Calculus Students' Graphical Constructions of a Population Growth Model

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One of the major goals of university mathematics education is the knowledge of the main themes of calculus: limits, differentiation, and integration. Each of these themes has intuitive definitions rooted in the graphs of functions. Understanding the concepts of calculus requires a thorough understanding of the relationship between functions and their graphs (Dunham & Osborne, 1991). Dunham and Osborne maintain that students must learn how to see graphs. These graphs may seem obvious and trivial to mathematics educators who have forgotten any difficulties and misconceptions about graphs that they may have held in their student days. In order to find a satisfactory solution to the problem of misinterpretation of graphs, we need to know what conceptions students hold. Maher and Davis maintain that "trying to make sense of what students are doing and why they are doing it is prerequisite... to gaining insight into the nature of development of children's representations" (1990, p.89). This process of sense-making is no less important at the college level. Specifically, this study investigates calculus students' conceptual understandings of rate of change. We attempt to access their understanding by exploring their graphical representations of a dynamical population model.

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Theoretical Framework

The characterization of individual knowledge development as a constructive process provides the theoretical environment for this research. Constructivism posits that "conceptual knowledge cannot be transferred ready-made from one person to another, but must be built up by every knower solely on the basis of his or her experience" (Steffe, 1990, p.169). Within this paradigm, knowledge construction is interpreted as an incremental process based on the learner's interaction with his or her environment and pre-existing knowledge. Students constantly construct an understanding of their experiences. As such, the mathematics teacher must always give consideration to the possibility that the student's constructs may seem perfectly valid to the student, regardless of how they differ from the teacher's own constructs (Confrey, 1990). The research on "students' misconceptions, alternative conceptions, and prior knowledge provides evidence of this constructive activity" (Confrey, 1990, p.111) in which students engage. Understanding these misconceptions, inconsistencies, and cognitive conflict with respect to the mathematical knowledge of the students and how that knowledge might be modified can save mathematics teachers from wasting time in mutual misunderstandings (Clement, 1989). According to the *Curriculum and Evaluation Standards* (National Council of Teachers of Mathematics, 1989), teachers must be able to adapt instruction so that it meshes with students' thinking. Constructivist teachers are those who anticipate misconceptions and plan activities that will lead students to challenge their own faulty conceptions (Davis, Maher, & Noddings, 1990). All teachers need to look for students' misconceptions and alternative conceptions and strive to help students modify them.

The distinction is made in this study between alternative conceptions and misconceptions. Alternative conceptions are identified as variant conceptions that follow a reasonable rationale, while misconceptions are those conceptions built on the student's faulty logic. Specifically in regard to graphing, many misconceptions have been documented. Clement (1989) reviews much of the literature on misconceptions in Cartesian graphing. He divides observable graphing errors into two types: 1) links to incorrect graphing features and 2) graph as picture. Errors of the first type are subdivided into four categories: a) height for slope, b) slope for height, c) height for difference in height, and d) slope for curvature. The second type of error, graph as picture, is separated into two subcategories: a) global correspondence and b) local correspondence. In a global correspondence error the shape of the entire problem scene is matched to the entire graph in a global fashion. For example, the speed versus time graph of a bicycle going down hill might resemble a picture of the hill itself. A local correspondence error refers to a local visual feature of the problem scene being matched to a specific feature of the graph. For example, the intersection of two lines representing interest rates over time might be seen as where the two investments have made the same amount of money.

Bell, Brekke, and Swan (1987) suggest that traditional methods of instruction do not facilitate high school student interpretation of graphs. Dunham and Osborne (1991) report that university precalculus students, given the choice between solving a problem by using a relatively direct graphical interpretation or by solving a more complex algebraic inequality, found the complex solution process of the algebraic inequality easier than interpreting the graph. They suggested that a new emphasis on visual representation must be accompanied by an increased awareness among mathematics educators of student misconceptions. Therefore, the questions that this research proposes are 1) What misconceptions and alternative conceptions do beginning calculus students have regarding rates of change? and 2) How do these conceptions change after eight weeks in a calculus classroom using a conceptually-motivated curriculum?

