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Guest Editorial: Understanding Numerical Concepts and Skills—Some Differences

By Edward J. Davis

In one of the classic articles in our field entitled: "Meaning and Skill: Maintaining the Balance" the late William A. Brownell (1956/1987) argued for teaching mathematics so that students acquire both meaning (concepts) and computational proficiency (skill). This seems like a sensible goal. Why then do we still often see meaning and skill as competitive as *Pepsi* and *Coke* ? Why did the Third International Mathematics and Science Study, TIMSS, find the vast majority of time in United States mathematics classrooms being spent on computational matters (Forgione, 1996; U. S. Department of Education, 1996), while critics of the National Council of Teachers of Mathematics, NCTM *Standards* (1989) claim that skills are being ignored at the expense of exploring concepts and problems (Cheney, 1997; Ratnesar, 1997; Romberg, 1997)? Why can't concepts and skills be seen as allies—both being necessary such as carbohydrates and protein and both receiving appropriate attention in classrooms and in the *Standards* ?

Perhaps we could find common ground and avoid some unnecessary and unproductive conflicts over number skills and concepts if we heed Brownell and advance the position that our goal is for students to understand *both* concepts and skills. If we adopt Brownell's reasonable stance we should also consider that *understanding number concepts is different in some ways from understanding number skills.*

Understanding Number Concepts

Number concepts, such as division, fractions, decimals, and percent have properties and should be modeled, explored, and discussed in problem contexts before being defined and symbolized. Most number concepts are not difficult. Teacher literature abounds with hands on activities for mathematics concepts. Children can experience division by sharing and by arranging transportation in vehicles with equal capacity. Imagining various objects "cut up" into 100 parts and estimating the portion indicated by 90, 50, or 15 of these parts begins a journey with percent. Finding and naming locations on the number line (perhaps as a clothesline) between whole numbers is an engaging and enlightening context for fractions and decimals. We see ample attention—in NCTM *Standards* , addenda and journal publications for helping students to understand numerical concepts by:

- acting on models (examples),
- drawing and interpreting pictures (more examples),
- resolving and modifying story situations,
- describing and comparing their experiences (identifying properties)
- constructing analogies and,
- estimating.

The *completeness* of the knowledge students have for a concept determines the degree to which a concept is understood. The knowledge students construct is dynamic and growing and can be assessed in contexts such as 1-6 above. Understanding concepts is a student's foundation for success in mathematics. Concepts are the building blocks of mathematics and like Travelers Checks one should not leave home without (understanding) them!

Students rarely become "comfortable" with a concept in one encounter (lesson)—no matter how well constructed. Be prepared for students to need a dozen or more experiences in forming or constructing a reasonably correct concept. A reasonable understanding of a concept should occur *before* students begin to learn skills associated with the concept.

Understanding Number Skills

Number skills, to a considerable extent, include the algorithms of school mathematics such as multiplying and dividing fractions and whole numbers. Many students experience difficulty with skills. They find skills boring and formidable. In contrast to concepts, which may seem alive and growing, skills are frequently perceived as static. They can also be carried out by electronic aids more rapidly and accurately than by hand.

Estimating is important in teaching number skills. The ability to reasonably estimate the answer to a computation is a good indicator of a student's understanding of the concepts involved in the skill. *The ability to give a reasonable estimate for a computation should be prerequisite to developing skill in that computation.*

There are at least three aspects in understanding a number skill. One can understand:

1. *what* the skill is doing,
2. *how* to do the skill, and
3. *why*, or the justification, of each step.

Conceptual knowledge is the foundation for understanding *what the skill is doing* (Skemp, 1978). If a student has a healthy concept of fraction and a good concept of subtraction, then this student has a basis for subtracting fractions conceptually—with manipulatives, a model, or a drawing. On the basis of conceptual knowledge the student should also be able to estimate the answer to a reasonable degree. Then the child is ready to learn *how to do the skill* (the algorithm) rapidly and accurately. This is one place where direct teaching can be appropriate. There are three principal tools for teaching how:

- Demonstrations for and with students
- Identifying the steps, and
- Practice with quick feedback.

Motivation and variety in practice are especially welcome in learning how to do a skill. Motivation can include the notion that the skill is a shortcut for doing something students have already been doing the long way with a manipulative or model. A reasonable amount of practice distributed over time is essential in making practice effective. In contrast to the teaching of concepts, however, it appears that NCTM publications may need to spend more attention to the effective and efficient teaching of number skills. The last NCTM Yearbook (Hamrick & McKillip, 1978) that focused on number skills is now over 20 years old!

Teaching the why or justification of each step in a skill can come *after* students understand what they are doing and how to do it. This may seem surprising, but upon reflection, many of us have experienced this in our journey in mathematics. Asking students to learn both how to do the steps and why each step works at the same time puts a difficult task before them. There seems little risk in waiting a few days or longer before teaching the significance of each step. Limited research (Cooney, Hirstein, & Davis, 1981) indicates that "How" followed by "Why" is a feasible strategy for teaching number skills. The question "Why are doing it this way?" has come naturally from many of my students *after* they felt comfortable doing a skill. Student invented algorithms are natural extensions and possible explorations after the standard algorithm is learned. Student generated algorithms can then be compared and contrasted to the standard algorithm.

The strategy of separating how to do a skill and why the skill works has the advantage of not asking students to deal with two different learning tasks at the same time. It also has the advantage of investigating mathematical structure in a later context where students feel comfortable knowing what is happening and feel successful in knowing how to do it. The structure, or mathematical reasons bring a completeness and resolve the mystery of "Why did that shortcut work?" Mathematical structure can now be a friend—it is powerful and perhaps can be seen as beautiful!

So?

Concepts and skills should both be understood. *Understanding a concept involves different actions than understanding a skill* (Davis, 1978). If we respect these differences in our teaching, students may be helped in understanding both concepts and skills. While concepts take precedence over skills, understanding concepts and skills are complimentary goals. Concepts and skills are helpers not competitors. Teachers spend a great deal of time teaching number skills. Explicit attention to effective and efficient teaching of number skills could result more time being allotted to students developing number concepts.

Edward J. Davis is Professor Emeritus at the University of Georgia. He is the director of operations for the Eisenhower Professional Development Project. He has conducted a number of studies on mathematics teaching and learning. His e-mail address is edavis@coe.uga.edu

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