Article Review…

A Critical Question: Why *Can't* Mathematics Education and History of Mathematics Coexist?

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Fried's abstract of his article (p. 391):

Despite the wide interest in combining mathematics education and the history of mathematics, there are grave and fundamental problems in this effort. The main difficulty is that while one wants to see historical topics in the classroom or an historical approach in teaching, the commitment to teach modern mathematics and modern mathematics techniques necessary in pure and applied sciences forces one either to trivialize history or to distort it. In particular, this commitment forces one to adopt a "Whiggish" approach to the history of mathematics. Two possible resolutions of the difficulty are (1) "radical separation" – putting the history of mathematics on a separate track from the ordinary course of instruction, and (2) "radical accommodation" – turning the study of mathematics into the study of mathematical texts.

Michael Fried makes a confusing case that combining history of mathematics with mathematics education is inherently difficult, if not impossible. Initially Fried's argument seems to rest on the argument that mathematics educators' commitment to modern mathematics makes mathematics education incompatible with the history of mathematics. However, the bulk of Fried's discussion concerns the purposes of the historian: Fried asserts that the concerns of the historian render education incompatible with the history of mathematics. In this review I attempt to expose Fried's unsupported assertions by posing questions that need to be addressed in order to fill out his argument. Specifically, I question his assumption that mathematics educators are unavoidably committed to so-called modern mathematics and his claim that the historian is committed to limit his research to understanding idiosyncrasies in the thinking of historical mathematicians.

Fried states that there is no room for the history of mathematics in mathematics education; he also claims that mathematics educators are so committed to modern mathematics and modern mathematical methods that any attempt to either incorporate historical topics or take an historical approach to mathematics teaching is bound to compromise the "true" history of mathematics. For him, most discussions that support an historical approach to teaching mathematics are guilty of endorsing bad history, or more correctly a "Whiggish" history—an anachronistic reading of history in which modern mathematical concepts are ascribed to ancient thinkers.[i] But, Fried asserts, even if this problem of bogus history could be overcome, the commitment to modern mathematics, mathematics which have been proven best for solving modern problems, leaves no room for examining what true history of mathematics must focus on, namely the idiosyncratic thinking of historical figures. This idiosyncratic thinking not only established mathematics completely different from our own, but often led to many "dead ends" and numerous mistakes by historical mathematicians. The historian of mathematics wants and is duty bound to examine the differences between historical mathematics and modern mathematics. The mathematics educator wants and is duty bound to explain and justify modern methods. The purposes Fried assumes for the historian and the mathematics educator are at such odds as to render any reconciliation impossible, or gravely difficult. Fried takes a such a strong stance against reconciliation throughout his argument that one has to wonder what is left for him to support when he closes by saying that his critical examination of "attempts to introduce the history of mathematics in mathematics education should not be interpreted as *opposing* such attempts" (p. 406, his emphasis).

I find Fried's argument confusing and often contradictory. His discussion is deceptively simple and it is in attempting to analyze his position that confusions arise. For example, Fried proposes two solutions to linking mathematics education to the history of mathematics which are based on current attempts that he assumes early in his discussion to be doomed to failure. The apparent contradictions of his argument lie in the many unsupported claims he makes about the nature of mathematics education and the history of mathematics. In this review, rather than argue against his position, I try to expose these unsupported claims and raise questions that I feel Fried should answer before I, or any one else, consider his position well developed and worth accepting.

Fried takes the failure to implement historical approaches in school mathematics as a signal to consider whether it is possible to combine history of mathematics and mathematics education; in this paper he does not consider the possibility of external sources of failure to implement those approaches. Such sources might include the socio-political contexts of mathematics curricula development, the depth of familiarity of mathematics educators with historical topics, the availability of historical materials for educators, etc. I agree that Fried's concern about the possibility of combining history of mathematics with mathematics education is important. However, he takes the position that there is an inherent difficulty in attempting such a combination. I question this position because it rests on so many unsupported claims.

Fried begins by classifying the recommendations made by advocates of incorporating history into mathematics curricula into two basic strategies. As a result of the educational commitment that Fried claims, both of the two basic strategies for introducing history into school mathematics programs are bound to fail. The first strategy, what he calls the *strategy of addition*, involves "historical anecdotes, short biographies, isolated problems, and… does not alter a curriculum except by enlarging it" (p. 392). This strategy is bound to fail because mathematics programs are already over-crowded and allow little or no room for historical enhancement. The *strategy of accommodation* is more thoroughgoing and demands a restructuring of the mathematics curriculum based on historical development and circumstances. This strategy is bound to fail because it forces either an anachronistic reading of history or a history so deeply edited as to be similarly bastardized.

