



On Teaching

Roy Smith

Nothing teaches like experience

-John Bunyan.

Although we in mathematics accept that meaningful discussion is impossible without precise definitions, it seems to me that in discussing teaching we often depend upon our own unstated assumptions as to what teaching is or should be, and therefore waste some breath debating how to do it well. So for help, I looked in Webster's dictionary (copyright 1943, *The World Publishing Company*), where I found, among others, the following:

Definitions and Synonyms for To Teach (lit. to show):

1. to enlighten (illuminate, enable to see truth, lit. shed light on),
2. to educate (nurture, develop, initiate, lit. draw forth),
3. to guide in studies,
4. to instruct (inform, admonish, lit. pile up),
5. to exhibit so as to impress on the mind,
6. indoctrinate (imbue someone else with one's own ideas),
7. train (drill; exercise; break, tame and reduce to docility, as to train dogs or monkeys; lit. to draw, pull).

I find it interesting that "to inspire" and "to increase interest in" are not among Webster's meanings or connotations of "to teach". Furthermore, in all but the first listed definition, truth plays no role; one can teach evil, falsehood, and sloth, just as well as their opposites. Let us begin by observing that the different meanings of "teach" above imply different behaviors by the teacher and aim at different results in the student: to open the mind, to prepare for further growth, to show the way, to explain the facts, to display spectacularly, to enforce one's own opinion, to destroy initiative. Value judgments enter, it seems to me, in deciding which of these is good teaching and which is bad, or in asserting that good teaching is doing any one of them, but doing it effectively. The existence of differing value systems is thus another source of disagreement, beyond imprecision of terminology, as to what constitutes

good teaching. It is my own opinion, or bias, that the various aspects of teaching in the list above occur there in order of importance.

Rather than use such language as "good teacher," (*cf.* the caveat in the Gospel according to St. Luke, chapter 18, verses 18-19) I believe it is more illuminating to use descriptive terms. I would suggest that to inspire the students, to continually seek fresh insight and look for ways to explain it more clearly and vividly, to insist on critical thinking and high standards, and to exhibit in oneself a continuing thirst for knowledge, are the most desirable qualities of a teacher. The worst possible result would be to convey the idea that it is preferable to be a punctual, predictable, and available, but unimaginative, uninspiring, and uncurious teacher, than to be the opposite. Nonetheless, I think we are often in danger of giving this impression. Indeed many professors may recall the only traditional advice to a new teacher: "begin and end on time, and start writing in the upper left hand corner of the board."

My own heroes of teaching did not conform to this standard profile of the good teacher. I consider Maurice Auslander, a mathematics professor at Brandeis University, to have been one of the finest teachers I have ever known, although he was persistently criticized for his lack of organization. His classroom exploded with spontaneity and excitement. His was dynamic teaching which was hard to follow, but which gave us powerful precepts (instead of details) and made us want to rush out and apply them, teaching which made our powers of comprehension and problem solving more trenchant. He was clearly a powerful mathematician who was letting us watch him do mathematics, and showing us how to do it too. Auslander once gave us a homework problem that I still do not know how to settle over 25 years later. He would not tell us. I felt Auslander was teaching us that mathematics is exciting, challenging and fun, that it deserves the best we have to give, and will repay the greatest effort we can muster.

I was subsequently drawn into my present specialty of algebraic geometry, by the teaching of Alan Mayer, an extremely strong and imaginative geometer, also at Brandeis, whose lectures were so rapid-fire, so jam-packed with information, so lengthy and wide ranging, that I sometimes got headaches, and felt a need to recuperate after them for an hour in the sauna! They also made the subject so fascinating that there was nothing for me to do but to try to join the company of algebraic geometers.

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Others among the best teachers I have had from that field, Herb Clemens (Utah), David Mumford and Phil Griffiths (Harvard), sometimes gave me wonderful instruction simply by expressing ignorance of a topic and saying that they wished somebody would explain it to them. I of course rushed off to learn it and become the first to teach it to them. When I had spent days or weeks mastering it, they never seemed to need more than a few minutes to catch up and go well beyond what I had told them. I admit that as a professional, in constant need of up-to-the-minute insights, I no longer scorn pedestrian explanations of useful information. But I still find it difficult to sit through such a lecture if I do not understand why the information is useful. So for me motivation is still necessary.

