Snapshots from Mathematics Education in Sweden Thomas Lingefjärd

Sweden, like many other western countries, has undergone dramatic changes in the way it prepares its mathematics teachers. The training of prospective mathematics teachers in Sweden was drastically changed during the latter parts of the 1980s. Until then, Sweden had three categories of teachers. There were elementary teachers who taught most subjects in grades 1-3 and middle school teachers who taught most subjects in grades 4-6. These two groups of teachers were generalists, often without any specialization-especially not in mathematics. A five- or ten-week course in mathematics was equivalent to a specialization. The third category consisted of secondary teachers for grades 7-9 or the gymnasium (grades 10-12), many of whom had a masters in mathematics or physics.

The prerequisites for entering the elementary or middle school teacher programs were historically very modest—in fact, the students could come from gymnasium programs where there was only a very limited amount of mathematics involved. On the other hand, the students who wanted to study in the secondary teacher program were forced to choose the natural science program (a program with an emphasis on mathematics, physics and chemistry) in the gymnasium. These two traditions—the seminar tradition, that trained the elementary and middle school teachers in small colleges and the academic tradition, that trained the secondary teachers at the universities—caused a distinctive gap between the two groups of teachers.

The students in the compulsory school (i.e. grade 1-12) normally experienced this distinction when entering grade 7. Coming from 6 years of school experience with (mostly) the same teacher for the first 3 years and then another teacher for most subjects the next 3 years, many students entering grade 7 experienced something often described as a shock. They had to give up the safety of a classroom of their own, where they were taught most subjects by one teacher, and enter a school situation with maybe a dozen or more teachers, all teaching different subjects in different classrooms. The students were also for the first time experiencing new subjects like physics, chemistry, and biology, just to mention a few, and those subjects were all taught by teachers who had specialized in these subjects. It was like going from one world to another, with teachers in the two worlds not understanding each other's teaching situations and not easily communicating with each other.

To equalize the preparation of teachers aiming at different levels in the compulsory school (grades 1-12), Sweden decided in the late 1980s that all teachers

Thomas Lingefjärd is a doctoral candidate of the University of Georgia and is a senior lecturer in mathematics and mathematics education in the department of Education, Section for Mathematics at the University of Gothenburg. He has been involved in several educational reform initiatives in Sweden. His dissertation focuses on college students' use of technology in mathematical modeling. His e-mail address is Thomas.Lingefjard@ped.gu.se. should meet the same prerequisites. The concept of elementary and middle schoolteachers was abandoned. Instead, two new categories were created, namely, teachers for grades 1-7 or grades 4-9. Three branches were offered: mathematics and natural science, Swedish language and social science, and English and foreign language. It meant that no student could enter a teacher-training program to become a mathematics and natural science teacher without first graduating from the natural science program at the gymnasium. This gymnasium program contains mathematics throughout all its three years. Areas like algebra, functions, trigonometry, derivatives, integrals and statistics are covered. As a result, prospective mathematics and science teachers in programs for grades 1-7, 4-9, or the gymnasium all enter the program with the same background in school mathematics and may even take the same introductory courses at the university level. Therefore they also have the possibility to change the direction during their program. They could, for example, enter the program for grades 4-9 and then switch to the grades 10-2 program if they do it early enough in their studies. The new policy also gives the University of Gothenburg a fairly solid body of students who enter the teacher education program for mathematics and natural science.

It is clear that the mathematics teachers for grades 4-9 will teach more advanced mathematics in grades 8 and 9 compared to their future colleagues studying in the program for grades 1-7. It is also clear that both will teach substantially less mathematics than those who study for a gymnasium teacher diploma. As a result, the amount of mathematics taken at the university level varies. It is worth noting that the students in the mathematics and natural science program for grades 1-7 at the University of Gothenburg take college courses in abstract algebra, Euclidean geometry, statistics, and advanced problem solving with the support of computer packages like Microsoft Excel and the Geometer's Sketchpad.

