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The Consequences of a Problem-Based Mathematics Curriculum

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Implementation of a problem-based mathematics curriculum, the *Interactive Mathematics Program* (IMP), at three high schools in California has been associated with more than just differences in student achievement. The outcomes that distinguished students who participated in the IMP program from students who followed a conventional algebra/geometry syllabus were the students' perceptions of the discipline of mathematics, of mathematical activity and the origins of mathematical ideas, of the mathematical nature of everyday activities, and of school mathematics and themselves as mathematicians. A coherent and consistent picture has emerged of the set of beliefs, perceptions and performances arising from such a program. Students who have participated in the IMP program appear to be more confident than their peers in conventional classes; to subscribe to a view of mathematics as having arisen to meet the needs of society, rather than as a set of arbitrary rules; to value communication in mathematics learning more highly than students in conventional classes; and to be more likely than their conventionally-taught peers to see a mathematical element in everyday activity. These outcomes occurred while the IMP students maintained performance levels on the mathematics portion of the SAT at or above those of their peers in conventional classes. If student achievement outcomes are comparable, the mathematics education community must decide whether it values these consequences of a problem-based curriculum.

Among the debates engaging the energies of the mathematics education community, one of the more energetic has concerned the role of problem solving in mathematics instruction. This debate has encompassed issues from what constitutes a problem to whether problem solving should be the medium or the message of the mathematics curriculum (cf., Clarke & McDonough, 1989; Lawson, 1990; Owen & Sweller, 1989; Schoenfeld, 1985). Claims and counter-claims have been made regarding the advisability and the feasibility of basing a mathematics syllabus on non-routine mathematics tasks. Attempts to evaluate the success of such curricula have typically employed achievement tests to distinguish student outcomes. The authors of this study felt that a problem-based curriculum would be characterized more appropriately by the belief systems which the instructional program engendered in participating students than by the students' achievement on conventional mathematical tasks.

It is students' belief systems that are likely to influence the students' subsequent participation in the study of mathematics, to structure their consequent learning of mathematics, and to guide and facilitate the application of mathematical skills to everyday contexts. If it could be demonstrated that student achievement on conventional mathematics tasks was enhanced by a problem-based program, and if student performance on non-routine problem-solving tasks was heightened by such a program, the ultimate value of the instruction would depend still on whether the student chooses to continue to study mathematics, develops a set of beliefs which supports and empowers further learning, and sees any relevance in the skills acquired in class for situations encountered in the world beyond the classroom. Conventional instruction does little to address such concerns, and research has commonly ignored such outcomes.

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The evaluation of teaching experiments currently in progress must address these other consequences of instruction. In discussing their work on "one-on-one constructivist teaching," Cobb, Wood, and Yackel (1990) drew attention to non-conventional learning outcomes. This instructional approach provides opportunities for the children to construct mathematical knowledge not found in traditional classrooms. The difficulty for researchers evaluating innovative classroom practices is that many of the conventional research tools are insensitive to the behaviors and the knowledge that distinguishes such instruction. This concern is also relevant where the goals of the program are affective as well as cognitive. Since studies such as that of Erlwanger (1975) drew attention to the significance of a student's belief system regarding mathematics and mathematical behavior, research into effective teaching practice has had an obligation to address student belief outcomes. This obligation is linked to the recognition of "cognition as socially situated activity" (Lave, 1988, p.43). While the subject of student beliefs has been discussed usefully in a variety of forums (for instance, Clarke, 1986; Cobb, 1986), research studies have still to accept a responsibility to address student belief and perception outcomes routinely in the evaluation of instructional programs. The study reported here is one attempt to do so.

The Instructional Program

In 1989, the California Postsecondary Education Commission (CPEC) released a request for proposals that would drastically revamp the Algebra I-Geometry-Algebra II sequence. The curriculum envisioned in the guidelines would set "problem solving, reasoning and communication as major goals; include such areas as statistics and discrete mathematics; and make important use of technology" (CPEC, 1989, p. 4). The Interactive Mathematics Project (IMP) Curriculum Development Program obtained funding to develop and field test three years of problem-based mathematics that would satisfy six of the University of California requirements for high school mathematics.

