the questionnaire, there were two open-ended questions. The first inquired about the pupils' good and bad experiences with mathematics teaching, and the second was concerned with their wishes for mathematics teaching.

To structure the questionnaire, a modification of the Underhill groupings (1988) was used. The
statements of the questionnaire were classified into three types of views (the numbers given in parentheses correspond to the number of the statement on the questionnaire.)
View of Mathematics
(items 1, 2, 3, 5, 6, 9, 10, 14,16, $17,19,20,21,22,23,24,28)$, View of Learning Mathematics (items 11, 12, 18, 29, 30), View of Teaching Mathematics (items 4, 7, 8, 13, 15, 25, 26, 27, 31, 32).
The first and third category can be easily divided into two parts. The View of Mathematics category can be separated into Mathematical Content (items 1,3,6, 9, 14, 17, $19,22,28)$ and the Way of Doing Mathematics (items 2, 5, 10, 16, $20,21,23,24)$. The View of Teaching Mathematics category can be subdivided into Pupils' Involvement (items 4, 7, 8, 13, 27, 31) and Teachers' Involvement (items 15, $25,26,32$ ). Some statements were difficult to classify in one category only; e.g. statement 6 could also have been in the group Way of Doing Mathematics and statement 20 in View of Learning Mathematics.

## The Structure of the Survey and its Implementation

The questionnaire was translated from Finnish into English, and further into Estonian, Hungarian and Swedish by the local coordinators. In each country, the questionnaire was administrated during 1989-91 to about 200 seventh graders in the capital cities and their surroundings. The local coordinators in the study were as follows: Dr. Lea Lepmann (University of Tartu, Estonia), Dr. Klara Tompa (Budapest, Hungary), Mr. Arne Engström (University of Örebro, Sweden), and Dr. Tom Cooney (The University of Georgia, United States).

In Finland, the questionnaire was administered in the fall of 1989 within the framework of the research project Open Problems in Mathematics. There were pupils from 15 classes of grade seven in Helsinki and Järvenpää (a small town about 40 km to the north of Helsinki), altogether 260 pupils.
$\left.2 \begin{array}{lccccc} & \text { F } & \text { E } & \text { H } & \text { S } & \text { US } \\ 5 & \text { getting the right answer is always } \\ \text { more important than the way of } \\ \text { solving the problem } \\ \text { everything ought to be expressed } \\ \text { always as exactly as possible }\end{array}\right)$



Figure 3: View of mathemtics (way of doing mathematics).

Ludvika (a small town about 200 km to the northwest of Stockholm). Teachers administered the questionnaire during an ordinary mathematics lesson in April 1991.

In November 1991, the questionnaire was distributed in seven classes of grades six and seven in schools around the state of Georgia in the United States. There were schools from large cities (Atlanta, Savannah) as well as schools in the countryside. Altogether 204 pupils completed the survey. The data was gathered by the author with the help of Dr. Tom Cooney

In most countries, seventh grade pupils are approximately $12-13$ years old. Only the Hungarian and U.S.pupils are a bit younger, about 11-12 years old.

## On Differences in Pupils' Conceptions

Preliminary results of the survey will be described in this article. We will restrict ourselves to pupils' responses to the statements in the questionnaire and concentrate on the revealing differences in the responses. A deeper discussion of the results will be available at a later date.

In general, the results give a very unified view of mathematics teaching in all the participating countries. But when we take a closer look, there are interesting differences between the countries. In fact, there is no item without a statistically-significant difference between the means of at least two countries tested with the $t$-test. It is remarkable that all the differences discussed here are statistically significant at the $0.1 \%$ level. (With this number of test subjects ( $n \approx 200$ ), a difference is statistically significant at the $0.1 \%$ level if the difference is larger than 0.3.)

The survey results will be examined according to the three-fold grouping of the statements on the questionnaire described in the previous section: the View of Mathematics, the View of Learning Mathematics, and the View of Teaching Mathematics. Pupils' conceptions are discussed using the means of the items (in all cases, the standard deviations are of order 1). This form of dealing with the results has been consciously chosen, although all the formal prerequisites for its use were not fulfilled (e.g. the questionnaire is actually an order scale).

We will use the following approach: If the mean of responses for a statement lies within the interval 1.0 to 2.5 (interval 2.5 to 3.5 or interval 3.5 to 5.0 ), we will say that pupils agree (are neutral or disagree) with the statement. In all figures, the following abbreviations are used: F -


Figure 4: View of learning mathematics.

Finland, E-Estonia, H-Hungary, S - Sweden, US - United States.

## View of Mathematics

Mathematical Content. First, we will consider the content component of the View of Mathematics, which consists of nine items (Figure 2). The differences are largest in items $1,3,6,9,14$ and 28. The U. S. pupils did not stress mental calculation as did the others. The Estonians took remarkably more of a neutral attitude than the pupils from other countries towards both mechanical computing (3) and word problems (9). The Hungarians emphasized more clearly than the others the drawing of figures (6). The Estonian pupils were more in favor than the others of using calculators (14). Furthermore, the U.S. pupils differed from the others in emphasizing the construction of concrete objects (28).