If we take the point of view of some physicists, that a concept is defined by how it is measured, we learn what is considered good teaching in our own department by examining our teaching evaluation forms. These suggest that good teaching is a quantity which can be best appreciated by an average student in the class who has not yet even finished the course. I cannot fit my own examples of great teachers into this paradigm. Take Raoul Bott, for instance, who was regarded as an outstanding teacher during my time at Harvard. Once, in his class on algebraic topology, he remarked after giving a detailed proof of the Brauer fixed point theorem that one could see that all we had really needed was a “homotopy invariant functor that doesn’t vanish on the sphere.” I did not follow anything at all about this remark. I did not even grasp what it meant right away, much less why it contained the essence of the proof of the theorem. To me the theorem had a lengthy and complicated proof, and I did not understand how he could even pretend to be summarizing the whole argument in one phrase. Since he did not write his comment down, I even remembered it wrong as specifying a “homotopy invariant functor that vanishes on the sphere.”

To some people the previous incident might be evidence that Bott was a bad teacher. We might say, more descriptively, that Bott was a teacher who indulged himself in making deep, succinct statements which would encapsulate an entire discussion, even though he may have known very well that the statement was not immediately comprehensible to the class at large. But is this a good quality or a bad one? You may grant me that the answer depends on how many in the class find the statement comprehensible: the more the better. But I will suggest that another point of view is possible and even preferable. Namely that this behavior of Bott’s is valuable teaching even if not a single student understands the statement! In fact it is all the more valuable to the student who does not understand it, because that student is being helped the most. That student has already been taught all he can take at the moment, and is being pointed to the higher ground

which he will eventually be able to tread. That student is actually receiving instruction not just for the moment, but also for the future; he is being given something to think about which will last him some significant amount of time, and which will repay all the thought he will give to it.

In my case, several years passed before I understood Bott’s statement above. It occurred during a period when I was beginning to appreciate the difference between building a tool and using it, that is between definitions and existence theorems. The details that had obscured my vision in Bott’s first proof were the nuts and bolts of the construction of the tool, and his lightning summary contained just the key features of the finished tool. As I finally understood his comment (simultaneously correcting my memory of it) I savored fully the knowledge in it, his generosity in making the remark, and the satisfaction of finally resolving by my own efforts a puzzle of many months standing. When reading student evaluations of a teacher, how often does one encounter the grateful comment, “He really gave us some provocative questions to think about. I still have not settled them all!”? I would ask, if this comment is missing, can the teacher really be excellent? As another example, when I took Freshman calculus from John Tate (at Harvard) I was aware that his lectures were very dynamic, that his course was very difficult, that he always knew instantly the answer to any question, and that he could prove any statement in full detail apparently without even a moment’s reflection. But I did not know if he was a good teacher. At Christmas I compared notes with a friend who had gone to a well known engineering school in the South. It was immediately clear that calculus was not the same everywhere, that my course was much more demanding, and that I was being given far more by Tate than my friend was getting from his professor. I began to realize that Tate was a good teacher.

The quality of a teacher was often measured by my undergraduate student acquaintances, and more often as we became graduate students, according to the seeming quality and depth of the material being presented. The professor was praised for his reputation of possessing a distinguished vision of the subject as much as, or more than, for his facility at making it easy. It was understood that deep material cannot be made easy. In Bott’s course, for instance, his proof of the homotopy lifting property of covering spaces was sketchy and incomplete, and he seemed not to have any interest in writing out the details. However I had no trouble finding it in every book on the subject, and eventually in working it out for myself. What I could not get for myself was the grasp of the big picture, the sense that it was possible to view all these things from a perspective from which they were quite trivial, and the inspiration to achieve that perspective. A teacher can be

considered good in that sense only if he gives you something beyond what is in the books, and perhaps insists that you try to understand it. For example, how often does one encounter comments like the following in a teaching evaluation:

“She really made this course hard for me by including many points of view more sophisticated than those of the book, which afterward appeared mundane in comparison. I feel I have grown intellectually more in this quarter than ever before! Great teacher! The students in the course last year were really shortchanged by a professor who just plodded straight through the syllabus, assigning only the easier problems.” Should we expect them?