Program Goals

The goals of IMP were to:

- broaden who learns mathematics, by making the learning of core mathematics accessible to groups previously underrepresented in college mathematics classes;
- expand what mathematics was learned, consistent with the recommendations of the Curriculum and Evaluation Standards (NCTM, 1989), emphasizing problem solving and the communication of mathematical ideas;
- change mathematics instruction, by requiring students to be active learners and investigators, by integrating the study of mathematical domains, such as algebra, geometry and statistics, with each other and with areas of application, and by making use of current technology;
- change how teachers perceive their roles, by emphasizing the role of the teacher as guide and model learner and by changing dominant modes of classroom communication from teacher explanation to student interaction;
- change how mathematics learning is assessed, by assessing students' use of mathematical knowledge to solve complex problems, and by diversifying assessment strategies to include student portfolios, self-assessment, teacher observations, oral presentations, and group projects, as well as written homework and tests.

Pupil Selection

Methods of selection of pupils for participation in the IMP classes varied. The principal criterion was student self-nomination. One high school collected information on student performance, instructional preferences, and academic history and then selected "60% of the group who would have been placed in Algebra and 40% from those below." It was the opinion of the various school administrations that the academic standing of the sample of IMP students arising from the various selection criteria was certainly no higher than that of the students in conventional Algebra classes. In fact, in the case of the high school just mentioned, the overall academic standing of students commencing IMP was almost certainly lower than that of commencing Algebra students.

Teacher Selection

Teachers were also self-nominated.

The IMP Materials

The IMP materials consist of modular units, each requiring approximately five weeks of instructional time. These units employ historical, literary, scientific and other contexts to provide a thematic coherence to the pupils' exploration of mathematics. For instance, in one unit the Edgar

Allan Poe short story *The Pit and the Pendulum* is used to facilitate student investigation of variation, measurement uncertainty, normal distribution, graphing, mathematical modeling, and non-linear functions. The instructional sequence of each unit addresses mathematical concepts and skills and mathematical problem solving in a context that provides both the rationale for the skills being acquired and a means of integrating newly acquired knowledge within a coherent structure.

An IMP Classroom

Class size averaged around 32 students. Classroom activities were typified by group work, writing, and oral presentations. Graphing calculators were available at all times. The characteristics of IMP and Algebra classrooms, as perceived by the pupils, were documented in the course of this study, and are detailed in the results presented later in this paper.

Assessment Practices

Priority was given in IMP classrooms to a diversity of assessment strategies, consistent with the program goals. For example, in one IMP class, grades were calculated from student performance on homework (30%), classwork and class participation (30%), problems of the week (30%), and unit assessments (10%). It appeared that most assessing of Algebra students was through weekly quizzes and chapter tests.

Method

Subjects

The subjects of this study were 182 students at three Californian high schools participating in the IMP program outlined above. In addition, matching data were collected on 74 Algebra 2 students and 143 Algebra 4 students from the same schools. Data on an additional 52 Algebra 2 students were collected from a fourth high school to provide a comparable sample of students at the same level as the IMP pupils.

Procedures and Measures

During June, towards the end of the academic year, all students completed two questionnaires. The student questionnaire was constructed in large part by combining items developed and tested in a study of student mathematics journal use and a further study of student self-assessment.

The *Mathematics Belief* questionnaire examined student perceptions of their mathematical competence, and student beliefs about mathematical activity and the origins of mathematical ideas. Students were asked to report their perceptions of those valued activities, which, in their opinion, assisted their learning of mathematics, in addition to their perceptions of what constituted typical classroom activities in mathematics and their attitudes towards mathematics. The *Mathematics World* questionnaire required students to identify the extent to which specific everyday activities were mathematical. At the time of administration of the questionnaires, IMP students had completed almost one year in the program.

