Contract and Custom: Two Registers of Didactical Interactions[1]

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Translated and edited[2] by Patricio Herbst

A Social Problématique

Research in *didactique* of mathematics[3] is founded on the constructivist thesis from Piaget's genetic epistemology: "The hypothesis that the subject explores actively his or her environment, and actively participates in the creation of space, time, and causality." (Inhelder & Caprona, 1985, p. 8). Likewise the subject, who for us is the student, actively participates in the construction of his mathematical knowings.[4]

These psychological bases are, however, insufficient for at least two reasons. On the one hand, the acquisition of mathematics[5] requires the development of situations that are specific to the nature and functioning of the discipline. Today it is a truism to say that it is impossible to expect the child to reconstruct mathematics spontaneously from free interactions with his or her environment: "There is no natural method for the teaching of mathematics" (Brousseau, 1972). The development of those learning situations requires, above all else, an epistemological analysis in order to determine the suitable conditions for the constitution of the meanings associated to a given mathematical notion.

On the other hand, the psychological bases are insufficient because the teaching of mathematics has a social dimension that is revealed through two sorts of constraints, one specific to the knowledge-to-be-learned itself and the other related to the organization of the knowledge taught:

- Mathematical knowledge has a very strong social status. Students must acquire items of knowledge[6] which are already known (and recognized as such) by the society at large or by some of its divisions, and which are being used by these groups. This constraint is particularly obvious when one considers the learning of notations and other means of representation that are specific to a given item of mathematical knowledge.
- Even when it is agreed that what is at issue is the learning of each individual student, the teacher also has the responsibility of ensuring the homogeneity of the construction of the items of knowledge and their coherence at the level of the class in general. Without that homogeneity, the didactical[7] functioning of the class would be impossible. As the constructivist hypothesis entails that recourse to the authoritarian imposition of a given item of knowledge is impossible, such homogenization can only be produced in social interaction. Brousseau (1972, 1981) has described the fundamental forms that this social interaction can take in the teaching of mathematics within the framework of the theory of didactic situations.[8]

These two social dimensions place the teacher in a very special position, at the junction of the system of societal knowledge and the system of the items of knowledge constructed in the classroom. And he or she has the responsibility of ensuring the adequacy of the second system in relation to the first. Moreover, the teacher has the responsibility of ensuring that the functioning of the knowledge and the intellectual productions of individuals are regulated; the teacher can do that either by direct intervention or by means of using some specific learning situations.

This article addresses the study of phenomena related to those social constraints on the didactic functioning in the context of my research on proof, and in particular on mathematical proof. Elsewhere I have reported on a study about the characterization of situations of validation. I offered an analysis of the relations between the characteristics of these situations and the nature of the processes of proof used by students (Balacheff, 1987).[9] These analyses had essentially addressed situations of validation largely isolated from the teacher, that is to say *adidactical situations*[10] in the sense of Brousseau (1984). In this paper I draw on the results of an experimental study (Balacheff, 1988) to examine the didactic constraints attached to the social characteristics of didactic situations, particularly the problem of the nature and means of their regulation.

Genesis of a Conjecture and Devolution of the Problem of Proving It

In order to illustrate my proposal, I have chosen a portion of an experimental study that I conducted with 7th grade students. The study dealt with the students' construction of the conjecture that the sum of the angles of a triangle is 180 degrees and the *devolution*[11] of the problem of proving this conjecture. The *devolution of the problem* not only means that each student makes it his or her own problem, but also that the class as a whole recognizes the problem of proving the conjecture and thereby adopts a collective responsibility. This socialization is a necessary condition for the existence of a proving debate as characteristic of a type of didactical situation; it is the classroom community that is engaged.[12] Additionally, what is understood as *conjecture* in this paper is more than a mere speculation or a statement that is only plausible: The truth of the statement must be perceived by the class as sufficiently

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interesting and problematic that a process of ascertaining it is felt to be in order by the class.

