Mathematics Placement Test: Helping Students Succeed

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A study was conducted at Merrimack College in Massachusetts to compare the grades of students who took the recommended course as determined by their mathematics placement exam score and those who did not follow this recommendation. The goal was to decide whether the mathematics placement exam used at Merrimack College was effective in placing students in the appropriate mathematics class. During five years, first-year students who took a mathematics course in the fall semester were categorized into four groups: those who took the recommended course, those who took an easier course than recommended, those who took a course more difficult than recommended, and those who did not take the placement test. Chi-square tests showed a statistically significant relationship between course grade (getting a C– or higher grade) and placement advice. The results indicate that students who take the recommended course or an easier one do much better than those who take a higher-level course or do not take the placement exam. With achievement in coursework as the measure of success, we concluded that the placement test is an effective tool for making recommendations to students about which courses they should take.

There is a widespread recognition of the need for appropriate placement in the mathematics courses for undergraduate freshmen. Many colleges and universities around the nation have used the Mathematical Association of America (MAA) Placement Test; others have designed their own exams or used a combination of placement exams and other measurements, such as ACT or SAT mathematics scores and high school GPAs. Since the MAA has discontinued its placement test program, the responsibility has been put on individual institutions to develop their own placement exam. The purpose of our study was to determine the effectiveness of Merrimack College's placement test by examining the connection between students enrolling in recommended courses and their success in those courses.

Literature Review

In this section, we investigate some of the specific methods reported in the literature for placing undergraduates in their first mathematics course. We point to some of the assumptions in these reports and suggest some of the drawbacks to the method of placement.

Cederberg (1999) reported on the three placement tests administered at St. Olaf College in Minnesota.

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She explained that the placement recommendations were based on a large number of regression equations that required considerable expertise in development and periodic redefinition. The placement test also required the coordination of numerous categories of student data used in the equations. Approximately 85% of students who enrolled in a calculus course based on the recommendations from the placement test at St. Olaf College received a grade of at least C–.

Cohen, Friedlander, Kelemen-Lohnas and Elmore (1989) recommended a placement procedure that was less technically sophisticated than St. Olaf's, but still required considerable background data about students. They recommended multiple criteria methods, which included a placement test customized to an institution's curriculum. They started with sixty variables, and found the eight best predictors: high school graduation status, number of hours employed, units planned, age, high school grade point average, mathematics placement test score, reading placement test score, and English placement test score. Cohen et al.'s study was based on (a) thousands of surveys completed by students and faculty members at eight California community colleges, (b) a comparison between student scores on assessment tests and grades in different courses, and (c) the relationship between student characteristics and their grades.

Krawczyk and Toubassi (1999) described a simpler placement procedure used by the University of Arizona. The University of Arizona used two placement tests adapted from the 1993 California Mathematics Diagnostic Testing Project (see http://mdtp.ucsd.edu/). Students chose which test they felt was most appropriate for their ability and choice of

major. One test covered intermediate algebra skills and placed students in one of three levels of algebra or a liberal arts mathematics course. The second test covered college algebra and trigonometry skills and placed students in courses from finite mathematics through Calculus I. In the fall of 1996, their data indicated that approximately 17% of freshmen placed in College Algebra through Calculus I failed or withdrew from their respective courses, compared with a 50% attrition rate in the early 1980's before the mandatory testing and placement. Apart from a test, they also considered other factors, such as high school GPA.

A number of studies have investigated the use of standardized tests, such as ACT and SAT. Bridgeman and Wendler (1989) found that the mathematics SAT score was a relatively poor predictor of grades compared to placement exams. Their results were based on grades from freshman mathematics courses at ten colleges. Odell and Schumacher (1995) showed that a placement test used in conjunction with mathematics SAT scores could be a better predictor than SAT scores alone. Their conclusion was based on data from a private business college in Rhode Island. Callahan (1993) studied the criteria followed at Cottey College in Missouri to place students in the appropriate level course, as well as their placement success rates. As with the studies mentioned above, Cottey College used several variables to achieve their results – the MAA Placement Test, ACT and SAT mathematics scores, and years of high school mathematics taken. Each of these studies assumed that the success rates were based on students following the placement advice. Mercer (1995), on the other hand, conducted a study to compare pass rates in a college-level mathematics class for mathematically unprepared students who enrolled in developmental courses and those who did not. The results of this study showed a statistically significant relationship between passing the course and following placement advice.

Background

Merrimack College, located in North Andover, Massachusetts, is a small four-year Catholic college offering programs in the liberal arts, business, the sciences, and engineering. Among the college's distribution requirements, students must complete three mathematics or science courses, with no more than two courses from the same department. Most of the students take at least one mathematics course. Most liberal arts majors usually choose Basic Statistics, Finite Mathematics, or Discrete Mathematics to satisfy the mathematics/science requirement. During data collection for this study, business administration majors were required to take Applied College Algebra, Calculus for Business, and one other mathematics course. Students majoring in science or engineering generally were required to take more mathematics; for example, engineering students were required to take three calculus courses and one course in differential equations. They also took Precalculus if they did not place out of this course on their placement exam.

Since some incoming freshmen are not prepared to take a college-level mathematics course, a non-credit developmental mathematics course, Math I, has been offered at Merrimack College since the fall of 1994. Before 1994 we administered a mathematics diagnostic exam to incoming students, but were unable to accommodate students who were not ready to take a college-level mathematics course. Instead, they enrolled in a mathematics course at a higher level than their exam score indicated they should. There was a high failure rate among these students.

