Guest Editorial: Mathematics Education in an Age of Uncertainty

Hardy Grant

Mathematics education has a long and noble history. In the 5th century B.C. the wandering Greek teachers called sophists took the then revolutionary step of viewing the subject as an incomparable tool for the training of the mind; this, wrote the great classical scholar Werner Jaeger, marked "a turning point in the history of the world." A second enduring motif was sounded a little later, by Plato. The famous pedagogical program in the *Republic* stressed the study of mathematics as the best way to lead the mind upward from the deceptive and perishable world of the senses to the contemplation of eternal reality and absolute truth. Compared to these lofty aspirations, the goal of merely training people for the practical calculations of daily life tended (at least in theory) to finish well up the track.

Two "higher" purposes, then, for education in mathematics: the cultivation of rigorous reasoning and the perception of eternal truths. How fare those ideals in our own time, two millennia after their Greek beginnings? I can testify that the first was alive and well in the schools of my native Ontario in the 1950s, when the senior-high geometry course was Euclid's *Elements* and the frankly avowed purpose was mental gymnastics. But what of that other, subtler value that the ancients saw in mathematics, the revelation of timeless verities — does that survive?

Let me try to set the question in a wider context. We live in an age of collapsed certainties. Skepticism is rampant about the "truths" that underwrote our civilization for centuries — in particular, the "revelations" offered by the great religions. Various other systems of thought, ostensibly secular but often seeming to share many traits with those same religions, have rushed into the resultant void, only to be widely discredited in their turn; Marxism and Freudianism are the obvious examples. The Western tradition's long-held confidence that political and social questions have unique, recognizable answers has drastically eroded. The legacy is all around us: a withering of broad consensus, an ascendancy of relativism, of pluralism, of subjectivism at every turn. A loss of faith in absolutes is nearly the defining characteristic of our modern condition.

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Some would regard mathematics as a particularly poignant casualty. Until the early nineteenth century it stood out as the model science, utterly certain and perennially true; but the subsequent years have dealt severe blows to that flattering image. Every schoolgirl can tick off some of the milestones on her fingers. The discovery of "non-Euclidean" geometries destroyed forever the status of the *Elements* as a unique repository of timeless truths. The work of Kurt Gödel made clear that any attempt to prove the consistency even of ordinary arithmetic must founder. Paul Cohen showed that the continuum hypothesis holds or does not hold, depending on what you assume. A sobering picture! Morris Kline summed it all up in a book with the melancholy title *Mathematics: The Loss of Certainty*.

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But the queer thing about this gloomy conclusion is that mathematicians — or anyway, most mathematicians, most of the time — persist in behaving as if they do not really believe it. Giants among them, of the stature of Gödel himself, of David Hilbert, of René Thom, have declared a "Platonist" conviction that mathematics does attain objective and certain knowledge. And the whole vast enterprise of modern research still very largely proceeds from the same assumption, in a manner so palpable as to need no underlining. Everybody assumes that (say) the existence or nonexistence of odd perfect numbers is a *fact about the world*, to be *discovered*; the idea that the matter could be settled by our arbitrary decree would seem grotesque. Of course mathematicians do *invent* things, they play with creations of pure thought; that is one of their discipline's great glories. But even their flights of imagination operate in delicate counterpoise with constraints built stubbornly into "the nature of things." The concept of a group, for example, is certainly a pure abstraction, a human invention; but once the basic definitions are in place, the theorist who wishes to seek out and classify actual *specimens* of groups must explore a reality given from without, in no different spirit from the biologist who would discover and catalogue the several species of flying fish. Who doubts that (as they say on *The X-Files*) the truth of such things is "out there," independent of us?

It follows — I submit — that mathematics retains an immense cultural value in a fragmented and tendentious age. It remains what it has always been, and what made it pedagogically central for centuries: our nearest and surest

intimation of objective and permanent truth. Somehow, the "Queen of the Sciences" survives both the legitimate unease over its own foundations and the crisis of confidence that looms over modern culture in general. It thus offers itself as one of the most benign of all responses to humanity's ineradicable hunger for the absolute and enduring. In mathematics the exhilarating sense of contact with realities larger and more lasting than ourselves still awaits the devoted inquirer. And so the highest purpose and benefit of mathematics education remain very much what Plato declared them to be, long ago.

But with one cardinal difference. In our time, in contrast to the Greeks' — and this is the central point I want to make — mathematics retains its Platonist character, its belief in sure knowledge of an objective reality, amid a massive retreat from that vision in other spheres. That makes for a cultural divide which can seem enormous. I have said that many other departments of modern intellectual life flounder in the relativism and subjectivism left behind by widespread breakdowns of certainty and consensus. The murky theorizing, the repulsive jargon, that now infest these rudderless domains are notorious — and sometimes their consequences are downright scary. The scholarly world was recently titillated by a delicious scandal. A physicist named Alan Sokal submitted to a leading journal of "social thought" a paper impressively decked out in the buzzwords of "postmodernist" claptrap but actually a deliberate farrago of factual absurdities and bizarre nonsequiturs. The journal's five-person editorial board pondered, then accepted this outrageous hoax for publication, treating it to all appearances as a serious contribution to scholarship. (A grabby account of the whole instructive episode can be found in the *Times Literary Supplement* of December 13, 1996.) Bemused kibitzers were left to guess at the editors' motives: perhaps they felt that anything so learnedly incomprehensible *must* be profound, or perhaps they hoped cynically that this highfalutin nonsense would *sound* like good grist for their particular ideological mill. In any case mathematicians who got wind of the affair could be pardoned for feeling a warm pride in their own discipline, where the premium on clear thinking and rigorous proof — that *other* durable goal of ancient education — makes such travesties all but inconceivable.

I am not saying — God knows! — that mathematics is a realm blissfully immune to all intellectual controversy and misadventure. Nor would I minimize for a moment the very real anxiety that some feel over the foundations. That there has been some loss of innocence is incontestable. The mathematical enterprise can seem like a dance on a wire over a yawning abyss. But I have tried to urge that, even so, the dance goes on, undiscourageably, and that it offers joys and values in bracing contrast to many of the bleaker and shabbier tendencies of our time. Students who are mature enough to grasp what is at stake are shortchanged by teaching which does not proclaim and celebrate those joys and values. Let the professional philosophers of mathematics worry about the abyss; make sure the students savor *all* the delights and rewards of the dance.

About the Cover

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$$(x+2)^2+(y-8)^2=2$$

$$(x-9)^4(y-6)^4=\frac{1}{3600}$$

$$y=\cos(\cos(\cos(\cos x)))x$$

$$y=\cos(\cos(\cos(\cos(\cos x))))(x^5-5)-2$$