The cornerstone for the construction of this sequence is the student's conception that all measures associated with a triangle increase as the size of the triangle increases. Such a conception is known to be common among French seventh grade students. The origin of this conception can probably be found within the practices of measuring area or perimeter, and actually holds for the usual geometric forms in general. The conception leads to the following theorem-in-action:[13] "The larger a triangle, the bigger is the sum of its angles." Encountering a contradiction to this theorem-in-action will become the source for the conjecture to be constructed.[14]

The teacher is not to indicate the objective of the sequence of activities at the beginning of their implementation; If that were done, it would obviously run the risk of losing the conjectural character of the target proposition. Therefore, in designing the sequence it was necessary for us to find conditions under which the measure of the angles of a triangle and the manipulation of those numbers could be introduced without rendering it necessary to make explicit the learning objective that was being sought. At the grade level where the experience was to take place—the French equivalent to 7th grade—the lessons on angles (as prescribed in the curriculum) constituted one natural frame. Activities that involved measuring angles in a triangle could be easily introduced as a novelty because it is normal for teachers to update situations by making them a little bit more complex or by changing the context. It did not seem necessary to justify the introduction of the triangle in any other way.

Within this frame, the first activity would require the students to trace a triangle, measure its angles, and add up the results obtained. The teacher would record the sums from the class and draw a histogram on the board. Up to this point all proposed results would be acceptable and should be recorded by the teacher without noting anything special about the differences. These differences do not have a particular meaning: For example, they may appear normal for students due to the fact that the sums correspond to different triangles.

In order to differentiate among the variety of results obtained between those that relate to the uncertainty of measurement and those that relate to students' erroneous conceptions, the whole class would then need to measure the angles of the same given triangle. Student conceptions of the relation between the size of a triangle and the sum of the measures of its angles must also be called into play.

Thus, in the second activity each student would receive an identical copy of a given triangle (without special geometric characteristics), but sufficiently large in comparison with the triangles that students usually trace in order to engage the expected conceptions. The second activity would ask students to predict the sum of the measures of the angles of this triangle before determining this number. Once each student has determined the sum of the actual measurements, the teacher would record the sums that each student has obtained and note them in a histogram on the board. These results are compared with the predictions, and each student is asked to comment on the comparison between his prediction and his result; then the class is asked to comment on the histogram. The requirement that each student comment on the comparison between his or her prediction and the result obtained is intended to underscore possible differences between these two numbers. The requirement that the class comment on the histogram aims at making explicit the necessity that all the students should have obtained the same measurements for the same triangle.[15] The differences that would arise would therefore be explained by the errors associated with measurement (errors related to the instruments or the practices of measurement).

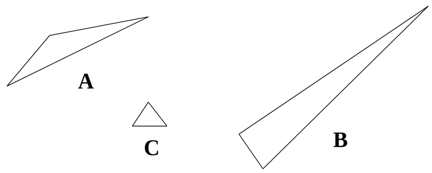
The situation thus arising would have the characteristics of a *situation of action*:[16] It would permit the engagement of conceptions that serve as models of action or of decision in the succeeding activities, and hence it would support the construction of the conjecture. Nevertheless, for the time being, any difference noted between prediction and measurements would not necessarily be seen as problematic: It could legitimately be seen as contingent, since it is related to a particular choice of triangles.

Thus far in the development of the sequence, only the conditions for the genesis of the conjecture would have been achieved:

- Student conceptions that would lead to the conjecture would have been activated and the milieu within which this genesis takes place would have been constituted;
- The uncertainty of the measurements would have been noted, making it possible to view knowing the sum of the angles of a given triangle as problematic. The disqualification of measurement as a way to determine this sum would legitimize the requirement of *intellectual proofs*[17] for the expected conjecture.

The third activity aims at formulating the problem of invariance in the sum of a triangle's angle measures; eventually it aims at enabling the emergence of the conjecture that states that this sum is equal to 180 degrees. To bring about the recognition of this invariance, it would be necessary that the students perform actual measurements and calculations for several triangles. But these experiences would not have any special meaning unless the students effectively engage their conceptions. The conjecture and the requirement of proving it should arise at the individual and at the classroom levels from a conflict between two sets of conceptions: On the one hand those that assert that the sum of the angles of a triangle depends on the shape of the triangle, and on the other hand those that support the empirical results of measurement that place this sum in the neighborhood of 180 degrees.