Way of Doing Mathematics. There are eight items in the Way of Doing Mathematics (Figure 3). In the case of statements 5, 10, and 16, the differences are remarkable. These three statements represent a formalistic view of mathematics which stresses the exact use of language and
rigorous reasoning, as well as the belief in the existence of such procedures which will lead to a solution. The Estonian and Hungarian pupils agree most strongly with these statements. The Swedes also favor exact reasoning (16), but they do not support the belief in the existence of such procedures which will lead to a solution (10). The Finns are furthest from agreement with these statements, except in exact reasoning (16) where the U.S. pupils are still more neutral.

## View of Learning Mathematics

Five items from the questionnaire pertain to the View of Learning Mathematics (Figure 4). In this group, the largest differences are in statements 11,12 , and 18. These statements represent understanding and learning through memorizing. All pupils agreed with statement 11 (understanding) except the U. S. students, whose view was neutral. In the case of memorizing (12), the Estonian pupils disagreed with the statement. The Hungarian and U. S. pupils took a clearly different attitude towards
repetition (18) than the pupils from other countries: they almost disagreed with the statement.

## View of Teaching Mathematics

Pupils' Involvement. In the component of Pupils' Involvement, there are six statements (Figure 5). In the pupils' responses, statements 7,8 , and 27 showed statistically very significant differences. In statement 7 (right answer quickly), the conceptions were distributed from neutral to disagreement with the viewpoint of the Estonian pupils differing the least from the statement; the Finnish and U. S. pupils disagreed most with the statement. With statement 8 (discipline), the Estonians and Finns agreed almost fully, whereas the Hungarian and Swedish pupils took a neutral attitude and the U. S. pupils disagreed. All were in favor of pupils' working independently (27) except the Hungarian and U. S. pupils who took a neutral attitude.

Teachers' Involvement. In the component of Teachers' Involvement, there are four statements (Figure 6).

Remarkable differences in the pupils' responses can be found in the case of statements 25 and 32. All pupils agreed with the use of learning games (25). However, the U. S. pupils supported them more clearly than the others, and the Hungarians took a neutral attitude. Only the Estonians supported the idea that the teacher should tell pupils exactly what to do (32); the others took a neutral attitude to the statement.

## Summary

The 17 statements discussed above exhibited the largest differences between the countries. These statements can be categorized into five groups:
Basic Calculations
(items 1, 3, 6, 9, 14),
Mathematical Rigor (items 5, 10, 16),
Mechanistic Learning (items 7, 12, 18), Pupil-Centeredness (items 11, 25, 27, 28), Teacher-Directedness (items 8, 32).
In the following figure (Figure


Figure 6: View of teaching mathematics (teachers' involvement).
7), a score for each country has been computed as an average of the item averages to enable us to compare the countries on these dimensions. Figure 7 shows that the largest differences in the pupils' conceptions in different countries emerge in the case of Teacher-Directedness. The Estonian pupils felt that mathematics is teacher-directed, whereas the U. S. pupils tended to disagree. Another large difference can be found in the case of Mathematical Rigor. Both the Finnish and U. S. pupils took a more neutral attitude, whereas the Estonians and Hungarians agreed that mathematics teaching involves rigor. On the other dimensions, the "averages" of the countries were similar to each other.

## Discussion

The reliability of the questionnaire is calculated using the halving method and the Spearman-Brown formula. This gave a test reliability of 0.61 for Estonia, 0.72 for Finland, 0.61 for Hungary, 0.63 for Sweden, and 0.67 for the U. S. which forms an estimate for the constancy of the results (internal reliability).

Although pupils' conceptions of mathematics teaching will not surface in a pure form in the survey, the results still reflect the conceptions that the pupils have. These conceptions are interesting because they will influence pupils' mathematical behavior. It is clear that the use of a questionnaire alone will not give an adequate picture of pupils' conceptions of mathematics teaching. Interviews are needed to enhance the picture.

When interpreting the results of the survey, the readers' conceptions of mathematics teaching and learning are very strongly involved. For example, the responses to statement 20 (only the talented can solve most of the tasks) might give us the picture that mathematics teaching has been successful since the pupils disagreed with the statement. We might think that they have learned to struggle with mathematics and not to think of it as being impossible. But there could be another contradictory interpretation of this same fact: Mathematics teaching could have failed because it deals with problems too simple and easy, since the majority of pupils consider mathematics to be accessible to everyone.


Figure 7: The comparison of the countries on the five dimensions.

## Concluding Note

Upon a first evaluation, the mathematics teaching experienced by pupils in Estonia and Hungary seems to be more formalistic than in Finland, Sweden and the U. S. In Finland and Estonia, pupils consider teacher-directedness a more integral part of mathematics teaching than in Hungary, Sweden and the U.S. But the main finding in this research is that differences between countries are significantly larger than within the country, e.g. those between girls and boys (cf. Pehkonen 1992). It seems that pupils’ conceptions regarding mathematics teaching are culturebound.