Method

Forty-two students enrolled in a first semester calculus sequence at a large southeastern university participated in the study. The course used the text *Calculus* (Hughes-Hallet, et al, 1994) produced by the Harvard Consortium as part of its calculus reform effort. The subjects, selected primarily due to researcher accessibility, were voluntary participants from calculus classes taught by the researchers. In a pre- and postassessment environment, subjects considered the following problem:

> *Suppose for a given population it is observed that the birth rate is increasing and the death rate is decreasing. Suppose also that after a few years a virus is introduced which slows down the growth until there is no real observable change. Draw a graph depicting population over time.*

Both written instruments were administered during scheduled class time to intact classes. The preassessment was given on the first scheduled class meeting prior to any instruction by the researchers and the postassessment was given eight weeks later. This type of assessment question was discussed in class after the preassessment was given. Specifically, mathematical concepts such as concavity and exponential growth were addressed between assessments.

All subjects' responses were classified according to a five point rubric developed by the researchers for the particular problem of study. The rubric was designed to reflect anticipated features of the population graph. In particular, pre- and postassessment scores were obtained by assigning one point value to each of the following rubric criterion:

- attempts a response
- function increasing (before the virus is introduced)
- function concave up (before the virus is introduced)
- population growth slows (after the virus is introduced)
- function shows no real observable change

For each criterion, a student received a score of one if the criterion was present and a score of zero if it was not. To differentiate between students who may have had the same total score but obtained that score from having different criteria present, each student's composite score was preserved in a 1x5 matrix. A paired t test using students' total scores was computed as one means of measuring improvement on the postassessment.

Student response profiles were also developed from the rubric and tested for improvement using a kappa analysis. In developing the individual student response profiles, the last four rubric criteria served as the classifying variables. The first ,"attempts a response", was eliminated because all but one student attempted a response. Because this student chose not to respond, we were not able to include him in the study. The resulting profiles are represented as 1x4 matrices. For example, a student profile of "1010" indicates that the student constructed an increasing function that was not concave up before the virus was introduced, and with a decreasing growth rate after the virus was introduced.

The data were also studied qualitatively by sorting participants' responses according to their possible alternative conceptions and misconceptions. Six categories were determined:

CATEGORY 1: The birth and death rates were graphically represented as separate variables.

CATEGORY 2: The concept of "no observable change" was represented by disproportionately large population increases for almost no change in time.

CATEGORY 3: The graph did not reflect a slowing population growth; rather, an abrupt break from increasing to no observable change.

CATEGORY 4: The graph did not reflect an increasing birth rate and decreasing death rate before the virus was introduced.

CATEGORY 5: The graph reflected its initial point at the origin.

CATEGORY 6: Graph as picture (graphical representations that pictorially follow the statement of the problem).

Results and Discussion

Quantitative Analysis

Students' preassessment and postassessment scores were analyzed using a paired t test to determine if the students had made a significant improvement on the rate of change problem. As shown in Table 1, there was a significant improvement in the scores. On the postassessment, more students moved to more desirable profiles (those named by letters closer to the beginning of the alphabet) and away from less desirable profiles (those named by letters closer to the end of the alphabet). The t value of 2.548 was significant at the .05 level, suggesting a significant improvement on the postassessment scores.

Table 2 addresses the student profiles for the preassessment and the postassessment. In order to see how the students' profiles changed from the preassessment to the postassessment, the scores were plotted in a matrix

