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# Teachers' Mathematical Beliefs: A Review

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This paper examines the nature and role of teachers' mathematical beliefs in instruction. It is argued that teachers' mathematical beliefs can be categorised in multiple dimensions. These beliefs are said to originate from previous traditional learning experiences mainly during schooling. Once acquired, teachers' beliefs are eventually reproduced in classroom instruction. It is also argued that, due to their conservative nature, educational environments foster and reinforce the development of traditional instructional beliefs. Although there is evidence that teachers' beliefs influence their instructional behaviour, the nature of the relationship is complex and mediated by external factors.

For the purpose of this paper, *teachers' mathematical beliefs* refers to those belief systems held by teachers on the teaching and learning of mathematics. Educationalists have attempted to systematize a framework for teachers' mathematical belief systems into smaller sub–systems. Most authors agree with a system mainly consisting of beliefs about (a) what mathematics is, (b) how mathematics teaching and learning actually occurs, and (c) how mathematics teaching and learning should occur ideally (Ernest, 1989a, 1989b; Thompson, 1991). Certainly, the range of teachers' mathematical beliefs is vast since such a list would include all teachers' thoughts on personal efficacy, computers, calculators, assessment, group work, perceptions of school culture, particular instructional strategies, textbooks, students' characteristics, and attributional theory, among others.

In this paper, the concept of *progressive* instruction is associated with a socio-constructivist view of teaching and learning mathematics. Socio-constructivism, which for the sake of brevity will be called just constructivism, gives recognition and value to new instructional strategies in which students are able to learn mathematics by personally and socially constructing mathematical knowledge. Constructivist strategies advocate instruction that emphasises problem-solving and generative learning, as well as reflective processes and exploratory learning. These strategies also recommend group learning, plenty of discussion, informal and lateral thinking, and situated learning (Handal, 2002; Murphy, 1997). In turn, *traditional* instruction is associated with a behaviourist perspective on education. Behaviourist practices are said to emphasise transmission of knowledge and stress the pedagogical value of formulas, procedures and drill, and products rather than processes. Behaviourism also puts great value on isolated and independent learning, as well as conformity to established one-way methods and a predilection for pure and abstract mathematics (McGinnis, Shama, Graeber, & Watanabe, 1997; Wood, Cobb, & Yackel, 1991). Leder (1994) stated that in the behaviourist movement "the mind was regarded as a muscle that needed to be exercised for it to grow stronger" (p. 35).

The study of teachers' instructional beliefs and their influence on instructional practice gained momentum in the last decade. Some research on teachers' thinking reveals that teachers hold well-articulated educational beliefs that in turn shape instructional practice (Buzeika, 1996; Frykholm, 1995; McClain, 2002; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1992). Examples of research, as reviewed in this paper, have also shown that each teacher holds a particular belief system comprising a wide range of beliefs about learners, teachers, teaching, learning, schooling, resources, knowledge, and curriculum (Gudmundsdottir & Shulman, 1987; Lovat & Smith, 1995). These beliefs act as a filter through which teachers make their decisions rather than just relying on their pedagogical knowledge or curriculum guidelines (Clark & Peterson, 1986). In fact, these beliefs appear to be cogent enough to either facilitate or slow down educational reform, whichever is the case (Handal & Herrington, 1993, in press). The literature also shows that there are internal and external factors mediating beliefs and practice (Pajares, 1992). This dissonance bears serious implications for the implementation of curricular innovations since teachers' beliefs may not match the belief system underpinning educational reform. Even if

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teachers' beliefs match curricular reform, very often the traditional nature of educational systems make it difficult for teachers to enact their espoused progressive beliefs. In contrast to linear and static approaches to curriculum implementation, modern perspectives look at how teachers make sense of educational innovations in order to re-appraise an ongoing and always flexible process of implementation (Handal & Herrington, 2003).