Fried assumes and asserts his position regarding the commitments of the modern mathematics educator without making a case for accepting that assumption: Mathematics educators have an "unavoidable commitment to the teaching of *modern* mathematics and *modern* mathematical techniques" (p. 392, his emphasis). I believe Fried needs to address at least two major questions: Why should we accept that mathematics educators are committed exclusively to modern mathematics? Why should we accept that such a commitment is unavoidable? Fried might respond that the answers to these two questions are obvious from the purpose of mathematics education; however what Fried suggests as the sole purpose of mathematics education is questionable. I will return to this point shortly.

In his closing comments, Fried quotes and accepts Thomas Tymocsko's claim that "pure mathematics is ultimately humanistic mathematics, one of the humanities, because it is an intellectual discipline with a human perspective and a history that matters" (p. 406).[ii] Fried compares mathematics to literature, art, and music; they are all expressions of "that vision and inventiveness so much part [sic] of the human spirit" (p. 406). He then states that the "study of the history of mathematics is an effort to grasp this facet of human creativity" (p. 406). His argument suggests that it is *only* through the history of mathematics that one can attempt to grasp mathematics as a creative endeavor. Would he also suggest that one can grasp art, literature, and music as creative endeavors only through studying their histories? I wonder what Fried conceives of as suitable education in art, literature, or music; would he demand a strong separation of studio arts and history of art in a way parallel to the cleft he sees between mathematics education and history of mathematics? Would Fried consider courses in studio arts to constitute art education and not those in art history? Would he claim that there is no room for art history in art education? While I can imagine that discussions and debates are waged over the roles and relative merit of studio arts and history of art courses, I suspect that most of us, and most arts educators, conceive of studio arts and art history as kinds of art education not as kinds of education. Fried seems to assume that learning mathematics and learning about history of mathematics require two separate kinds of education.

Questions arise, then, about Fried's conceptions of mathematics education and mathematics history. He is much clearer about the purpose of the history of mathematics than he is of the purpose of mathematics education. We have to infer both from his claims about the aims and commitments of the mathematics historian versus those of the mathematics educator. The historian is committed to "understand the thought of the past" (p. 398), to understand and examine the "idiosyncrasy of a mathematician's thought or of the thought of the mathematician's time" (p. 400). The mathematics educator, he suggests, is committed only to preparing future scientists and engineers. Ultimately, in Fried's view, these differing commitments prevent an amicable marriage of history and education in mathematics.

The historian's commitment forces the "serious" historian to always begin with an assumption that the past is different than the present and to focus on this difference. The purpose, then, of the historian of mathematics is to study peculiarities of mathematics—what, for instance, makes a text "*peculiarly* Apollonian or *peculiarly* Greek" (p. 400, his emphasis). This study of peculiarities is what makes the historian

> particularly interested in the "dead ends" mathematicians come to and the mistakes they make, for these are the kinds of things that reveal the peculiarity of the person's thought; these are the things that reveal the human character of doing mathematics. So, while one might succeed in making mathematics interesting, understandable, and approachable, or in providing insights into concepts, problems and problem-solving without history or with an "unhistorical" history, *humanizing* mathematics with history requires that history be taken quite seriously, not as a mere tool, but as something studied earnestly (pp. 400-401, his emphasis).

There are at least three issues at stake in Fried's claim. One is the foundational assumption Fried takes all historical work to rest upon—the assumption that the past is unlike the present. Another is the focus of study Fried seems to demand that all historians take—the study of idiosyncratic thinking peculiar to a particular historical figure in a particular historical time frame. Also at stake is the status of historical work in education—that proper history is *not* to be relegated to being a mere pedagogical tool.

If all historical work rests upon the assumption that the past is different than the present, and rests only upon that assumption as Fried suggests, of what value can history have? If we assume that the experiences of people in bygone times are completely different than our own, what could we hope to gain by examining their experiences? Couldn't anyone claim that dealing with our own present day experience is difficult enough without shouldering the burden of trying to understand the disconnected and unrelated experiences of someone in another era? And if their experiences, their times, and their way of thinking and understanding is completely disconnected and dissimilar from ours, how can anyone from our time claim to understand them in theirs? Fried draws an analogy between mathematics education and teaching literature by saying that "while one learns something about Elizabethan culture by reading Shakespeare, the main reason one reads Shakespeare's works is that they are great in their own right" (p. 401). If there is nothing to be found in common between the Merchant of Venice and the shopkeeper on Main Street, what is the basis for claiming that Shakespeare's works are *great in their own right*? While jokes about codpieces may be lost on the modern reader or viewer, certainly those same modern readers can understand Hamlet's frustration and sense of being betrayed by those around him. Without some sense of relevance to the modern viewer, *Hamlet* would simply be a collection of odd movements and sayings.

