Interpreting Teachers' Movement toward Reform in Mathematics[1]

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Call for Change

American classrooms are notoriously dominated by the teacher-centered activities of explaining and lecturing (Goodlad, 1984, p. 105) which often leads to learning by imitation. Recent calls for reform such as those advocated by the Mathematical Sciences Education Board [MSEB] (1989) and the National Council of Teachers of Mathematics [NCTM] (1989, 1991, 1995, 2000), however, have called for instructional approaches that move away from learning by imitation and toward student mathematical understanding that is conceptually based. MSEB (1989) asserts that "students simply do not retain for long what they learn by imitation from lectures" (p. 57). NCTM (1989) claims that "knowing mathematics is doing mathematics," and that "instruction should persistently emphasize 'doing' rather than 'knowing that'" (p. 7). According to the NCTM *Standards*, doing mathematics requires students to examine, explore, communicate, conjecture, reason, and argue.

This increased emphasis on student understanding has led to an increased interest in assessment (NCTM, 1995; MSEB, 1993). Recent journal articles and books have provided rationales for using alternative assessment along with practical suggestions on how to incorporate alternative assessment into the classroom (e.g., Hancock, 1995; Lambdin, Kehle, & Preston, 1996; Moon & Schulman, 1995; Kulm, 1994; Stenmark, 1991; and Webb & Coxford, 1993). Specifically, the use of open-ended questions has received considerable attention. For example, Moon and Schulman (1995) write,

> Open-ended problems often require students to explain their thinking and thus allow teachers to gain insights into their learning styles, the 'holes' in their understanding, the language they use to describe mathematical ideas, and their interpretations of mathematical situations. (p. 30)

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Similarly, Hancock (1995) noted that "because of the wider range of solution methods they allow students, open-ended questions are thought to be better at revealing students' thinking" (p. 496).

Issues related to the use of alternative assessment or open-ended tasks or items are complex but focus on the necessity for instruction and assessment to be aligned and mutually reinforcing. If teachers change their instruction to move toward NCTM's vision for school mathematics, their assessment practices must change as well. Conversely, changes in assessment can promote changes in instruction (Kulm, 1993). Because teachers are the agents of change in the mathematics classroom, attempts to implement reform must involve them in an integral way if such attempts are to be successful. The purpose of this article is to describe one approach to systemic assessment reform in a large suburban county.

The Projects

Description

Middle and high school mathematics teachers participated in professional development projects whose goals were to train teachers to create and use alternative assessment items. The participants taught in a large, suburban school district in Georgia where the students were diverse in terms of their ethnicity and socioeconomic status. The first project was conducted from April 1997 to June 1998 and involved 30 Algebra I and Geometry teachers; the second project began in April 1998, involved 26 Pre-algebra teachers, and concluded in June, 1999. The two projects shared a common structural approach and differed only in content. The 4-5 spring training sessions, each approximately 2 hours, included analyses of the rationale for and characteristics of alternative assessment, the development of strategies for writing open-ended items and projects, and analyses of students' responses to open-ended items. During the respective summers, teachers immersed themselves in the writing and editing of open-ended items that addressed the Algebra I, Geometry, or Pre-algebra objectives developed by the school district. The Algebra I and Geometry items were field-tested during the 1997-1998 school year. Project teachers examined student responses to the items and recommended revisions or deletions as deemed appropriate. The items were then evaluated by a research mathematician for (a) the quality of mathematics involved, (b) correct use of mathematical language and symbols, (c) clarity, and (d) appropriate use of technology. Based on comments from the project teachers and the mathematician, final revisions and deletions were made and the items were entered into a searchable database. In June1998, the school district provided training to all Algebra I and Geometry teachers on how to use the open-ended item bank in their lessons and in their evaluation of students. In June 1999, after the items were added to the database, similar training was provided for the county's Pre-algebra teachers.

> Describe where you can find a real-world example of each term listed. Sketch each of your examples.
>
> line segment — parallel lines
> plane — obtuse angle
> perpendicular lines

Figure 1. An item that was deleted from the bank.

Original Item

José notices the following pattern:

$$\frac{1}{2}-\frac{1}{3}=\frac{1}{6} \text{ and } (2)(3)=6$$
$$\frac{1}{3}-\frac{1}{4}=\frac{1}{12} \text{ and } (3)(4)=12$$
$$\frac{1}{4}-\frac{1}{5}=\frac{1}{20} \text{ and } (4)(5)=20$$

He creates the following rule, assuming neither *a* nor *b* equals zero: $\frac{1}{a}-\frac{1}{b}=\frac{1}{ab}$.

Is his rule correct? Why or why not?

Revised Item

José notices the following pattern:

$$\frac{1}{2}-\frac{1}{3}=\frac{1}{6} \qquad \frac{1}{3}-\frac{1}{4}=\frac{1}{12} \qquad \frac{1}{4}-\frac{1}{5}=\frac{1}{20}$$

Generate two additional equations that follow this same pattern and explain why your equations demonstrate the pattern.