Three triangles, like those shown in Figure 1, are chosen for this activity, under the hypothesis that they would enhance the probability of a conflict between conceptions. On the one hand, triangles A and B are of sufficiently uncommon shapes so that one could expect that the way to predict the sums of the measures of their angles would not be obvious. Rather, it would be expected that these triangles would lead the students to nontrivial reasoning that would engage their conceptions. On the other hand, triangle C is notably smaller than the other triangles that students are likely to have experienced so far. Hence, it would probably promote predictions that the sum is much less than 180 degrees.



To support the effective engagement of conceptions and the emergence of cognitive conflict, the students' activity should be organized in teams. Each team should receive a copy of the three triangles and would be required to agree on an a priori evaluation of the sum of the angles for each triangle. The requirement of this agreed-upon prediction for the group would satisfy the theoretical conditions of a *situation of decision* in which the confrontation of viewpoints is supposed to lead to the proposal of explanations based on the conceptions of each of the members. When this activity is complete, the teacher would be expected to collect and record the results obtained in a histogram drawn on the chalkboard. The collection of results presents the opportunity for a public discussion of contradictions between the results and the predictions. This discussion is expected to underline possible contradictions and encourage the spelling out of the conceptions.

Immediately following this discussion of the predictions and results, the teacher would ask students whether they have, after examining the three histograms and in light of the comments that have been offered, any particular observations to make. Such a question might seem too open, but the requirement that each triangle must have a precise value for the sum of the magnitudes of its angles—which is implicit in the question¾makes the question meaningful. The problem of knowing this precise value must now be posed.[18]

My hypothesis is that the robustness of the initial conceptions ensures that both positions in favor and against the envisioned conjecture would have actual supporters in the classroom. The students, in that case, would be in a situation of validation, where they are bound to produce one or more proofs[19] of the conjecture or else refute it.

The design of the described sequence of activities is, therefore, based on setting up a certain type of social interaction that, eventually, would require the students to take on *responsibility for truth* and, hence, to play a game in which the teacher is to some extent disengaged from the knowings. This type of social interaction can be described by a set of rules—almost always implicit—that organizes the exchanges among the students and between students and teacher so as to permit the knowings to function in a certain form. In this case, the form in which the knowings function would be allowing the construction of a conjecture. This set of rules constitutes the didactical contract in the sense of Brousseau (1981).[20]

Custom of a Mathematics Classroom and Didactical Contract

The pertinence of a concept of custom for didactical analysis

I have observed the implementation of the sequence of activities on the sum of the angles of a triangle in many 7th grade classes. Two of these were videotaped (in March 1983 and in January 1984) and I have analyzed them in detail relying on the complete transcriptions of the dialogues (Balacheff, 1988). The two videotaped classes—referred as D and E—present a particularly striking contrast of practices. Important differences between the two classes were noted in the negotiation of the didactical contract specific to the situation. In class E this negotiation faced an obstacle to be described below which I had previously interpreted as resulting from the initial "standing" contract for that class.

The differences between the classes were essentially noted with respect to some rules of social functioning that are described in the sequel. These rules concerned mathematical activity and had been made explicit in class E but not in class D. In fact, due to their very general legislative character, those rules no longer appear to me as belonging to the didactical contract but to a deeper and more enduring order. The distinction between customary society and legal society seemed to be a good model to account for and analyze the difficulties encountered in the negotiation of a didactical contract in class E and, consequently, the difficulties associated with the devolution of the problem.

The classroom is a society of customs. *Custom* is understood here as a set of obligatory practices (Carbonnier, 1971) established as such by their use, and which, in the majority of the cases, is established implicitly. *Custom* regulates the way in which the social group expects to establish relationships and interactions among its members and, therefore, it is initially characterized as a product of social practices. Within this framework, some sociologists of law consider the explicit formulation of rules as one of the mechanisms that allow a society of customs to become a legal society:

> What makes [customary and legal societies] different is a technical fact: As we have seen, while custom is spontaneous and 'unconscious,' law emanates from a specialized organism and through a procedure that within our present societies receives the name of *promulgation*. (Levy-Bruhl, 1964, p. 55, my italics)

Certain properties of the didactical contract (Chevallard, 1983, p. 11) can be reformulated mutatis mutandis so as to characterize custom. For example, in the same way as the didactical contract becomes visible when it is broken, "one only realizes the existence [of custom] when it produces its effects" (Levy-Bruhl, 1964, p. 44).