One of the moments I remember best in a course by David Mumford was when he turned to the class and remarked, “the way to read Grothendieck [author of a formidable tome on algebraic geometry, thousands of pages long] is to find the topic you want, read that section (tracing back through the pages for all the references) and understand it, then go home and write it up yourself in two pages.” This advice on how to extract information from tedious and lengthy source material is invaluable to the student who thinks he must slog through every book from the beginning.

To get some student viewpoints on these questions I asked my children, ages 14 and 17, what they thought made a good teacher:

17 year old: The teacher makes the material interesting, and knows a sufficient amount about the material to make it interesting.

14 year old: The teacher is able to convey his knowledge while keeping the material interesting.

I then asked the younger one what he would recommend as a way to find out how good a teacher someone was.

14 year old: I would go and sit in the class and find out for myself.

Me: What about passing out questionnaires to the students asking them how good the teacher was, not going to the class, but reading the questionnaires?

14 year old: That would be dumb, because you would get complaints from the students who didn't like doing the work.

17 year old: You would get a lot of students venting their frustrations because they didn't do well because they didn't study, or shouldn't have been in the course because they didn't take the prerequisites or other reasons.

It is particularly poignant to me that the ability to maintain interest in the subject is a crucial ingredient in both children's characterizations of good teaching, especially in view of serious discussions in our department, in the not too distant past, advocating dropping any attempt to measure that ability. How shall we progress beyond the minimal teaching skills associated with training people, displaying information, and instructing from a syllabus, to the deeper, more valuable ones of guidance, nurturing, illumination, and inspiration? I suggest that we begin by emphasizing that these latter qualities are actually more important than simple information transferral. My friend Steve Sigur, mathematics teacher at The Paideia School in Atlanta, has even asserted to me that there is no point in teaching only for factual content, since after one year essentially no factual content is retained! The truth of this brutal claim is verifiable by every teacher faced with verifying the presence of prerequisites in a new class. Indeed the lifetime of “learned” information often seems only weeks or days after the final, instead of one year. Therefore I suggest that advice to a new teacher include a reminder to volunteer to teach a variety of courses and to attend seminars and professional meetings, so that one's ability to inspire, enlighten, and draw connections between different topics continues to mature.

I agree emphatically with my colleague David Penney's warning that good teaching is zen-like; that there are as many potential manifestations of good teaching as there are teachers, and that we should each make use of our individual strengths. I agree too with our former department Head John Hollingsworth, that the success of the teacher in accomplishing her goal is more important than the means, and we should measure teaching by observing the subsequent trajectory of the students, rather than by exit polls from the course. He tells a story of a kangaroo court held at a well known state school to review complaints about the teaching of a certain famous but eccentric mathematics faculty member. The faculty member began by dividing the board in half by a line and writing a large number of names in one half. He was asked why. The response was that he felt it important that before reviewing of his teaching they establish their respective credentials, so he had listed on one side of the board the names of those among his former students who had been elected to the National Academy of Sciences. The other half of the board was empty to allow his inquisitors to do the same. The group quickly voted to dismiss the hearing.

I would like to add that a good teacher's goal must be one worthy of pursuit. I would even argue (to paraphrase Randall McMurphy from *One Flew Over the Cuckoo's Nest*) that to have tried and failed to accomplish a worthy goal, even one which was plainly impossible, is deserving of admiration. For those of us who lament that we do not

fully achieve our goals with all our students, I recommend (although, to his dismay, not clearly enough for his disciples).
the fourth chapter of the Gospel According to Saint Mark, where Jesus explains the importance of prerequisites

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