## Theoretical Conceptualisations

Theoretical conceptualisations of teachers' mathematical beliefs show that the range of these beliefs can be expressed in multiple dimensions (Kuhs & Ball, 1986; Renne, 1992; Ernest, 1991). Ernest (1991), for example, outlined a developmental sequence of five different mathematics-related belief systems that are hypothesized to be found amongst teachers: authoritarian, utilitarian, mathematics centred, progressive, and socially aware. Ernest's contribution showed that it is possible to relate these attitudinal representations to conceptions on the theory of mathematics, learning mathematics, teaching mathematics, and assessment in mathematics, as well as identifying beliefs on the aims of mathematics education. According to Ernest, the most important of these categories is the teacher's philosophy of mathematics, which might vary from absolutist to social-constructivist values. Teachers' theories of learning and teaching are said to relate to approaches used in class and are fundamental because they define the teacher's perception of the learner's role as active or passive, dependent or autonomous, or as receiver or creator of knowledge. Ernest also proposed three main philosophical conceptions of mathematics among teachers. In the instrumentalist view, mathematics is seen as a collection of rules and skills that are to be used for the attainment of a particular goal. Teachers adhering to the Platonist view will maintain that "mathematics is a static but unified body of certain rules" (p. 250) that are to be discovered and are not amenable of personal creation. The problem solving view presents mathematics as a continuous process of inquiry that always remains open to revision.

In turn, Kuhs and Ball (1986) characterised three different and dominant conceptions of the ideal teaching and learning of mathematics. The first is the learner-focused view that stresses the learner's construction of mathematical knowledge through social interaction. The second is the content-focused view with an emphasis on conceptual understanding. The third is the content-focused approach with an emphasis on performance which values performance as the key goal whose attainment depends on the mastery of rules and procedures.

Furthermore, Renne (1992) proposed a *Purpose of Schooling/Knowledge* matrix to conceptualise four different teachers' conceptions of teaching and learning mathematics. Two groups of teachers are identified in the purpose of schooling category, namely, *school-knowledge oriented* and *child-development oriented*. Teachers within the school-knowledge group believe that teaching is an act of passing information on to others while learning involves the process of reproducing that information. At the same time, school-knowledge oriented teachers place great emphasis on the syllabus and curricular guidelines to guide their instruction. In turn, child-development oriented teachers are more likely to consider children's needs and characteristics as the primary factors in instructional decision making. The second category in the matrix relates teachers' beliefs to the way teachers perceive knowledge itself. School-knowledge oriented teachers design activities that emphasise acquisition of knowledge in terms of "what" is going to be learned. As such, this type of knowledge is concerned more with rules, procedures, and drill. This type of knowledge is very fragmentary because it does not help the learner relate isolated pieces of knowledge to the whole framework. In contrast, child-development oriented teachers are more concerned with learning of mathematical concepts within an interrelated knowledge structure that is holistic and meaningful.

These three different conceptualisations of teachers' beliefs about the nature and pedagogy of mathematics (Ernest, 1991; Kuhs & Ball, 1986; Renne, 1992) constitute an analytical framework to discuss teachers' mathematical belief systems. In general, it can be argued that teachers' belief systems are complex networks of smaller sub-systems operating contextually. The following section attempts to explain the origin of these belief systems within the context of present and past educational environments that appear very traditional and resistant to change.

## The Cycle of Teachers' Mathematical Beliefs

How do teachers' mathematical beliefs originate? In part, teachers acquire these beliefs symbiotically from their former mathematics school teachers after sitting and observing classroom lessons for literally thousands of hours throughout their past schooling (Carroll, 1995; Thompson, 1984). This process parallels in many respects the apprenticeship style of

learning that takes place while learning a trade. Traditionally, tradesmen learn by observing a master doing a particular job (Buchmann, 1987; Lortie, 1975). In the schooling process, students learn not only content-based knowledge but also instructional strategies as well as other dispositions. By the time the aspirant is admitted to a teacher education program, these beliefs about how to teach and learn are deeply embedded in the individual, and very often are reinforced by the traditional nature of some teacher education institutions which may not have positive effects on preservice teachers' mathematical beliefs (Brown & Rose, 1995; Day, 1996; Foss & Kleinsasser, 1996; Kagan, 1992; McGinnis & Parker, 2001).

There is evidence that, in some cases, teacher education programs are so busy concentrating on imparting pedagogical knowledge that little consideration is given to modifying these beliefs (Tillema, 1995). Consequently, teacher education programs might have little effect in producing teachers with beliefs consistent with curriculum innovation and research (Kennedy, 1991). For example, Marland (1994) found that reasons given by inservice teachers regarding their classroom strategies were not related to what was actually taught in their college training. There is also some evidence confirming that teachers' decision making does not rely solely on their pedagogical knowledge but also on what they believe the subject-matter is and how it should be taught (Brown & Baird, 1993; Laurenson, 1995; Prawat, 1990). These beliefs are also difficult to change (Borko, Flory, & Cumbo, 1993) and very often conflict with educational innovations, threatening educational change (Brown & Rose, 1995; Fullan, 1993). As discussed in the next sections, there are also a number of external factors influencing teachers' beliefs.