However, the concept of contract as it has been described in the literature up to now is insufficient to account for the complete set of social phenomena that regulate the functioning of knowledge in a class. For example, the envisioned sequence of devolutions to the students and of resumptions by the teacher of the "responsibility for truth" in the case of the sum of the angles of a triangle, suggests a sequence of contracts that could imply an instability that is not confirmed in the observation of actual classes. Moreover, the problématique of the contract leads one to ask about its origin, to identify the parts that hold the contract, to determine the place and moment of its negotiation, to estimate its duration. To the question "when … who… has established the contract … instituted the rules?" Chevallard responds: "It is always already done and gone. (…) One subscribes to the contract when one enters into the kind of social relationships that the contract regulates" (1983, p. 11). The concept of custom is much better adapted to account for the modes of regulation of the social functioning in the class and at the same time it may circumscribe the domain of validity of the concept of didactical contract. It may also be possible that this differentiation between contract (a notion that I see more as having a local character and being a key element in the process of devolution) and custom (a notion that I see as regulating the social functioning of a given class across time) will permit us to do away with all the "misfortunes of the concept of didactical contract" (Brousseau, 1984, p. 45).

Insofar as it is *produced in and by practice*, the custom of a mathematics class is specific to the knowledge taught in that class. This is clear, for example, with respect to what constitutes a proof or the definition of "true." I refer to "*a* class" and not to " *the* class" because custom is specific to each level of schooling. Thus, it is likely that the custom of the 7th grade class will be different from that of the 8th grade class. In France, mathematical proof is among the practices of the 8th grade class. But before becoming a mathematical object or a mathematical tool, mathematical proof is, for a 7th grade student, this new thing that will have to be done and to which the student will have to be initiated. This leads us not so much to pose the question of the origins of the custom as much as that of the initiation of students to the custom of the class in which they enter. An initiation into a sort of system of new "rights and duties [of subscribers to an agreement] within the frame of a shared reference" (Chevallard, 1983, p. 11).

In contrast, the didactical contract has a local character. It is negotiated for a particular task that requires the rules for the social functioning of the class to be defined locally and in a new way. For example, the devolution to the class of the responsibility for truth in the solution of a problem where the student agrees to play a game with restrictions or artificial characteristics that are nevertheless necessary for a certain functioning of the knowledge (like the situations of communication aforementioned or the impossibility of using some material means). Custom matters at the moment of negotiation of the didactical contract, in particular in determining what is negotiable and what is not. When the contract vanishes, the class comes back to its usual custom.

Agreed-Upon Response and Knowing

In the implementation of the sequence described above, I observed with some regularity (across the various 7th grade classes) the emergence of a norm, from the moment of the analysis of the first measurements throughout the first activity. This norm excludes measurements that lie far outside the interval [170°, 190°], so that the announcements of measurements such as 231° would provoke the amusement of the class in general. The emergence of a norm is confirmed by the increase of predictions of 180° in the second activity, and moreover by the immense majority of effected calculations that happen to yield 180°. This last indicator is the most relevant because it entails that there had been a correction "towards 180°" of results initially obtained.[21] This does not mean, however, that the students had acquired the idea that the sum of the angles of any triangle would be 180°. For example, in class D, 21 students from a total of 25 "obtained" 180° for the proposed triangle, but at the time of the third activity and after discussion, some of them (12 students) stuck to a different value for one of the three triangles. The conformism that is at the origin of the first responses is essentially fragile because it is reached under the initial imposition of producing "agreed-upon" responses or assumed so by the students. But conformism by itself cannot generate true knowings and thus it does not allow the student to face a situation outside the frame in which the agreement has been reached.