As an implication of the last statement, we should know the effect of differences in pupils' conception, in order to use the research results concerning mathematics teaching obtained in one country. Therefore, more data should be gathered, and the results obtained with questionnaires should be examined in more depth with pupils' interviews. In the research project International Comparison of Pupils' Mathematical Conceptions, the first aim is to gather enough data that we can state what pupils' mathematical conceptions are in Europe. Then we will be able to compare these conceptions with the corresponding U. S. results.

## Appendix: Pupils' Questionnaire about Mathematics Teaching

Background data from the respondent
Circle one in each:
(a) boy / girl ,
(b) grade $6 / 7 / 8 / 9$

Consider the following statements concerning mathematics teaching. Circle the point corresponding to your opinion. Use the scale:

$$
1=\text { fully agree } \quad 2=\text { agree } \quad 3=\text { undecided } \quad 4=\text { disagree } \quad 5=\text { fully disagree }
$$

## GOOD MATHEMATICS TEACHING INCLUDES

| 1 | 2 | 3 | 4 | 5 | doing calculations mentally (1) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | the idea that getting the right answer is always more important than the way of solving the problem (2) |
| 1 | 2 | 3 | 4 | 5 | doing computations with paper and pencil (3) |
| 1 | 2 | 3 | 4 | 5 | the idea that the student can sometimes make guesses and use trial and error (4) |
| 1 | 2 | 3 | 4 | 5 | the idea that everything ought to be expressed always as exactly as possible (5) |
| 1 | 2 | 3 | 4 | 5 | drawing figures (e.g. triangles) (6) |
| 1 | 2 | 3 | 4 | 5 | the idea that one ought to get always the right answer very quickly (7) |
| 1 | 2 | 3 | 4 | 5 | strict discipline (8) |
| 1 | 2 | 3 | 4 | 5 | doing word problems (9) |
| 1 | 2 | 3 | 4 | 5 | the idea that there is always some procedure which one ought to exactly follow in order to get the result (10) |
| 1 | 2 | 3 | 4 | 5 | the idea that all students understand (11) |
| 1 | 2 | 3 | 4 | 5 | the idea that much will be learned by memorizing rules (12) |
| 1 | 2 | 3 | 4 | 5 | the idea that students can put forward their own questions and problems for the class to consider (13) |
| 1 | 2 | 3 | 4 | 5 | the use of calculators (14) |
| 1 | 2 | 3 | 4 | 5 | the idea that the teacher helps as soon as possible when there are difficulties (15) |
| 1 | 2 | 3 | 4 | 5 | the idea that everything will always be reasoned exactly (16) |
| 1 | 2 | 3 | 4 | 5 | the idea that different topics, such as calculation of percentages, geometry, algebra, will be taught and learned separately; they have nothing to do with each other (17) |
| 1 | 2 | 3 | 4 | 5 | the idea that there will be as much repetition as possible (18) |
| 1 | 2 | 3 | 4 | 5 | the idea that studying mathematics has practical benefits (19) |
| 1 | 2 | 3 | 4 | 5 | the idea that only the mathematically talented students can solve most of the problems (20) |
| 1 | 2 | 3 | 4 | 5 | the idea that studying mathematics could not always be fun (21) |
| 1 | 2 | 3 | 4 | 5 | calculations of areas and volumes (e.g. the area of a rectangular and the volume of a cube) (22) |
| 1 | 2 | 3 | 4 | 5 | the idea that studying mathematics requires a lot of effort by students (23) |
| 1 | 2 | 3 | 4 | 5 | the idea that there is usually more than one way to solve problems (24) |
| 1 | 2 | 3 | 4 | 5 | the idea that games can be used to help students learn mathematics (25) |
| 1 | 2 | 3 | 4 | 5 | the idea that when solving problems, the teacher explains every stage exactly (26) |
| 1 | 2 | 3 | 4 | 5 | the idea that students are led to solve problems on their own without help from the teacher (27) |


| 1 | 2 | 3 | 4 | 5 | the constructing of different concrete objects (e.g. a box or a prism) and working with <br> them (28) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 4 | 5 | the idea that there will be as much practice as possible (29) |
| 1 | 2 | 3 | 4 | 5 | the idea that all or as much as the student is capable of will be understood (30) |
| 1 | 2 | 3 | 4 | 5 | the idea that also sometimes students are working in small groups (31) |
| 1 | 2 | 3 | 4 | 5 | the idea that the teacher always tells the students exactly what they ought to do (32) |

What kind of experiences do you have until today (from the primary level up to now) about mathematics teaching? Can you explain with a couple of words!
good:
bad:

What wishes do you have for mathematics teaching? Can you explain with a couple of words!
${ }^{1}$ Bernd Zimmermann (Hamburg) developed the questionnaire for the research project "Open Problems in Mathematics" (Pehkonen \& Zimmermann, 1990).

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