## The Constraining Nature of Educational Environments

The context of school instruction obliges practising elementary and secondary teachers to teach traditional mathematics even when they may hold alternative views about mathematics and about mathematics teaching and learning. Parents and professional colleagues, for example, expect teachers to teach in a traditional way. Teachers are also expected to focus on external examinations, to adhere to a textbook, and to keep a low level of noise and movement in their classrooms. In such environments, even teachers with progressive educational beliefs are forced to compromise and conform to traditional instructional styles (Handal, 2002; Perry, Howard, & Tracey, 1999; Sosniak, Ethington, & Varelas, 1991). Other accountable factors are ethnic background, social class origins, experience living in other cultures, gender issues, and prior styles of teaching experience (Butt & Raymond, 1989; Raymond, Butt & Towsend, 1991). Thompson (1984) argued that teachers, in the exercise of their practice, and because of the large number and diversity of interactions, tend to develop quick responses to types of episodes, which in time become patterns in their instructional repertoire.

McAninch (1993) reviewed a body of literature showing that teachers are very practical in their approach to pedagogical tasks. Jackson's (1968) interviews revealed that teachers tend to be "confident, subjective, and individualistic in their professional views" (cited by McAninch, 1993, p. 7). In addition, Doyle and Ponder (1977) and Lortie (1975), both cited by McAninch (1993), described "teachers as pragmatic in their decision making…and intuitive in their approach to problem solving" (p. 7). Moreover, teaching is seen as a highly practical and utilitarian profession where teachers quickly label innovations as practical or impractical, depending on whether the teacher considers that the proposal will work for him or her. Success of innovations was also found to be related to a teacher's personality and teachers were found to emphasise the peculiarities of their classroom over the generalizations of innovations.

Nespor (1987) adds that, given the unpredictability and uniqueness of classroom events, teachers have to resort to their own beliefs, particularly in pedagogical situations when formal knowledge is not available, is disconnected, or cannot be retrieved. In Nespor's words, "When people encounter entangled domains or ill-structured problems, many standard cognitive processing strategies such as schema-abstraction or analytical reduction are no longer viable" (p. 325). This type of situation is characteristic of classroom teaching. In general, teaching is a decision-making based activity in which teachers have to make an interactive decision every two minutes (Brown and Rose, 1995; Clark & Peterson, 1986; Lovat and Smith, 1995).

In brief, the teaching job places great external demands on decisions that teachers have to make rapidly, in isolation, and in widely varied circumstances. These demands put teachers in the position of resorting to practicability and intuition as indispensable resources for survival in the profession. These demands in turn favour the development of beliefs about what works and what does not in a classroom. At the same time, it seems that teachers

generate their own beliefs about how to teach in their school years and these beliefs are perpetuated in their teaching practice. Thus, educational beliefs are passed on to the students.

### Teachers' Instructional Practice

If, as the adage says, "teachers teach the way they have been taught" (Frank, 1990, p.12), we need to ask ourselves: what type of mathematics teaching have our and past generations been exposed to? Studies conducted in American mathematics classrooms by Cuban (1984), Mewborn (2001), Sirotnik (1983), and Romberg and Carpenter (1986), Gregg (1995) indicate that most mathematics lessons follow a pattern of whole-class lecturing and "show and tell" style of teaching. Work in small groups is not common and students do not participate actively. Teacher questioning emphasizes right or wrong answers and students are often allocated to passive seatwork. Too much emphasis is given to rote learning, procedures, and facts. It was also found that excess teacher talk dominates in classroom communication and desks usually are arranged to face the teacher's desk. In sum, this pattern of lessons in American classrooms can be characterised as traditional oriented. Furthermore, the Third International Mathematics and Science Study (TIMSS) identified a similar pattern in Australian classrooms, "one of what might be called 'traditional approaches' dominating classroom instruction…particularly in relation to lesson sequencing and types of activities undertaken" (Lokan, Ford, & Greenwood, 1997, p. 231).