The emergence of a norm, phenomenon that was observed in all classes, is probably not avoidable. The source of the students' identification of "acceptable" responses after the first activity is most likely the effect of an expectation that every classroom activity entails the existence of expected behaviors or responses, reinforced by "an ardent and unshakable desire to be in accord with the group" which Watzlawick (1987, p. 88-95) has shown leads to conformist behaviors.

I suggest that principles of this level of generality (such as the injunction to find an agreed-upon solution so as to be in accord with the group) which are involved in regulating the activities of the class (as seen in the correction of measures toward 180 degrees) should be considered within the domain of the custom. Accordingly, I see custom operating in the way that Chevallard reserved for the contract, i. e., custom "operates as a code, a generating principle for some behaviors that are by the same token defined as orthodox" (Chevallard, 1983, p. 11).

An Impossible Legal Society

One of the main misfortunes of the notion of didactical contract as a fundamental concept for didactique seems to be the idea that one should be able to make explicit as thoroughly as possible a "good contract." Such a "good contract" would be made explicit to the students as defining what is accepted with regard to their rights and duties, "a sort of permanent object that should have been agreed upon" (Brousseau, 1984). But as Brousseau remarks, the notion of didactical contract has nothing to do with that idea of a "good contract", because the didactical contract is always (of necessity) invisible (p. 45). Everything happens as if there was a contract, but that contract can never be agreed upon and whenever it is effectively agreed upon, it cannot be enforced.

The model of custom should allow one to understand and explain what could appear to be a paradox of the contract: the necessary implicitness of its reality and the necessity of its failing when spelled out. This failure highlights the fact that the spelling out of the rules changes the nature of the social interactions in the classroom. Just as occurs in the passage from a customary society to a legal society, the transformation is more than a simple technical adjustment. It changes behaviors and, therefore, the meaning of the items of knowledge constructed in the new frame are also changed.

My observations also show with relative clarity that the proposed situations, and hence the meaning of the behaviors and productions of the students, are very sensitive to what could be identified as the custom of the class in which these situations are introduced. I had not envisioned such a central role for custom a priori.

In class E, the rules for debate had been explicitly adopted. The students would state propositions that would be written on the chalkboard in order to prove or refute them; if a statement was proposed it was supposed to be sufficiently general and interesting; if something was given as proof, it should be recognized as convincing; it was not improper for somebody to stick to the position of the minority if unconvinced; etc. (See Capponi, 1985, for the characterization of the general principles of this organization.) Even though these rules permit the regulation of social interaction, the rules themselves are a source of difficulty for implementing of a genuine situation of proof. In particular, a debate in which the proponents can contradict each other is possible and, to a large extent, legitimate by the explicit obligation to convince, but this, together with the legality of holding a solitary position regarding what is true, opens up the possibility of refusing to be convinced. The spelling out of the rules has the possible (and sometimes essential) consequence of causing the emergence of a juridical void. In this case, the absence of rules that would oblige one to admit to having been convinced.

Moreover, this legislation does not operate unless it is kept under vigilance; Laws make judges necessary. To satisfy the law might mean, in the first place, to satisfy the judge. In this case, the existence of explicit rules, even though envisioning a "scientific" way of working, could provoke a drift toward the production of proofs or arguments according to their capacity to satisfy the teacher instead of the intrinsic requirements of the problem of validity. And this could occur in spite of the injunctions to autonomy, since "the class" is not able to decide by itself on the conformity of its own behaviors with the rules that have been decreed. The class is not able to bypass the double role of the teacher as mediator and arbiter.

In any class, the teacher is always and completely engaged in the situation; in this way, as Brousseau (1984) remarks, the teacher is and remains accountable (p. 4). I have observed that students may hold positions that are very different with respect to the validity of the property being addressed: There are those that know it already, those that reject it, those that doubt about it, those that would be willing to adopt the opinion of the majority, those that are not interested. In opposition to a group of students that hold the view that the sum of the angles of a triangle is invariant, another group of students could claim that there exists a particular case that does not verify the property, even if they are not able to show which case that is. But this problématique of proof and refutation cannot develop if a very small minority holds one of the two positions: They must be defended in a sufficiently balanced way so that an authentic dialectic of validation will occur.[22] For this to occur, the teacher is responsible for encouraging the supporters of each position equally.