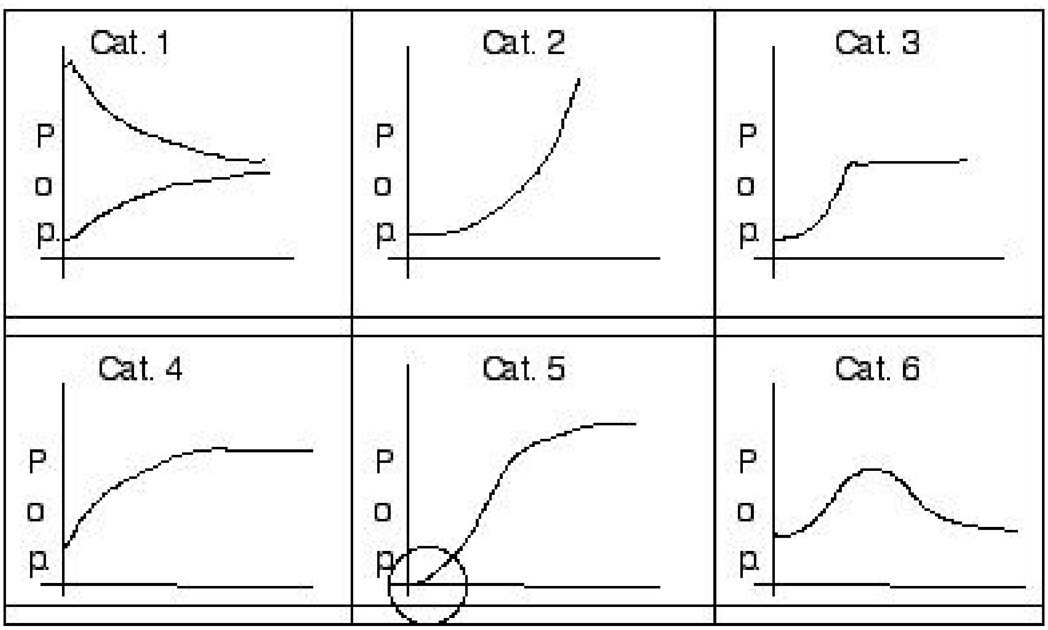

Figure 1. Typical responses for categories of misconceptions and alternative conceptions identified from the data sort.

A typical response for each category is given in Figure 1. Five participants whose responses reflected at least one of the categories identified by the data sort were selected for individual interviews lasting about 15 minutes each. A clinical interview format was used to explore their understanding of rate of change.

form. The matrix tallies the number of students who moved from one profile to another. Table 2 shows the changes in profiles that the students made from the preassessment to the postassessment. For example, the entry of 3 in row I and column B indicates that 3 students whose responses were categorized as profile I in the

Table 1. Pre- and Postassessment Frequencies of Student Response Profiles

preassessment gave responses categorized as profile B in the postassessment. Fifteen students' scores lie along the diagonal indicating that their profiles were the same for both assessments. Eighteen students made a positive shift (toward the lower triangular matrix). These students improved on the postassessment. A negative shift (toward the upper triangular matrix) was exhibited by eight students.

A kappa analysis was performed to determine if the shift in profiles was significant. This statistical test addresses the question of how much more agreement exists between assessments than that due to chance. It is based on the ratio of observed agreement not due to chance to the possible agreement beyond chance. A kappa value of zero would imply only chance agreement between the preassessment and postassessment profiles. A kappa value of one would imply perfect agreement. The kappa value was -0.315, suggesting even less than chance agreement between the preassessment and postassessment. That is, they are more different than would be expected by chance. The standard error was computed to get $\underline{z}$ = -3.389, which is significant at the .01 level. Therefore, we can conclude that the students made a significant positive shift in their profile descriptions.

Qualitative Analysis

The results of the preassessment sorting were tallied to determine percentages associated with students' alternative conceptions and misconceptions identified from the responses. We have defined alternative conceptions here to mean students' conceptions different from our own but which are justifiable by the student. For example, one student interpreted "no observable change" to mean the graph eventually returned to its original form. We feel that this unanticipated response reflected an alternative interpretation of the problem statement and not illogical thinking by the student. Also, some students' graphs showed an abrupt rather than gradual change in the population curve after the introduction of the virus. They explained this by the severity of the virus, which is not specifically addressed by the given problem. On the other hand, misconceptions were defined to be those conceptions produced by the student's faulty logic. For example, some students constructed separate curves for the birth rate and death rate on a coordinate system that requested a single function of population over time. Additionally, some students constructed a single function reflecting population over time but were unable to represent the combined effects of an increasing birth rate and decreasing death rate.