The students in the new compulsory school (increasing in number for every year) will experience a very different school situation. In the model school situation, not yet a reality in many schools, students will have three to five teachers for their first 4 or 5 school years. This group of teachers will consist of at least one mathematics and natural science teacher, one Swedish and social science teacher, and one English and foreign language teacher. Normally, one physical training teacher and one crafts teacher will also be a part of the team. All teachers will be specialized in their subjects, with the knowledge of how the subject continues through the school years. In order to remove the difficult transition from grade 6 to grade 7 (when Swedish students are age 13 or 14), a flexible and smooth "take over" will be organized. The take over begins by organizing the teaching so that the teacher for grades 1-7 and the teacher for grades 4-9 both meet and teach the class for some time. In that way, no

dramatic changes are expected between grades 6 and 7, and the teachers will have a mutual responsibility for all 9 years of mathematics (which is actually 75% or more of all the mathematics a student takes in grades 1-12). Students will no longer experience the "culture shock" experienced by students of the past.

During the 10 or so years that we have been educating teachers this way, the schools and the "old teachers" have changed from resistance to willing acceptance of the "new teachers," and today about 40% of the teacher population for grades 1-9 has been changed. The change has also created a huge need for special inservice training, so-called further education programs, for the elementary and middle school teachers who have more than 15 years left before retirement. At the University of Gothenburg, we run two such programs, one local and one via distance learning. All the students in these two programs are experienced teachers with little or no training in mathematics. Many of them must start with studies of the gymnasium mathematics they lack and then take courses in college mathematics. Because many of these teachers teach during the day, teaching them advanced mathematics requires special sensitivity in both instruction and evaluation. In order to help these teachers experience mathematics at this level as both interesting and challenging, we have developed a variety of special mathematics courses in what we call a *didactical mathematical theory*.¹ It should be mentioned that these courses nowadays are an integral part of the regular program for prospective mathematics teachers at the University of Gothenburg.

The didactical mathematical theory is built upon the difference between how students build their mathematical knowledge and how mathematical theory is built. The latter serves to consolidate and give a general description of the mathematics we know, as well as to serve as a basis for the development of the subject. The didactical mathematical theory, on the other hand, should be an instrument to support the teacher's need to understand children's ideas about mathematics as well as to describe the way children build their knowledge out of intuitive beliefs and experiences. It should serve as a foundation for understanding which domains of school mathematics are possible to construct and which are necessary to define. The didactical mathematical theory should, first of all, be grounded in research about how children and students learn mathematics, as well as their beliefs and ideas about mathematics.

As it seems, Sweden has strengthened the mathematical knowledge within the teacher population for grades 1 to 6. It is unclear whether or not the fact that teachers for grades 7 to 9 also are educated for grades 4 to 6 will weaken their ability to teach advanced mathematics. The strong policy of including the "nursery" (i. e. be more concerned about the children's general well-being than their progress in different subjects) in the elementary school will be changed to a more subject-driven school. How will this affect the students? On the other hand, students will now stay with a small group of teachers through the first 9 years, an idea that seems to be common in the Nordic countries but not in other parts of the world. Maybe the next international study will tell us if Sweden has chosen the right track or not, although I am aware that extensive research will be needed in order to answer this question.

> Thomas Lingefjärd University of Gothenburg Department of Mathematics Education

Reference

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15* (2), 4-14.

Note

¹ Which in the U. S. is called pedagogical content knowledge (Shulman, 1987, pp. 9-10). A direct translation of the Swedish terms would be "didactical content theory."

News From the National Council of Teachers of Mathematics

Call for papers

For the 2001 NCTM Yearbook, *The Roles of Representation in School Mathematics*. The book will be edited by A Couco, senior scientist and codirector of the Mathematics Initiative at Education Development Center, Newton Massachusetts. The goal of the yearbook is to create a forum for current thinking and practice related to the representations (such as graphs, tables, diagrams, manipulatives, equations, and so forth) used to record and communicate mathematical ideas, and for developing mathematical thinking throughout the grades K-14. In particular the editorial panel is interested in papers addressing instructional implications for using various representations of mathematical ideas and contexts, the use and construction of symbol systems in school mathematics, and the tools students use for thinking mathematically. The panel also welcomes brief articles on classroom experiences with representations that reflect cultural, affective, and linguistic influences. To obtain authon guidelines write to General Editor Frances R. Curcio, Department of Teaching and Learning, School of education New York University, 239 Green Street, Mail Code 4741, Washington Square, New York, NY 10003 USA; e-mail frcl@is2.my.edu.