The "truth" thus becomes problematic not because the teacher does not take a position, but because the teacher appears ready to support or provide a status both to the thesis and the antithesis. The teacher provides a certain status to uncertainty and at the same time underscores how relevant it would be "to know." This ensures that the statement at stake is not just a mere speculation but a conjecture. But this role for the teacher can not be assumed by the teacher unless "the class" accepts it. The students know that the teacher knows the answer, as usual, but the negotiated contract allows the teacher to retreat from her or his usual responsibilities of imparting knowledge. However, at the end of this contract, everyone will recover his or her position in relation to knowledge; the process of institutionalization will indicate this has occurred. The class, and the teacher with it, will return to ordinary custom until the next contract needs to be negotiated so as to let new knowings be put at work.

As a product of practices, custom evolves with those practices. Custom is modified as long as learning progresses and new mathematical practices appear. Some customary rules will become obsolete during the school year, but also, and more radically, the negotiation of the didactical contract against the custom can create ruptures that boost the meaning of a knowing by uncovering certain rules and their relation to knowledge. The model of the contract and the custom provides a framework for describing and explaining both the dynamic character of social interactions in class inasmuch as they are related to knowledge, and the stability and permanence of those social interactions that are indispensable for the functioning of the didactic system.

References

Balacheff, N. (1987). Processus de preuve et situations de validation [Processes of proof and situations of validation]. *Educational Studies in Mathematics, 18*, 147-176.

Balacheff, N. (1988a). Une etude des processus de preuve en mathématiques chez des eleves de college. [A study of the proof processes in mathematics of the college students]. Doctoral dissertation. Grenoble: Université Joseph Fourier.

Balacheff, N. (1988b). Aspects of proof in pupils' practice of school mathematics. In D. Pimm (Ed.), *Mathematics, teachers, and children* (pp. 216-235). London: Hodder and Stroughton.

Balacheff, N. (1990). Towards a problématique for research on mathematics teaching. *Journal for Research in Mathematics Education, 21*, 258-272.

Brousseau, G. (1972). *Processus de mathématisation La mathematique a l'ecole elementaire.* [Processes of mathematization. The mathematics of the elementary school]. Paris: Association de Professeurs des Mathématiques de l'Enseignement Publique.

Brousseau, G. (1981). Problèmes de didactique des décimaux [Didactical problems with decimals]. *Recherches en Didactique des Mathématiques, 2*, 37-127.

Brousseau, G. (1983). Etude de questions d'enseignement. Un exemple: La géometrie. [Analysis of teaching questions. An example: Geometry] *Séminaire de didactique des mathématiques et de l'informatique*, 45. Grenoble: IMAG.

Brousseau, G. (1984a). *Le rôle du maître et l'institutionnalisation* [The role of the teacher in the institutionalization]. Actes de la III Ecole d'Ete de didactique des mathématiques. Grenoble: IMAG.

Brousseau, G. (1997). *Theory of didactical situations in mathematics: Didactique des mathématiques 1970-1990*. (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Trans. & Eds.). Dordrecht, The Netherlands: Kluwer.

Capponi, M. T. et al. (1985). Apprentissage du raissonement [The learning of reasoning]. Grenoble: IREM de Grenoble.

Carbonnier, J. (1971). *Flexible droit: Textes pour une sociologie du droit sans rigueur* [Flexible right: texts for a sociology of right without rigor]. Paris: Pichon & Durand-Anzias.

Chevallard, Y. (1983). *Remarques sur la notion de contrat didactique* [Remarks on the notion of didactical contract] Marseille: IREM d'Aix-Marseille.

Inhelder, B. & de Caprona, D. (1985). *Constructivisme et creation de noveautés* [Constructivism and creation of novelties]. Archives de Psychologie, 53 (204), 7-17.

Kieran, C. (1998). Complexity and insight—A review of *Theory of didactical situations in mathematics: Didactique des mathématiques 1970-1990*—Guy Brousseau. *Journal for Research in Mathematics Education, 29*, 595-601.

Levy-Bruhl, H. (1964). *Sociologie du droit* [Sociology of right]. Paris: Presses Universitaires de France.