This sorting procedure was repeated for the postassessment. No new categories were observed. Percentage results are summarized in Figure 2. The preassessment and postassessment percentages reflect those students who held the alternative conception or misconception of a particular category at the relevant time of assessment. The unchanged percentages represent those students with the particular conception that persisted over the entire assessment period.

The results indicate that many students use the intersection of the coordinate axes, or the origin, as the initial

Table 2. Changes in Student Responses from Preassessment (Vertical) to Postassessment (Horizontal) Based on Student Response Profiles

point of the population graph. While students may view this intersection in a general sense as simply a starting point, the interpretation of it as (0,0) would contradict the reality of the given problem. It is also interesting to note that approximately one-half of the responses in both assessments did not reflect an increasing birth rate and decreasing death rate before the introduction of the virus. Mathematically speaking, those students represented this graphically with downward concavity. Twenty-two percent of the subjects held this misconception over time. It is encouraging to note that in the postassessment, less than 3 percent of the students produced graphical representations that pictorially followed the statement of the problem, the phenomenon known as graph as picture (Category 6).

The interview data provided a more detailed picture of the conceptual understanding of students possessing alternative conceptions and/or misconceptions. Aspects of the five individuals' interview responses are included because they were judged to be representative of a particular misconception or alternative conception. The examples are presented through summary and transcript.

Julie. Julie's preassessment attempt (see Appendix) was representative of categories 3, 4, and 5. When asked to explain her thinkingin her preassessment, she immediately realized that she had made changes and indicated that the preassessment was wrong because it did not account for a slowing in the population growth. When asked about the section of her graph before the virus was introduced, Julie explained her choice of representation as follows:

Julie: If there is more births and less deaths, then the population will increase. If there were the same amount of births as deaths, it wouldn't increase as rapidly, ummmm, or something. (pause) Or, it might stay the same if the birth and death rate were the same. (She indicates the last section of the graph where there is no change.)

Figure 2. Percentages associated with students' misconceptions and alternative conceptions identified from the data sort.

Julie clearly believed that her representation was correct, but was unclear about the relationship between the slope and the population. The slope was either a constant positive for when the population was increasing, or zero for when there was no change. As the interview continued, Julie explained her choice of a starting point for her graph.

Julie: It starts at zero, but (pause) ...I don't know...It is at zero time and zero population.
I: Is that what you mean here? This is (0,0)?
Julie: Yeah, I guess.

It seemed that Julie had given very little thought as to why she started her graph at the origin. She did know that the origin was the point (0,0), but she could not explain why she had chosen to start there and did not see that the point could have any value besides (0,0). However, she was hesitant in giving her answer, suggesting that she did not have confidence in her answer.

Jim. Jim's preassessment was representative of categories 3, 4, and 5, while his postassessment reflected categories 4, 5, and 6. (See Appendix.) In describing his preassessment response, Jim explained as follows:

Jim: Ummm, I think what I did was that I said that, ummm, the population would be increasing constantly over time and then it's gonna level out and stay the same.

Although this explanation represented a fairly accurate representation of the population over time, it did not allow for upward concavity before the virus was introduced. This misconception persisted in spite of further probing in the interview:

I: What is happening in this portion (of the graph)? You gave me an overall idea of what was going on, but can you be a little more detailed here? (The student's attention was directed towards the part of the graph that he had identified as before the virus was introduced).
Jim: I think that (pause)...I really don't remember.
I: Yeah, it's been a while.
Jim: Ummm, I think I thought that, like, since there was, ummm, that the birth rate was increasing and the death rate was decreasing that, I don't know, I thought that somehow that would like make the population growth constant.
I: In other words this (Jim's attention was again directed to the portion of the graph before the virus was introduced) to you is linear?
Jim: Not constant, I mean increasing at a constant rate.

When questioned about the initial point of his preassessment graph, Jim explained that this represented at time t=0 no population:

Jim: At time zero I thought there would be, like, zero population.
I: O.K. So that point (indicating the origin) represented to you (0,0)?
Jim: Yeah.
I: That was your beginning? (Jim nodded affirmation).

This persisted in Jim's postassessment.