Based on the above arguments it is possible to suggest that the educational system may act as a vehicle to reproduce traditional mathematical beliefs. Teachers seem to pass on these beliefs in subtle ways in school classrooms. By the time candidates enroll in a teacher education program, these ideas are so solidified and entrenched in their personal philosophy that they will be passed on to their students once the candidates commence their teaching careers, thus carrying on a cycle. The following section attempts to explore the character, intensity, and diversity of these mathematical beliefs as conveyed by schoolteachers.

### Teachers' Beliefs about Mathematics and the Learning and Teaching of Mathematics

Teachers' mathematical beliefs are personal and are therefore mental constructs peculiar to each individual (Brown & Rose, 1995). A number of studies have been conducted to obtain "typical" teachers' mathematical beliefs. Teachers' mathematical beliefs have been analysed statistically and in many instances judgements were passed on a right-and-wrong criteria by researchers. Although patterns are identifiable within representative samples, these studies have at the same time revealed a broad diversity in the direction and intensity of these beliefs (Carpenter, Fennema, Loef, & Peterson, 1989; Moreira, 1991; Schmidt & Kennedy, 1990). This fact led some researchers to think that these differences could be alternatively interpreted either as stages of a developmental process, individual cognitive differences, or simply due to differences in socio-economic status, educational systems, or cultural environments (Moreira, 1991; Stonewater & Oprea, 1988; Thompson, 1991; Whitman & Morris, 1990).

The studies described below show that a large population of teachers still believe that teaching and learning mathematics is more effective in the traditional model, thus suggesting a historical correspondence between teachers' mathematical beliefs and the teaching practices described in the previous section. What follows is a summary of the main studies conducted to explore mathematical beliefs in preservice and inservice teachers.

#### *Mathematical Beliefs of Preservice Teachers*

A growing body of literature suggests that preservice teachers, that is, student teachers attending teacher education institutions, hold sets of beliefs more traditional than progressive with respect to the teaching of mathematics. Research findings reveal that preservice teachers bring into their education program mental structures overvaluing the role of memorization of rules and procedures in the learning and teaching of school mathematics. For example, Benbow (1993) found that preservice elementary teachers thought of mathematics as a discipline based on rules and procedures to be memorized, and that there is usually one best way to arrive at an answer. Most of the teachers also saw mathematics as dichotomized into "completely right or completely wrong" (p. 10). A similar conservative trend in teachers' beliefs was reported by Nisbert and Warren (2000), who surveyed 398 primary school teachers with regard to their views on mathematics as a subject, and on teaching and assessing mathematics. Civil (1990) interviewed four prospective elementary teachers and found that they believed that mathematics required neatness and speed, and that there is usually a best way to solve a problem. Frank (1990) surveyed the mathematical beliefs of preservice teachers and found a high level agreement in items such as: (a) "Some people have a

mathematical mind and some don't", (b) "Mathematics requires logic not intuition", and (c) "You must always know how you got the answer" (p. 11). Moreover, Foss and Kleinsasser (1996, p. 438) surveyed, observed, and interviewed preservice elementary teachers and found that the participants placed great emphasis on practice and memorization. Teachers also were of the opinion that ability in mathematics was innate. Southwell and Khamis (1992) surveyed 71 preservice teachers and found that most participants perceive that mathematics learned in school should be based on memorization of facts and rules. Lappan and Even (1989) and Wood and Floden (1990) report similar findings.

*Mathematical Beliefs of Inservice Teachers*

Results from research on inservice teachers show a broader spectrum of responses than with preservice teachers. This is partially the result of more flexible research designs allowing the collection of a broader set of responses in the samples. A number of these studies also show a more varied scope of research questions rather than just simply characterizing teachers' mathematical beliefs in a dichotomy.