Watzlawick, P. (1978). *La realité de la realité* [The reality of reality]. Paris: Seuil.

Vergnaud, G. (1996). The theory of conceptual fields. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp.219-239). Mahwah, NJ: Erlbaum.

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Notes

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[2] The editorial notes (EN) provide additional references and clarification of the usage of certain terms for the English-speaking audience. Translation notes (TN) are included as well. Otherwise, the structure and content of the original paper has been kept as it was in 1988. It is expected that forthcoming pieces in English will further clarify some conceptual aspects of the notions of contract and custom that are alluded here. Many thanks to David Pimm, Jack Smith, and most especially to Janet Barnett for useful conceptual and editorial comments on a previous draft of the translation.

[3] *Didactique of mathematics* is the "science of the communication of [mathematical] knowings and their transformations: an experimental epistemology that aims at theorizing the production and circulation of knowledge in a similar way as economics studies the production and distribution of material goods" (Brousseau, 1990, p. 260, Herbst's translation). To avoid the unfortunate associations with the English pejorative adjetive *didactic*, the original French noun *didactique* is adopted as a neologism. (EN)

[4] *Knowing* is hereafter used as a noun to translate the French *connaissance*, whereas *knowledge* is reserved to translate the French *savoir*. *Knowings* are the results of the cognitive adaptation of a subject to a milieu. The interaction provides for the situated nature of the knowings—in terms of contextualization, personalization, and temporalization (see Herbst, 1998, pp. 34-38). Balacheff, Cooper, Sutherland, and Warfield (in Brousseau, 1997) say that "[knowings] refer to intellectual cognitive constructs, more often than not unconscious; [knowledge] refers to socially shared and recognized cognitive constructs, which must be made explicit" (p. 72). (TN)

[5] To bypass fashionable ideological disputes about *acquisition* and *mathematics*, it is hereafter assumed that mathematics education as a social project at least demands the students' acquisition of mathematics—understood in a broad sense as the practices and discourse historically developed by working mathematicians. (EN)

[6] An *item of knowledge* is the chosen translation for the French *(un) savoir*, and its plural, *items*, for *(des) savoirs*. *Knowledge* is used to translate *(le) savoir*. (TN)

[7] The didactical functioning of the class is the functioning of the class as a system whose goal is for the teacher to transmit and for the students to acquire some culturally recognizable items of knowledge. (EN)

[8] See Brousseau (1997). (EN)

[9] See also Balacheff (1988b). (EN)

[10] An adidactical situation is one in which the student is enabled to use some knowings to solve a problem "without appealing to didactical reasoning [and] in the absence of any intentional direction [from the teacher]." (Brousseau, 1997, p. 30; see also Kieran, 1998). (EN)

[11] The notion of *devolution* is taken from Brousseau (1997) to mean "the act by which the teacher makes the student accept the responsibility for an (adidactical) learning situation or for a problem, and accepts the consequences of this transfer of this responsibility" (p. 230). (EN)

[12] The classroom community is the community engaged in debating the common system of validation against which a given explanation will be compared to decide whether this explanation counts as proof. "We call *proof* an explanation accepted by a given community at a given moment of time. The decision to accept it can be the object of a debate whose principal objective is to determine a common system of validation for the speakers" (Balacheff, 1987, p. 147, Herbst's translation). (EN)

[13]The notion of theorem-in-action is taken from Vergnaud's (1996) theory of conceptual fields. "A theorem-in-action is a proposition that is held to be true by the individual subject for a certain range of the situation variables (…). The scope of validity of a theorem-in-action can be different from the real theorem (…). A theorem-in-action can be false." (p. 225). (EN)

[14]See also Balacheff (1990). (EN)

[15]The histogram is neither an instrument for the correction of actual individual measures nor a way to produce a correct measure by aggregation of those individual measures. Rather, the practice of producing a histogram is a condition that permits all independent (individual) measuring practices to be associated into a single practice (that of measuring the same triangle), hence enabling the emergence of the problematic character of such measuring. (EN)

[16]See Brousseau (1997) pp. 65-66. (EN)