Jim's comments concerning his postassessment response revealed a weakness of using only written instruments to measure students' conceptual understanding. We had interpreted his response as "graph-as-picture". The interview showed that this was not Jim's intention:

Jim: I think I was, like, basically saying the same thing but I think I had a, like a better understanding of derivative. I don't know...Ummm...This one (referring to the postassessment graph) is not so, ummm, it's not increasing, like, at a constant rate. It's kind of like not really going up and down but it's still going up. I don't know how to say it... In this one (again referring to the postassessment graph), I had the population go down and then level out.
I: O.K. So where would you say the virus was introduced?
Jim: I think I had it like right at, ummm, the top of the curve. I was thinking that, like, the population would drop off quickly.
I: O.K. So in some sense you're reflecting the devastation of the virus?
Jim: Yeah.

What we had interpreted as Jim's representation of a decreasing death rate was actually his representation of the effect of the virus.

Rob. Rob's pre- and postassessment responses were representative of categories 4, 5, and 6, and categories 4 and 5, respectively. (See Appendix.) His explanation of his preassessment response showed that he did not control simultaneously for the increasing birth rate and decreasing death rate; rather, he initially ignored the decreasing death rate:

Rob:	I just put, like, over time, like, the birth rate is increasing so, like, I thought it (referring to the increasing portion of the curve) was going up like that. And then the death rate is decreasing.... I don't think I really showed that.

He also failed to represent "no observable change", but represented instead a population that eventually decreased:

I:	Where did you show that there was no observable change?
Rob:	(Pause.) I don't think I did that.

Rob's comments concerning the origin of his pretest graph revealed a reasonable alternative conception:

I:	What does the origin represent to you?
Rob:	Ummm... The place where it starts?
I:	O.K. Would you put any particular values with that?
Rob:	Not really.
I:	O.K.
Rob:	It just gave me a place to start.
I:	O.K. So it could be (0,0) or some other point? It's just a starting place?
Rob:	Uh huh. Right.

Rob demonstrated an improved understanding of rate of change between pre- and postassessments, although his misconceptions regarding a simultaneous increase in birth rate and decrease in death rate persisted in the postassessment. His interpretation of the origin of his graph would be categorized as an alternative conception rather than a misconception.

Jackie. Jackie's preassessment was representative of categories 1, 2, and 5. Her postassessment was indicative of only categories 2 and 5. (See Appendix.) She explained her preassessment representation as follows.

Jackie:	It increases like an exponential graph and then when the virus has an effect on the graph, the birth rate slows down the slope of the graph and then it resumes after a period of time the path that it was following. So, it is concave up in the beginning and then it concaves down, and then back up at the end.

Jackie had two curves in her first representation. She went on to explain her second curve:

Jackie:	That would be my death rate and how it is decreasing (indicates the line with negative slope) and the other would be my birth rate (indicates the line with the positive slope). At a certain point the death rate and the birth rate come together and there is an effect on the birth rate.
I:	What is happening here where the two lines intersect?
Jackie:	That would be where my birth rate and death rate are equal, that their rate of change would be the same.

This student was unable to control for two variables. She treated the birth rate and the death rate separately, rather than as a single function.

Jackie had an alternative conception of what was meant by "no observable change." She described how she shows "no observable change" on her graph.

I:	How do you show there is no observable change?
Jackie:	Just by making it a smooth type curve, not making any drastic changes.
I:	So, where is the "no observable change" on your graph?
Jackie:	I tried to make it just between two points. This is the point 1 (points to her second change in concavity) and this is point 2 (points to the end of the graph.) ... It continues along the path that it was before. (Pause) It (the virus) is introduced, and once it is introduced it really drops the death rate down a lot. And then after a period of time, the death rate doesn't really have any effect and it goes back to the way it was before.
I:	So, that is the "no observable change?"

Jackie: Right.

To Jackie, "no observable change" meant that the population rate of change returned to its original state, as it was before the introduction of the virus. This interpretation made perfectly good sense to Jackie and she could explain her rationale. Therefore, Jackie has an alternative conception, not a misconception. Based on her interpretation of the scenario, her construction seems reasonable.