In addition, the next fall, the Mathematics Scholastic Aptitude Test (SAT) was administered to the school populations, facilitating comparison of the mathematics performance of IMP students with their peers in conventional classes.

Mathematics belief. The mathematics belief questionnaire was adapted from an instrument employed to measure the student belief outcomes of an innovative program employing student journals (Clarke, Stephens & Waywood, 1992; Clarke, Waywood, & Stephens, 1994). Every item was validated through interviews with students. Minor changes in phrasing were made for administration in American schools. Some sample items were:

1. If I had to give myself a score out of 10 to show, honestly, how good I think I am at math, the score I would give myself would be…

3. The ideas of mathematics:

   A. Have always been true and will always be true.
   Agree Disagree

   D. Developed as people needed them in everyday life.
   Agree Disagree

   F. Are most clearly explained using numbers.
   Agree Disagree

5. When I am doing mathematics at school, I am likely to be:

   A. Talking
   Always Often Sometimes Seldom Never

   C. Writing words
   Always Often Sometimes Seldom Never

   F. Working with a friend
   Always Often Sometimes Seldom Never

   I. Listening to other students
   Always Often Sometimes Seldom Never

   K. Working from a textbook
   Always Often Sometimes Seldom Never

7. An adaptation of the IMPACT instrument (Clarke, 1987) was included as item 7, including such sub-items as:

   Write down one new problem that you can now do.
   How could math classes be improved?

Student attitudes towards mathematics classes were measured explicitly through the sub-item:

How do you feel in math classes at the moment? (circle the words which apply to you.)

A. Interested	B. Relaxed	C. Worried
D. Successful	E. Confused	F. Clever
G. Happy	H. Bored	I. Rushed

J. (Write one word of your own) _____________

The response alternatives provided in this sub-item arose from extensive interviewing of high school students in the course of a study of student mathematical behavior at the point of transition from primary school (elementary school) mathematics to high school mathematics (Clarke, 1985, 1992). The IMPACT instrument, from which the sub-item was drawn, was extensively field-tested with 753 grade 7 students over a period of one year (Clarke, 1987).

Mathematics world. The mathematics world questionnaire was adapted for American administration from an instrument employed in a study of community perceptions of mathematical activity (Clarke & Wallbridge, 1989; Wallbridge, 1992). In this questionnaire, students were asked to indicate whether they thought specific everyday activities were highly mathematical, quite mathematical, slightly mathematical, barely mathematical, or not mathematical. The activities listed included:

4. Cooking a meal using a recipe
7. Playing a musical instrument
9. Buying clothing at a sale

A complete listing of all questionnaire items is available in Clarke, Wallbridge, and Fraser (1992).

Results

The results that follow make reference to three groups of students to whom questionnaires were administered:

i. 180 IMP students – mean age 15.3 years
ii. 126 Algebra 2 students – mean age 15.4 years
iii. 137 Algebra 4 students – mean age 16.9 years

Comparing the Algebra 2 and Algebra 4 Samples

In all, 48 student measures were generated through the two questionnaires. Algebra 2 and Algebra 4 samples (n = 126 and n = 137, respectively) did not differ significantly on any of the 48 measures except the use of worksheets, for which the Algebra 4 students recorded an even lower incidence than did the Algebra 2 students, and the importance accorded to the teacher's explanations: Algebra 4 students attached lower importance to these than did the Algebra 2 students. It seems reasonable to summarize these findings by observing that, with respect to the beliefs documented here, conventionally-taught students adhere to a specific set of beliefs with a high level of stability over time. These beliefs and the associated perceptions of classroom practice were sufficiently distinct from those held by IMP students to clarify the characteristics of both class types. Results are given as comparisons between IMP and Algebra 2 students since these represent the most similar sample populations.

In each table where comparisons are made between groups the corresponding p value is given. Differences between groups which achieved statistical significance are asterisked.