Figure 2. A revision based on teacher feedback

Item Generation

During the review process, the authors made the decision to delete, substantially revise, or edit items based on student responses and feedback provided by the project teachers and the mathematician. The following paragraphs showcase examples of this review process.

Items were evaluated by criteria taken from Cooney, Badger, and Wilson (1993). These criteria require the item to involve significant mathematics, be solved in a variety of ways, elicit a range of responses, and require students to communicate. The item in figure 1 was deleted from the bank because project teachers did not think it involved significant mathematics. Responses to this item revealed that it was trivial for students.

The item in figure 2 was revised as a result of the project teachers' inspection of student responses. The intent of the original item was to encourage students to investigate the conditions under which the given rule was valid. Responses indicated that students either were unclear about the task or were unable to provide appropriate answers. Although the authors view the original item as involving significant mathematics, it was determined that the project teachers' revised item was more likely to be understood by their students.

Some items were edited simply to enhance clarity. Consider the revision in figure 3, which we felt would be clearer for students and would allow them to devise their own means of justifying their response—which may or may not involve the generation of examples.

Original Item

Two drink containers have equal surface areas. If both are filled completely, do you think it is possible for them to hold different amounts of liquid? Explain why or why not. Give examples to support your answer.

Revised Item

Two drink containers have equal surface areas. If both are filled completely, is it possible for them to hold different amounts of liquid? Justify your answer.

Figure 3. A revision made to enhance clarity.

Teachers' Reactions

Surveys and interviews were conducted throughout the project to determine teachers' reactions toward the project and their views of assessment. When asked about their participation in the training sessions, 95%

of the teachers rated their experience as excellent or very good (excellent to poor, 5 point Likert scale). Algebra I and Geometry teachers' written comments included "I look forward to using the material next year and sharing it with others" and "this project gets one thinking about the concepts of mathematics, not just the rules." After field-testing the items, typical comments from the Algebra I and Geometry teachers were "this project helped me assess better in my classroom" and "I can sense a difference with my instruction and listening to the students."

During the 1998 summer writing session, Pre-algebra teachers were asked to express their views about alternative assessment. Many teachers were excited about the idea of having a resource for open-ended questions; they reported attempts to include these types of questions in their assessment of students but found it "difficult to do." One participant felt that the open-ended item bank was "just the best idea there ever was." Another teacher said that it would be difficult for students "to explain something in writing" but thought that it was "good for them and their future … not just being so rote about a lot of things."

Teachers voiced several common concerns about alternative assessment, including time management and grading. They were concerned about the time needed for students to respond to open-ended questions and for the time involved in grading the items. Too, they were concerned about consistency in scoring items and worried that, in the interest of fairness, students' responses might not be scored uniformly across teachers. Another concern focused on the difficulty of getting students to write; one teacher claimed "some students just won't bother." Finally, while the Pre-algebra teachers in the project overwhelmingly felt that alternative assessment was very important, they were concerned about how their use of alternative assessment items in the classroom would be accepted by administrators, parents, and fellow teachers. Project participants worried that teachers who placed a primary emphasis on the mastery of skills to the exclusion of conceptual understanding might be opposed to the use of open-ended items.

Teachers identified numerous benefits that they felt they derived from their use of alternative assessment items. Primary among them was the insight they gained about their students' understanding of mathematics. One teacher put it this way, "I learned more about how much the students actually understand the topic as opposed to just knowing how to work the problem." The teachers expected alternative assessment to foster higher-order thinking skills in their students as indicated by the following comment: "Students will think on a higher level because [they] will have to justify their answers." Many of the project teachers who taught quite diverse students felt that alternative assessment would foster equity among the students. They felt that the use of open-ended questions provided opportunities for students to be creative, to demonstrate their strengths, and "to show what they know in a variety of ways."

What We Have Learned

We have come to understand that creating open-ended questions is a formidable task for many teachers. Senk, Beckmann, and Thompson (1997) found that teachers realized that "creating worthwhile assessment tasks is a time-consuming and difficult enterprise" (p. 213). In reviewing the items teachers generated, we noticed that often teachers would place a traditional item in a different context and believe that the item was "alternative". For example, teachers wrote items similar to the following: *Kelly claims that the solution to* $2x-6=10$ *is 8 while Jessica claims it is 2. Who is correct and why?* This question is not essentially different than simply asking students to solve $2x-6=10$. This finding is consistent with what Cooney (1992) found, viz., that most teachers have difficulty creating items that assess a deep and thorough understanding of mathematics. Teachers often mistakenly equate difficult items with items that test a deep and thorough understanding of mathematics. For example, one teacher saw solving the equation $2x+3=15$ as a typical problem that tests a minimal understanding of equation solving and $3x+5=4x-2$ as a problem that tests a deep and thorough understanding of equation solving. Although the second equation is clearly more difficult than the first, both are computational in nature and fail to focus on any conceptual understanding associated with equation solving. Perhaps teachers' difficulty in creating open-ended questions stemmed from their perception of what it means to have a deep and thorough understanding of mathematics. Teachers need to have strong understanding of the nature of conceptual knowledge in mathematics if they are to create effective assessment that focuses on this area of their students' knowledge.