The Third International Mathematics and Science Study (TIMSS) (Beaton, et al., 1996), conducted in selected countries around the world, revealed that most teachers believe mathematics is essentially a vehicle to model the real world, that ability in mathematics is innate, and that more than one representation should be used in explaining a mathematical concept. With respect to the emphasis on drill and repetitive practice, teachers around the world did not show a consistent response. Anderson (1997) surveyed and interviewed 25 primary teachers and found that the majority of the participants believe in the value of whole-class discussion, teacher's modelling, and the use of manipulatives in the classroom. However, it was found that teachers were of the opinion that calculators should not be an important component in teaching mathematics in the primary school. Grossman and Stodolsky (1995) surveyed and interviewed 399 teachers of mathematics, sciences, social studies, and foreign languages. The authors found that mathematics teachers, compared with those of the other subjects, consider their subject highly sequential, static, and have stronger consultation within their faculty for coordinating course content and common exams. The findings also showed that mathematics teachers prefer students to be grouped by prior academic achievement in order to get better benefits from instruction. Schubert (1981), quoted by Brown and Rose (1995), in questioning 123 educators, found that most teachers believe that pupils learn "in a passive manner by reacting to forces external to them, rather than in an active manner as producer of their own knowledge" (p. 21), a conclusion also supported by Desforges and Cockburn (1987).

Finally, Howard, Perry and Lindsay (1997) surveyed 249 secondary mathematics teachers in Sydney, Australia, and found two different patterns of beliefs. The first is identified with the "transmission" profile, that is, a traditional categorization of teaching and learning as the transmission and verification of information in which memorization of rules and procedures is fundamental. This group was larger in number than the constructivist profile, where teachers believe that students are capable enough of constructing their own mathematical knowledge in an atmosphere of negotiation and relevance. The evidence that a large number of inservice teachers hold a diverse collection of mathematical beliefs associated with traditional instruction is also documented in studies conducted by Handal, Bobis, and Grimison (2001), Kifer and Robitaille (1992), Middleton (1992), Perry, Howard, and Conroy (1996), and Perry et al. (1999).

## Teachers' Mathematical Beliefs And Instructional Practice

Studies on the relationship between pedagogical beliefs and instructional behaviour have reported different degrees of consistency (Frykholm, 1995; Thompson, 1992). While the nature of this relationship seems to be dialectical in nature (Wood et al., 1991) it is not clear whether beliefs influence practice or practice influences beliefs (McGalliard, 1983). It is in fact a complex relationship (Thompson, 1992) where many mediating factors determine the direction and magnitude of the relationship. This section reports a number of studies that have explored the relationship between teachers' mathematical beliefs and instructional practice.

Benbow (1995) conducted an intervention program to deliberately modify the beliefs and instructional practices of 25 preservice mathematics elementary teachers. Findings showed that there was no change in teachers' mathematical beliefs at the end of the program. However, the researcher stated that instructional behaviour in terms of selection of curriculum content and learning activities, teacher's role, and teachers' beliefs on self-efficacy were modified as a result of the program. Lack of pedagogical knowledge and subject-based content were found in some cases to be an obstacle to transfer progressive oriented beliefs into practice.

Brown and Rose (1995) conducted an interview study with 10 elementary mathematics teachers in order to determine their theoretical orientations. Teachers' responses showed a varied range of theories of teaching and learning mathematics. Teachers also said that these orientations influenced their instructional behaviour. The analysis of data revealed that teachers do not implement fully their ideal conceptions of mathematics education because of perceived pressure from parents and school administrators to implement traditional teaching. Other identified mediating factors were the need for more preparation time to satisfy instructional and curricular demands, and the challenges of mixed ability classes. Erickson (1993), in a study with two experienced middle school mathematics teachers, concluded that teachers' ideal beliefs have a strong influence on their instructional practice. However, obstacles to fully implement their ideals included lack of preparation time and lack of collaboration among peers; size of room; availability of technology, materials, and money; non-supportive administration and parents; need for lengthened class periods; and personal opportunity for growth.

Foss and Kleinsasser (1996) studied the behaviour and instructional practice of 20 elementary mathematics preservice teachers. At the end of a one-semester methods course participants had not changed their beliefs about teaching and learning mathematics, which were found to be traditional-oriented and heavily influenced by previous traditional learning experiences in diverse educational settings. Participants' instructional behaviour replicated or modelled activities learned in the methods course, but not to the extent that reflected an adoption of innovative approaches to teaching and learning mathematics in an articulated and consistent way. In addition, Cooney (1985) studied a beginning mathematics teacher who was committed in belief and in practice to problem solving instruction. The author described the conflict between the teacher's struggle to teach problem solving and students who preferred a more content-based instruction, a friction that sometimes led to classroom management problems. Perry et al. (1999) studied the beliefs of Australian head secondary mathematics teachers and classroom secondary mathematics teachers as independent samples. Head teachers said that curriculum demands were an obstacle to implementing innovative teaching. In the respondents' words:

> We try to make the work relevant but we are constrained by the syllabus. Sometimes, I feel, pressure of the syllabus tends to force us to cut corners with the kids…If I sound cheesed off, it's just that I may be a disillusioned mathematics teacher. (p. 14)

Raymond (1993) investigated beliefs and practices of six beginning elementary mathematics teachers and found diverse degrees of consistency. Two teachers displayed a high degree of correspondence between belief and practice, two teachers showed a moderate level, while the other two showed a low level. Reasons for the inconsistencies were found to be lack of resources, time limitations, discipline, and pressure to conform to standardized testing. The author concluded that there is a dialectical relationship between beliefs and practice. According to the researcher, teachers' mathematical beliefs influenced their practice more than their instructional practices influence their mathematical beliefs. The researcher also found that previous school experiences, teachers' current practice, and, importantly, teacher education courses also influence teachers' mathematical beliefs. Teachers also identified their own mathematical beliefs, students' abilities, the particular topic to be taught, the school culture, as well as the mathematics curriculum as factors that influenced their instructional practice.

Taylor (1990) attempted to assist a high school teacher to modify his beliefs through a process of conceptual change. However, there were conflicting beliefs, such as the teacher's belief that he had to teach for constant assessment and for covering the syllabus given that he did not want to jeopardize students' learning with alternative strategies. Consequently, change in instructional behaviour was restricted.

Van Zoest, Jones, and Thornton (1994) interviewed and observed six elementary preservice mathematics teachers participating as students in an intervention program to enhance their teachers' mathematical beliefs. The authors found that participants acquired beliefs consistent with socio-constructivist views of learning and teaching mathematics, although they were not able to translate these views into practice in the early stages of instructional episodes. The reason for this inconsistency was found in teachers' lack of pedagogical skill to guide students through the whole problem solving process, time needed to go through a task, teachers' and students' tension on how to go about a problem solving situation, and teachers' concerns about students' ability to solve the problem. Other studies not showing consistency include Grant (1984) studying secondary mathematics teachers, Kessler (1985) investigating four senior high school

mathematics teachers, Brosnan, Edwards, and Erickson (1996) researching four middle school mathematics preservice teachers, and Desforges and Cockburn (1987) studying seven experienced mathematics primary school teachers.

Thompson (1985) studied two relatively experienced mathematics teachers in their teaching of problem solving and found a high level of consistency between their beliefs and instructional practice. Phillip, Flores, Sowder, and Schapelle (1994) reached the same conclusion while studying four "extraordinary" mathematics teachers. Other studies reporting a strong relationship between teachers' beliefs and practices have been conducted by McGalliard (1983) investigating senior high school mathematics teachers, and Steinberg, Haymore, and Marks (1985) studying novice teachers. Shirk (1973) working with preservice elementary teachers and Stonewater and Oprea (1988) working with inservice teachers also reported similar consistencies.

In general, inconsistencies between teachers' beliefs and practices are due to constraining forces out of a teachers' control, such as parental and administrative pressure to follow traditional oriented methods of instruction. Other factors include the traditional oriented mathematical learning style of the students as well as a lack of time and materials. These factors seem to act as major barriers for some teachers in implementing their progressive beliefs, constraints that current approaches in mathematics education do not take into account (Nolder, 1990).

**Incongruities Between Teachers' Beliefs And Practice**

The incongruity between beliefs and practice can also be explained through the agitation and unpredictability of classroom life and the external pressures put on teachers. Thompson (1985) affirmed that these incongruities might be due to the frequency of unexpected occurrences which teachers face in the classroom. The high frequency of these incidents does not permit the teacher to reflect on alternative responses; rather, teachers have time only to react. Jackson (1968) suggested that elementary teachers engage in more than one thousand interactions with students in a single day.

Another source of incongruity lies in the personal resolution of conflicting beliefs. Orton (1991) suggested that teachers' commitment to progressive beliefs is not always a guarantee that these beliefs are going to be translated into practice because sometimes teachers have to compromise their progressive beliefs for the crude reality of traditional oriented educational environments. For example, a teacher might be motivated to provide rote-learning activities in class when that teacher knows that his or her students will be tested on basic skills in a district proficiency exam. In this case, the teacher might perceive that drill and repetitive practice is the best strategy to attain a temporary goal. Consequent to this strategy, the teacher suspends his or her own progressive beliefs for others that are more central at that particular time.