Student Mathematics Achievement on Conventional Tests

Where comparison was possible between IMP and Algebra students at the same school, mean SAT scores for IMP classes were higher than mean SAT scores for traditional Algebra/Geometry classes. Pair-wise comparison of group means (t test) was used to identify any statistically significant difference under a conventional null hypothesis assumption. At one high school the difference in performance was statistically significant. These results are documented in Table 1.

Table 1
SAT Scores for Algebra and IMP Students on Two School Sites

School	Class type	Mean SAT score	SD	p value
A	Algebra (n = 83)	420.48	82.96	.0372*
	IMP (n = 74)	443.37	77.21	
B	Algebra (n = 86)	367.56	57.02	.1003
	IMP (n = 67)	373.88	60.95	

Student Perceptions of Their Mathematics Competence

IMP students were significantly more likely to rate themselves highly on how good they were at mathematics than were Algebra 2 students (Table 2).

Sample Item:

> If I had to give myself a score out of 10 to show, honestly, how good I think I am at math, the score I would give myself would be:

Two comments should be made concerning this higher self-rating by IMP students. First, SAT scores indicated that where comparison was possible IMP students tended to be more capable at conventional mathematics tasks than were their peers in Algebra classes, which suggests that these self-ratings had some basis in fact. Second, the difference in self-ratings can also be interpreted as a difference in confidence. We would suggest that heightened self-confidence in mathematics is likely to lead to increased participation in further mathematics, and a greater likelihood that the student will make use of the mathematical skills acquired. Both are desirable outcomes.

Table 2
Self-rating Scores for IMP and Algebra 2 Students

Class type	**Mean**	**SD**	***p* value**
Algebra 2 ($n = 125$)	6.86	1.2	.0012*
IMP ($n = 173$)	7.5	1.38	

Student Attitude Toward Mathematics Classes

IMP students were significantly more likely to feel positive about mathematics classes (Table 3).

Sample item:

> How do you feel in math classes at the moment? (circle the words which apply to you.)
>
> A. Interested B. Relaxed C. Worried
> D. Successful E. Confused F. Clever
> G. Happy H. Bored I. Rushed
> J. (Write one word of your own) _____________

A student attitude index was calculated by scoring each positive response +1 and each negative response –1, and summing for each student.

Table 3
Student Attitude Index for IMP and Algebra 2 Students

Class type	**Mean**	**SD**	***p* value**
Algebra 2 ($n = 126$)	–.52	1.85	.0001*
IMP ($n = 174$)	.97	2.14	

Student Perceptions of Mathematical Activity

The distinguishing characteristic between the problem solving students and the Algebra 2 students was the degree to which they perceived mathematics to be a mental activity (Table 4).

Sample Item:

> Mathematics is something I do (circle one or more):
>
> A. Every day as a natural part of living
> B. Mostly at school
> C. With a pencil and paper
> D. Mostly in my head
> E. With numbers

Table 4
Student Perceptions of Mathematical Activity

Response alternatives	**Class type**	**Proportion (%)**
Every day as a natural part of living	Algebra 2	49
	IMP	52
Mostly at school	Algebra 2	63
	IMP	64
With a pencil and paper	Algebra 2	41
	IMP	42
Mostly in my head	Algebra 2	27
	IMP	39
With numbers	Algebra 2	51
	IMP	46

Table 4 is significant in the context of this paper in that it was only in these perceptions of mathematical activity that the IMP and Algebra students responded in a similar fashion. The marked differences in beliefs and perceptions reported by the two groups, which constitute the essential findings of this study, are only evident in Table 4 in the significantly greater inclination for IMP students to report mathematics as being a mental activity.

Student Perceptions of Mathematical Ideas

IMP students were more likely to agree that mathematical ideas could be clearly explained using every day words that anyone could understand, than were Algebra 2 students. IMP students were also less likely to view the ideas of mathematics as ones that can only be explained using numbers and language specific to mathematics. The IMP students were more likely to view mathematics as having developed in response to people's needs. The IMP students were also less likely than the Algebra 2 students to view mathematics as having been invented by mathematicians or to hold that the ideas of mathematics have always and will always be true. Figure 1 and Table 5 document these differences.