Teachers also found it difficult to score students' responses to open-ended questions. The creation and use of the scoring rubric in Figure 4 by the project leaders helped alleviate some of the problem but the problem did not vanish entirely. Variations in scoring might be attributed to differences among the teachers in what they stressed in their classrooms. For example, one item asked students to draw and label a concave pentagon. One student drew an appropriate pentagon but mislabeled the figure by not putting the letters in order (ABCED rather than ABCDE for consecutive vertices). One teacher placed a lot of value on correct labeling and gave the student a low mark while other teachers scored the student quite high because the figure drawn satisfied the stated conditions. Although this issue might seem trivial, on the one hand, it was a much-discussed issue among the project teachers as it generated considerable debate about what was really important. This and other instances of variation in scoring led us to question our original rubric and subsequently generate the new four point scoring rubric in Figure 5.

0	No evidence of appropriate mathematical reasoning
1	Incorrect response but indicates some evidence of appropriate mathematical reasoning
2	Correct response but explanation is unclear or incomplete OR Incorrect response but explanation is clear and complete
3	Correct response and explanation is clear and complete

Figure 4. Original rubric for assessing open-ended items.

0	Response indicates no appropriate mathematical reasoning
1	Response indicates some appropriate mathematical reasoning, but fails to address the item's main mathematical ideas
2	Response indicates substantial and appropriate mathematical reasoning, but is lacking in some minor way(s)
3	Response is correct and the underlying reasoning process is appropriate and clearly communicated

Figure 5. Revised rubric for assessing open-ended items.

We found that the original rubric placed too much emphasis on the correct answer. Many items in the database asked questions such as "who is correct and why" or "do you agree—why or why not?" When the correct answer was equivalent to a coin toss, teachers who were using the original rubric (Figure 4) were inclined to score a response as a 2 even if there was no appropriate mathematical reasoning, merely because the student provided a correct answer. Teachers felt more comfortable with the revised rubric (Figure 5), as it helped them focus less on the correctness of the answer and more on the validity of the mathematical reasoning in the response.

Teachers expressed a variety of reasons why the use of open-ended items would be good for their students. Keith, for example, reasoned that "it gives possibilities for different types of answers. It shows a lot of understanding." Anita expressed the view that "I think it covers a lot of mathematics … it just requires you to think and I think that is important." For Keith, effective assessment tasks should inform teachers about student understanding; for Anita, good assessment promotes student thinking. Keith and Anita appeared to have captured the spirit of alternative assessment as it was presented during the training sessions. However, even after training, some teachers seemed to interpret alternative assessment differently. Jennifer, when asked to critique her use of open-ended items, felt that their use was entertaining for students and "they could all eventually solve the problem if they had the patience." Terri, in defending an item she wrote, said, "if you give them enough time to do it…they'll be successful doing it. There's not really a wrong answer." Keith and Anita seemed to be emphasizing the cognitive aspects of assessment; they wanted to know how their students thought about mathematics. Jennifer and Terri, on the other hand, seemed more concerned with the affective dimension of using alternative assessment. They wanted their students to have success with and feel good about assessment tasks.

A Final Word

Our work with these teachers has informed us about the struggles teachers go through when creating alternative assessment tasks. Senk et al. (1997) found that "virtually no teachers used open-ended items on tests" (p. 202). With the open-ended item bank as an available resource, perhaps project teachers will increase their use of open-ended items. Kulm (1993) found that "when teachers used alternative approaches to assessment, they also changed their teaching. Teachers increased their use of strategies that have been found by research to promote students' higher order thinking. They did activities that enhanced meaning and understanding" (p. 12). With the development of an accessible item bank and the training on how to use the bank, we feel that a foundation has been laid for teachers to include more learning activities that promote student mathematical understanding as suggested by Kulm. We have seen the excitement our project teachers exhibited when using the items. Their enthusiasm, even amidst concerns, leads us to believe that reform in the ways teachers assess their students and, concomitantly, in the ways they teach their students is an achievable goal.

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[1] This article is based on projects directed by Thomas J. Cooney and Laura Grounsell with the assistance of the other authors. The names of teachers are pseudonyms. All quotations are taken directly from teachers' written or oral statements. The observations presented in this article were a result of grants under the Higher Education Portion of the Eisenhower Professional Development Act but do not necessarily reflect the views of the funding agency. A previous version of this article was published in Sweden (Cooney, Ice, & Sanchez, 1999).

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