With Fried's insistence that the historian's primary concern is with time dependent peculiarities, what prevents history from becoming little more than a collection of exotic trinkets? (Perhaps though, that is all history is for Fried.) If the only criteria for, or if the fundamental assumption of, the historian is difference, what counts as different? What, and where, is the line that demarcates the past and the present? Do we rely on arbitrary boundaries in terms of years—those who lived within, say, 200 years of the present are assumed to be sufficiently like us to be considered *us*? Or is there some objective measure or process by which the historian can establish *the* difference between *them and us*? It seems that for history to have any relevance, we must assume that the past is, after all, *somewhat* like the present: there must be something from the past that is translatable to the present. While I sympathize with Fried's concern about anachronistic interpretations, I suggest that any historical work that ignores either the similarities or dissimilarities between the past and present and focuses exclusively on one or the other will be severely limited and dramatically incomplete.

Fried claims that any history in which the present is a measure of the past is bad history, or worse, hardly even history at all. My question is how can the present *not* be a measure of the past? Fried rejects the search for the origins of ideas as a major and fundamentally misconceived task for history; in seeking the origins of concepts used in modern mathematics, we will inevitably take away the thoughts of the historical mathematician and "make him think our own" (p. 396); which is to say that we will read into ancient texts modern concepts that were inconceivable at the time. For Fried, it is the Whig history that traces paths (or a single path) from the past to the present, while the truly historical perspective "is the zigzag path of a wanderer who does not know exactly where he is going" (p. 396). But Fried's opposition to reading direction in history seems to blind him to the fact that we cannot *but* examine the past from our own position in the present. The very things that Fried claims are of most interest to the historian of mathematics—"the 'dead ends' mathematicians come to and the mistakes they made" (p. 401)—can *only* be determined as "dead ends" and "mistakes" from the stance of present day mathematical theory and practice. Similarly, what Fried calls modern methods and approaches (that the mathematics educator is unavoidably committed to) and justifies as "the most *powerful* means to solve problems of interest and of importance *in the modern world*" (p. 405, his emphasis) can only be judged "the most powerful" within a historical context. Whatever non-modern methods might be, they proved less fruitful only for the kinds of problems Fried assumes students will eventually face. How can he be sure that they might not be fruitful in the future? Shouldn't students be aware, then, of the mathematics that did not survive in order to enhance their appreciation of the mathematics that educators demand they know?

Fried seems to agree that perhaps they should, but he sees only two possibilities, both of which are extensions of the strategies he previously determined are doomed to failure. One solution is a *radical accommodation*—students would learn mathematics by engaging directly with historical texts (as is done in certain "great books" programs.) The other is a *radical addition or radical separation*—students would have a history of mathematics track parallel to their standard mathematics courses. These "radical" solutions seem to have their basis in Fried's aversion to the "use" of history in mathematics education. Fried is very concerned that history is to be studied in its own right—he seems to deny the same for mathematics.

Fried's conception of mathematics education appears to be limited to delivering the useful, powerful mathematics that will prepare competent scientists and engineers. In fact Fried most strongly implies this limited view of mathematics education when he questions whether the humanizing of mathematics through the radical accommodation approach "satisfies the other component of the mathematics teacher's commitment, namely, that students learn to do the mathematics of science and engineering" (p. 401). Does Fried really expect us to accept that the sole purpose of mathematics education is to prepare potential scientific workers in the applications of mathematics? Are we to accept both that mathematics is a human endeavor, with a history that matters, *and* that mathematics is only for the use of scientists and engineers? Would Fried expect a mathematician not to be offended by the implication of his claim that history is not to be used but that mathematics is? Fried claims (probably rightly so) that the commitments of the mathematics educator and of the historian of mathematics make the relationship of each to the history of mathematics quite different (p. 398). Can we not make the same claim about the differences of commitments (and hence the differences in relationship to mathematics) between the mathematician and the mathematics educator? If we take Fried's suggestion that we radically separate mathematics education from the history of mathematics (and hence separate the mathematics educator from the historian), should we demand a similar separation between the mathematics educator and the practicing mathematician?

Ultimately, Fried's argument seems to more about territorial boundaries than about the possibility of infusing mathematics education with historical understanding. Fried examined the mathematics of Apollonius in his doctoral dissertation, and when all is said and done in his case against history in mathematics education, he appears to be a historian trying to preserve the sanctity of his esoteric work from being directed toward any kind of utilitarian purpose. Fried's article abstract and opening remarks lead the reader to expect his case to be that the foundational commitments of the mathematics educator render the history of mathematics incompatible with mathematics education. As I read his argument, struggling to make sense of what appear to be his contradictory leanings, I suspect that the case he actually wants to make is that the commitments of the historian render mathematics education incompatible with the history of mathematics.

[1] Fried adopts the term "Whig history" from Butterfield, H. (1931/1951). *The Whig Interpretation of History*. New York: Charles Scribner's Sons.

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