Sample Item:

The ideas of mathematics

A. Have always been true and will always be true.
Agree Disagree

B. Were invented by mathematicians.
Agree Disagree

C. Were discovered by mathematicians.
Agree Disagree

D. Were developed as people needed them in daily life.
Agree Disagree

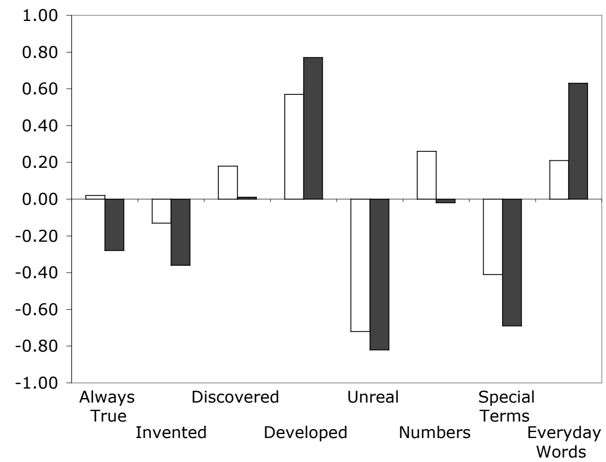
E. Have very little to do with the real world.
Agree Disagree

F. Are most clearly explained using numbers.
Agree Disagree

G. Can only be explained using mathematical language and special terms.
Agree Disagree

H. Can be explained in everyday words that anyone can understand.
Agree Disagree

In summary: IMP students were more likely to hold a socially-oriented view of the origins and character of mathematical ideas rather than a Platonist belief in the existence of mathematical absolutes awaiting discovery.



Algebra students = light bars, IMP students = dark bars; a positive mean value indicates agreement; a negative mean value indicates disagreement

Figure 1. Students' perceptions of the ideas of mathematics.

Table 5
Students' Perceptions of the Ideas of Mathematics

Sub-items	Class Type	Mean
Have always been true and will always be true.	Algebra 2 IMP	.02 –.28
Were invented by mathematicians.	Algebra 2 IMP	–.13 –.36
Were discovered by mathematicians.	Algebra 2 IMP	.18 .01
Developed as people needed them in daily life.	Algebra 2 IMP	.57 .77
Have very little to do with the real world.	Algebra 2 IMP	–.72 –.82
Are most clearly explained using numbers.	Algebra 2 IMP	.26 –.02
Can only be explained using mathematical language and special terms.	Algebra 2 IMP	–.41 –.69
Can be explained in everyday words that anyone can understand.	Algebra 2 IMP	.21 .63

Student Perceptions of School Mathematics

The IMP students were significantly more likely to agree that writing was important in helping them to understand mathematics. The IMP students were also more likely to see value in talking to other students than were the Algebra 2 students. The IMP students were significantly less likely than the Algebra 2 students to view drill and practice as the best way to learn mathematics.

Sample Item:

Circle the alternative which best describes how true you think each statement is (SA = Strongly Agree, A = Agree, D = Disagree, and SD = Strongly Disagree):

1. Explaining ideas clearly is an important part of mathematics.
   SA A D SD
2. Mathematics does not require a person to use very many words.
   SA A D SD
3. Writing is an important way for me to sort out my ideas in mathematics.
   SA A D SD
4. Talking to other students about the mathematics we are doing helps me to understand.
   SA A D SD
5. Drill and practice is the best way to learn mathematics.
   SA A D SD

The distinguishing characteristics between the IMP students and the Algebra 2 students were:

- the importance attached by IMP students to writing in mathematics ($p = .04^*$)
- the degree to which IMP students perceived talking to other students as useful in helping them to understand mathematics ($p = .0005^*$)
- the relative importance attached to drill and practice by the Algebra 2 students ($p = .0001^*$)