Teacher's resistance to adopting new approaches in the teaching of mathematics may be part of a defense mechanism that teachers adopt to avoid changes in their own mental structures (Clarke, 1997) because "changing beliefs causes feelings of discomfort, disbelief, distrust, and frustration" (Anderson & Piazza, 1996, p. 53). Orton (1991) stated that it is not easy to change a long-cherished mathematical belief since this belief proved before to be rewarding and useful to the teacher in the performance of his or her professional duties. Furthermore, changing a particular belief implies a re-structuring of the whole network of one's belief system, a feeling that might cause anxiety and emotional pain (Rokeach, 1968). Concerning teachers' resistance to change, it has been observed that teachers holding more relativistic orientations to teaching mathematics are more likely to consider and adopt new ideas (Arvold & Albright, 1995).

School cultures also influence teachers' mathematical beliefs (Anderson, 1997). This is particularly true when teachers are found holding beliefs different from the school culture in which they work. For example, a certain school environment might effectively foster values associated with progressive practices and this influence might be stronger than in other schools. In many instances, teachers are caught in a conflict of interest between their "technical-positivist" and their "constructivist" beliefs and therefore they compromise (Taylor, 1990). Moreover, teachers know that although administrators and supervisors promote reform efforts, professional assessment is in terms of the traditional paradigm and therefore they tend to conform to the status quo to minimize disturbance and professional risk in an ethical-practical way (Anderson & Piazza, 1996; Doyle & Ponder, 1977).

Research also shows that teachers may not hold consistent belief systems. Sosniak et al. (1991) analysed mathematical beliefs and self-perceptions of practice of US teachers representing 178 typical eighth grade classes. Based on those responses, the researchers attempted to profile teachers in either a

traditional or progressive orientation to the curriculum. However, it was found by statistical analysis that teachers lack a consistent theoretical orientation towards the curriculum. According to the authors, within each teacher's belief system there are beliefs that appear to be ideologically incompatible with the others. Andrews and Hatch (1999), working mainly with secondary mathematics teachers in the United Kingdom, and Howard et al. (1997) in Australia, reached similar conclusions.

Finally, Richardson (1996) adds that in some cases teachers cannot articulate a particular belief because they are unfamiliar with a specific educational innovation. According to Richardson (1996):

> … it cannot be assumed that all changes in beliefs translate into changes in practices, certainly not practices that may be considered worthwhile. In fact, a given teacher's belief or conception could support many different practices or no practices at all if the teacher does not know how to develop or enact a practice that meshes with a new belief. (p. 114)

## Summary

This paper argued that despite many educational reforms, a large number of teachers still perceive mathematics in traditional rather than in progressive terms; that is, as a discipline with a priori rules and procedures, "out-there," that has to be mechanically discovered rather than constructed. As such, students have to learn mathematics by rote and removed from human experience. The discussion also shows that the relationship between teachers' mathematical beliefs and their instructional practice is dialectical in nature and is mediated by many conflicting factors. Teachers' beliefs do influence their instructional practice; however, a precise one-to-one causal relationship cannot be asserted because of the interference of contingencies that are embedded in the school and classroom culture. Even teachers holding progressive beliefs find it difficult to render their ideas into practice due to mediating factors such as the pressure of examinations, administrative demands or policies, students' and parents' traditional expectations, as well as the lack of resources, the nature of textbooks, students' behaviour, demands for covering the syllabus, and supervisory style, among many others. In addition, the teaching profession appears to mould the nature of beliefs because teachers have to make decisions and make meaning of situations quickly, in solitude, with a diversity of subjects, based on empirical knowledge, and under the pressure of external factors. Pedagogical knowledge therefore is not a total predictor of instructional behaviour because beliefs appear to mediate between theory and practice as a powerful interface. Teachers' mathematical beliefs are seen as self-perpetuating within the atmosphere of a system that promotes progressive teaching but in fact helps in maintaining traditional beliefs and practices. It was also argued that by the time an individual enters a teacher education program, these traditional conceptions are so solidified and entrenched in their personal philosophy that change to alternative beliefs is difficult although not impossible.

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