Jackie's second graph had the second curve, representing the death rate, dashed. She explained:

Jackie: ...when I was originally planning to draw this, I drew my exponential curve and I continued with a dotted line to see how it would go, and I did the same thing with the death rate curve and where they hit each other I just picked a point and that is where I said the virus was introduced and then I brought this over to where it is no observable change.

She realized that population is a composite of the death rate and the birth rate. Then she visually tried to compose the two functions. However, she still maintained her alternative conception regarding "no observable change."

Mary. Mary's preassessment was representative of categories 1, 4, and 5. (See Appendix.) Even though she has two curves, she refers mainly to just one of them. She explains her reasoning for her preassessment response.

Mary: I broke it up into like two graphs. I didn't take it as like one. I put that (ugh) the birth rate is increasing so I drew an exponential graph increasing. And, I put that the death rate is decreasing, and that after a few years a virus is introduced and it levels off. It was going up and then levels off.

Mary described the exponential curve as being her birth rate and the other as representing her death rate. She explained that the birth rate and the death rate are equal at the point of intersection:

Mary: For every person that is born, one dies.

She pointed only to her death rate curve as showing "no observable change." She indicated that the no observable change area was parallel to the horizontal axis. Therefore, she does not share Jackie's alternative conception.

When asked about where the virus is introduced, Mary became frustrated. It is clear that she was not happy with her first representation.

Mary: Instead of there being a large amount of deaths still occurring when the virus is introduced...(long pause). The graph doesn't make sense because it looks like population is increasing and decreasing at the same time.

This student realized the need to control for two variables. She was then able to explain her second graph correctly. The postassessment response was correct in every aspect. In addition, regarding the initial point on her graph, Mary explains why she did not start at the origin:

Mary: It shouldn't be at the origin unless there were always people. There had to be people before this. (Points to her starting point on the graph)

Overall, Mary made a substantial improvement. She revised her misconception regarding the two variables. In addition, she was able to accurately represent the introduction of the virus and to represent no observable change.

This investigation supports student thinking as a constructive activity. As students worked through both assessments, participated in class, and, in some cases, revisited the problem through interview, their thinking generally reflected a building process. Some addressed their pre-existing misconceptions, either individually or through interview prompts, and were able to achieve a more appropriate understanding as they blended what they knew with other stimuli. Although some misconceptions persisted, the results show that most students moved toward a deeper understanding of this problem during our investigation.

The evidence of misconceptions in this group of students supports Dunham and Osborne's (1991) suggestion that educators emphasize visual representations. Graphical construction and interpretation can indeed be problematic for students. As such, teachers should be aware of potential student difficulties with rate of change problems and create learning environments in which students can confront these difficulties. This investigation suggests an emphasis on visual representations through construction and interpretation in conjunction with teacher-student analysis as a meaningful environment for student change.

Conclusion

Students do indeed have misconceptions as well as valid alternative conceptions with respect to rate of change problems. These conceptions, partially identified by the written assessment, were clarified through interviews. As such, dialogue with students on an individual basis proved to be an invaluable tool for investigating students' ideas. Indeed, interviews revealed that what may have appeared to us to be a misconception was actually a faulty diagnosis on our part. Although interviews are time-intensive, they offer insight into students' thinking that written responses don't always capture. The student may have had a valid argument for his or her interpretation. As educators, we cannot assume that our conception is the only valid one. We must seek to understand students' ideas and through the constructive process of learning, help students use their existing knowledge as a bridge to deeper understanding.

It was further observed that students' misconceptions were at least partially resolved at a statistically significant level after eight weeks in a calculus classroom. It should be mentioned that the subjects of this investigation did not comprise a random sample. Even so, this study provides informative results that teachers can use to help their students overcome misconceptions associated with rate of change problems. In particular, we attempted to categorize various ideas students had concerning the construction of a dynamical population model and through this process, came to see teacher-student interviews in conjunction with written assessments as an important means of better understanding students' ideas. It is only when we can meet the student at his or her present level of understanding that we can hope to positively impact that understanding.

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APPENDIX

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