[17]For an explanation of *intellectual* proofs in contrast with *pragmatic* proofs see Balacheff (1987, 1988b). (EN)

[18]Recall that the class must pose the problem. This open question could create the opportunity for the problem to be posed at this point. (EN)

[19]*Proof* is the chosen English translation for the French *preuve* and corresponds to the acceptance of an explanation by a community (see note 15). Balacheff (1987) addresses the distinction between proofs (Fr. preuves) and mathematical proofs (Fr. demonstrations) as follows: "Within the mathematical community, only those explanations that adopt a particular form can be accepted as proof. They are sequences of statements organized according to determined rules: A statement is either known to be true or deduced from those that precede it using a rule of deduction from a set of well-defined rules. We call these sorts of proofs *mathematical proofs*" (p. 148). (EN)

[20]The didactical contract is the "system of reciprocal obligations [between teacher and student that] resembles a contract [and] which is specific to the 'content', the target mathematical knowledge." (Brousseau, 1997, pp. 31-32). (EN)

[21]The emergence of a norm is a rational hypothesis that, insofar as it entails that actual measurements would be "corrected toward 180º," helps the researcher understand why, in fact, the announcement of 231º would be deemed amusing by the students and why the bets on 180º would increase. (EN)

[22]See Brousseau (1997) pp. 69-72. (EN)

PRIME—Partnerships and Reform in Mathematics Education: A Preservice Mathematics Teacher Education Model

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Project PRIME is a University of Georgia teacher education project funded by the Eisenhower Professional Development Program. Project PRIME has as its mission to establish and develop new partnerships with mathematics teachers and schools that will improve the preparation of secondary mathematics teachers and enhance the professional life of cooperating teachers who mentor students from the University of Georgia. PRIME is under the direction of Dr. Patricia S. Wilson. The purpose of PRIME is to provide the best field experience for pre-service and mentor teachers by enhancing professional relationships between them, and providing mentor teachers opportunities to dialogue with one another and a knowledge base to insure a consistent experience for preservice teachers.

Often the best source in the development of a professional identity is a colleague or other professional. A PRIME mentor teacher serves as a lifeline for the preservice teacher seeking to become an effective educator. Mentors are committed to providing support to potential teachers and motivated by dedication to the profession, caring attitude, and engagement to professional growth. The roles of a PRIME mentor teacher include that of being an advocate, a consultant, and a confidential support provider. A PRIME mentor teacher is an advocate by establishing credibility with faculty, students, and parents, encouraging involvement in extracurricular activities (for example, clubs, athletic events, school improvement team, SST committee), and acquainting preservice teacher with standards of professional and personal conduct.

A PRIME mentor teacher is a consultant who models appropriate instructional methods, practices, and approaches to classroom management, assessment, planning, and self-assessment as envisioned by the National Council of Teachers of Mathematics. He or she acts as a resource person sharing materials, experiences, knowledge, and insights gained throughout the teaching experience, guiding in the design and implementation of teaching plans, giving constructive guidance in analyzing teaching behavior, and encouraging the revision of instruction based on appropriate evaluation procedures.

A PRIME mentor teacher is a confidential support provider by developing a sense of trust, by encouraging open communication, listening to concerns and ideas, and promoting a time for reflection, evaluation and the development of a personal teaching philosophy and practices.

PRIME Mentor Teachers will enjoy several benefits. First, being a PRIME Mentor Teacher will allow professional growth that will be stimulated by establishing and promoting collegiality and personal reflection on practice and philosophy, sharing of insights into teaching and learning, and providing opportunities to contribute to new professionals.

Second, PRIME Mentor teachers need to interact with university personnel and student teachers on a continual basis both on and off the school campus. Therefore, administrators should work with PRIME mentor teachers in their schools to arrange needed release time from classes or exclusion from routine time-consuming duties, such as duty hall.

Third, in order to enable PRIME mentor teachers to keep abreast of changes in mathematics education, Project PRIME will facilitate participation in courses, workshops, or seminars as needed. Courses that could be offered deal with the appropriate use of technology, expertise in grant writing, Teacher Support Specialist programs, and how to improve instruction using teacher evaluation instruments. For participation in these courses, teachers could be

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