Student Perceptions of Mathematical Activity at School

The greatest degree of difference between IMP students and the Algebra 2 students was evident in their perceptions of mathematical activity at school. Table 6 illustrates the differences in student perceptions of their mathematics classrooms. In these statistics, the differences between the two class types are most clearly illustrated. Key differences between IMP and Algebra 2 classes can be summarized as follows:

- IMP students were significantly more likely to be writing words and drawing diagrams, and less likely to be writing numbers.
- IMP students were significantly more likely to be working with a friend or with a group, and less likely to be working on their own.
- While there was no difference between IMP and Algebra 2 classes in the relative frequency of listening to the teacher, IMP students were significantly more likely to be listening to other students than were students in Algebra 2 classes.
- IMP students were significantly more likely to be working from a worksheet and less likely to be copying from the board or working from a textbook.

Students were asked to respond on a four-point scale to the cue "When doing mathematics at school, I am likely to be..." The mean values in Table 6 should be read as students' perceptions of the relative frequency (on a 5-point scale) with which they engaged in each of the listed activities.

Table 6
Mean Relative Frequency of Student Engagement

Sub-items	Class type	Mean	SD	*p* value
Talking	Algebra 2 IMP	2.25 2.59	1.0 .88	.002*
Writing numbers	Algebra 2 IMP	3.13 2.66	.86 .83	.0001*
Writing words	Algebra 2 IMP	1.70 2.68	.93 .94	.0001*
Drawing diagrams	Algebra 2 IMP	1.85 2.70	.82 .83	.0001*
Working on my own	Algebra 2 IMP	2.59 1.91	.90 .87	.0001*
Working with a friend	Algebra 2 IMP	2.10 2.69	.90 .80	.0001*
Working with a group	Algebra 2 IMP	1.96 3.17	1.0 .80	.0001*
Listening to the teacher	Algebra 2 IMP	2.80 2.75	.98 .96	.63
Listening to other students	Algebra 2 IMP	2.19 2.79	.96 .83	.0001*
Copying from the board	Algebra 2 IMP	2.40 1.91	1.02 1.04	.0001*
Working from a textbook	Algebra 2 IMP	3.15 0.25	.93 .70	.0001*
Working from a worksheet	Algebra 2 IMP	2.05 3.32	1.06 .97	.0001*

Student Perceptions of the Relative Importance of Course Components

IMP students placed more value on working with others than did Algebra 2 students (p = .0001*). By contrast, Algebra 2 students valued the teacher's explanations (p = .0005), and the textbook (p = .0001) more than did IMP students.

Student Perceptions of Mathematics in Everyday Activity

IMP students were significantly more likely to identify a mathematical component in everyday activities than were Algebra 2 students. This result is evident in Table 7.

Table 7
Mean Math World Index for Algebra 2 and IMP Students (Incomplete responses from some students led to a slightly smaller sample size for both groups.)

Class type	Mean	SD	*p* value
Algebra 2 (n = 113)	19.292	5.591	.0014*
IMP (n = 172)	21.477	5.598	

In particular, the IMP students were more likely to view as mathematical:

- using a calculator to work out interest paid on a housing loan over 20 years (p = .003*)
- planning a family's two week holiday (p = .006*)
- chopping down a pine tree (p = .007*)
- buying clothing at a sale (p = .03*)
- painting the house (p = .0001*)

Gender Differences

Comparison was made in this study of the attitudes to mathematics of boys and girls in IMP and Algebra classes, and of the boys' and girls' self-ratings of their mathematics competence. These results are shown in Table 8.

Girls in both class types were less likely than boys to rate highly their own mathematical competence. However this difference was only statistically significant for students in Algebra classes. Both boys and girls in IMP classes had similar positive attitudes towards mathematics. In Algebra classes, both male and female students felt negatively towards mathematics, however boys' attitudes were less negative than those of girls. On the basis of these findings, it appears that the IMP program was of particular value to female students. The statistical significance of the direct comparison of Algebra 2 girls with IMP girls is quite clear from Table 8, where the difference in mean attitude and self-rating for the two groups of girls is even more striking than in the comparison of the Algebra and IMP cohorts reported in Tables 2 and 3.

Table 8
Gender Comparison of Self-ratings and Attitude Measures for Algebra 2 and IMP Classes

Class type & Measure	Gender	Mean	SD	*p* value
Algebra 2 Self-rating	Male (n = 58)	7.333	1.875	.0101*
	Female (n = 67)	6.433	1.994	
IMP Self-rating	Male (n = 77)	7.636	1.297	.081
	Female (n = 96)	7.271	1.410	
Algebra 2 Attitude	Male (n = 58)	–0.345	1.821	.3176
	Female (n = 68)	–0.676	1.872	
IMP Attitude	Male (n = 78)	0.756	2.021	.2344
	Female (n = 96)	1.146	2.234	

Conclusions

For the purpose of drawing conclusions from the findings reported here, the inclusion of the Algebra 4 sample in the study encourages the extrapolation of conclusions from comparisons of class types at a specific grade level to more general conclusions comparing problem-based and conventional instruction for high school mathematics classes.

The conclusions that follow, however, relate specifically to the study sample.

The Students as Learners

1. IMP students rated themselves as significantly more mathematically able than did the Algebra students.
2. IMP students held a significantly more positive attitude towards their mathematics classes than did the Algebra students.
3. On school sites where comparison was possible, IMP students averaged higher SAT scores than did pupils of conventional classes.

4. IMP classes appeared to have less negative outcomes for girls than did conventional Algebra classes.

Student Perceptions of Mathematics

5. IMP students were significantly more likely to perceive mathematics as a mental activity.
6. IMP students held beliefs consistent with a view of mathematics as arising from individual and societal need; while Algebra students were more likely to view mathematical ideas as having an independent, absolute and unvarying existence.
7. The IMP students were significantly more likely to perceive mathematics as having applications in daily use.
8. IMP students were significantly more likely than Algebra students to believe that mathematical ideas can be expressed "in everyday words that anyone can understand."

Instructional Alternatives

9. IMP students attached significantly more value to interactive learning situations; whereas Algebra students valued "the teacher's explanations" and "the textbook."

10. IMP students valued writing and talking to other students as assisting their learning. Algebra students were significantly more likely to value "drill and practice."

11. (a) It is possible to identify a coherent and consistent set of classroom practices which can be associated with conventional instruction (cf. Clarke, 1984).

11 (b) It is similarly possible to identify a set of classroom practices which identify, in the students' view, the characteristics of the IMP classroom.

11 (c) The characteristics of these two instructional models are sufficiently distinct to represent clear alternatives.

In conclusion, the classroom practices of the IMP program, as reported by the students, placed greater emphasis on a variety of modes of communication and on facilitating student-student interaction than was the case with conventional instruction. By contrast, conventional instruction was perceived as solitary, text-driven, and typically expressed through special terms and numbers.

To what aspect of the IMP experience might we attribute the student beliefs documented in this study? The small-group, interactive classroom and the problem-based mathematics curriculum represent two key characteristics of the Interactive Mathematics Project. Whether such belief systems would arise in interactive classrooms lacking a problem-based emphasis or in more conventionally taught, problem-based classrooms is a matter for further research.

Certainly the IMP program has provided students with significantly different experiences from those found in conventional mathematics classes, and these experiences appear to have led to demonstrably different beliefs about mathematical activity, mathematics learning, school mathematics, and the mathematics evident in everyday activity. The findings of other studies suggest that students whose instruction has included experience with open-ended tasks can be expected to perform more successfully on both conventional and non-routine tasks than students lacking that experience (for instance, Sweller, Mawer & Ward, 1983). In combination, this research suggests that a problem-based curriculum is capable of developing traditional mathematical skills at least as successfully as conventional instruction, while simultaneously developing non-traditional mathematical skills and engendering measurably different belief systems in participating students. The nature of these different beliefs has formed the basis of this study.

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