Because students at Merrimack College often have difficulty in other courses if they have not completed Math I, proper mathematics placement has become important to all of our departments. For example, the Chemistry Department now requires the students who place into Math I to complete the course before they take several of their chemistry courses. Krawczyk and Toubassi (1999) have found similar results at the University of Arizona in which all chemistry students and 90% of biology students—whose placement test scores indicated they should be placed in intermediate algebra or lower—received grades below C– in their chemistry or biology courses.

This interest in correct mathematics placement extends beyond the chemistry department as evidenced by the many questions from the business and liberal arts faculty, as well as science and engineering faculty at the meetings in preparation for orientation advising. A major reason for our concern about student success is that successful students are more likely to remain in their studies. A high level of student retention is not only academically and socially desirable at a school, but it makes sense economically.

We do not place students according to their prior high school GPA or whether they have taken calculus. We do not assume that these factors indicate whether or not they need algebra. In fact, many students placed into Math I have had four years of high school mathematics, including precalculus (and occasionally calculus), but according to our placement test they do not appear to understand the basic concepts of algebra.

Mathematics Placement Test

All incoming freshmen at Merrimack College are expected to take a mathematics placement test developed by members of the Mathematics Department. There are two versions of the exam, one for students who will major in Business Administration or Liberal Arts, and one for students in Science and Engineering. The version for Business Administration and Liberal Arts consists of two parts, elementary and intermediate algebra. The version for Science and Engineering students contains a third part that tests students' understanding of functions and graphs. Students are instructed not to use calculators. From 1994 until 1999 the placement exam was taken at Merrimack College in June during orientation or at the beginning of the academic year. Since 2000 the exam has been mailed to students at home. A Scantron form with the students' answers is mailed back to Merrimack College and graded. Students are informed that using outside resources or calculators may result in their being placed in a course for which they are unprepared and may result in their failing or withdrawing from the course. There is not a difference between these mail-in results and the previous monitored exam results with respect to the percentages of students who place into the various mathematics courses. Thus, we believe that most students heed our warnings. The recommendations are available to students and advisors during June orientation. See Appendix A for some problems similar to those given in the mathematics placement test.

Part I of the placement test consists of seventeen questions on elementary algebra. If a student does not answer at least fifty percent of these questions correctly, then that student must take Math I. For students who score above fifty percent on Part I, liberal arts majors may take any mathematics elective course; business administration majors are placed into a college algebra or a business calculus course, depending on their overall score; and science and engineering students may be placed into a college algebra, Precalculus or Calculus I course, depending on their total score. The specific recommendations resulting from the mathematics placement exam are as follows:

Science and Engineering:
Score below 9 in Part I $\Rightarrow$ Math I
For those who score above 8 in Part I:
- Score 20 or lower in Parts I–III $\Rightarrow$ Applied College Algebra
- Score between 21 and 34 in Parts I–III $\Rightarrow$ Precalculus
- Score of 35 or higher in Parts I–III $\Rightarrow$ Calculus I

Business:
Score below 9 in Part I $\Rightarrow$ Math I
For those who score above 8 in Part I:
- Score below 30 in Parts I–II $\Rightarrow$ Applied College Algebra
- Score of 30 or higher in Parts I–II $\Rightarrow$ Calculus for Business

Liberal Arts:
Score below 9 in Part I $\Rightarrow$ Math I
Any other score $\Rightarrow$ Enroll in a mathematics elective course

Data Analysis

We have performed statistical analyses since 1997 to study whether there was a relationship between the score on the placement test and how well first year students did on the first mathematics course taken at Merrimack College. In order to determine this relationship, we first examined the correlation between total score on the placement test and students' grades. A preliminary study with $n = 372$ showed that the correlation between the grade earned in a mathematics course and the total score on the placement test was 0.466. For the same study, the correlation between grades earned and SAT scores was 0.334. A multiple regression model to estimate the final grade based on the SAT score and the placement exam score gave the equation: Final Grade $= 0.80 + 0.00122(SAT) + 0.0641(Placement Exam)$. In addition, a t test for each of the variables in the model indicated that the SAT had a t value of 1.01 ($p = 0.314$) and the Placement Exam had a t-value of 6.58 ($p < 0.0005$). Based on the t test values and the p-values, we removed the SAT variable from the model and concluded that the placement exam was a better predictor of student success.

Although we first used t-tests to compare the total score on the placement test with SAT scores as predictors of students' final grades, we ultimately decided against them as a means to further examine the effectiveness of our placement test as a placement tool for two reasons. First, there was not a true linear relationship between the total score on the placement test and placement. It was more like a step-function, with a *range* of scores in each part of the test being considered for placement. Second, there was a relatively weak relationship between total score on the placement test and final grades because students are placed into so many different levels of courses. For example, a student may have a very low placement test score, be properly placed into an elementary algebra class, and earn a high grade in that course. Thus, a simple correlation between total score on the placement test and grades earned was considered inappropriate to determine the effectiveness of the

placement test, and therefore we decided to categorize the data.

Each first-year student was categorized according to the level of mathematics course taken: (1) the course was easier than that recommended by the placement test score; (2) the course was the recommended course; (3) the course was more difficult than that recommended (higher-level); or (4) the placement test was not taken. Although the test was required, some students—usually transfer students—were allowed to take a mathematics course based on their mathematics grade(s) at a previous institution. This policy has not worked well and is being changed to require all incoming students to complete the placement test. Within these categories, students were counted according to whether they (a) did well (received a grade of C– or better) or (b) did poorly (received a grade below C–). Chi-square tests were performed to determine whether there was a relationship between the level of the course taken and the grade received in the class. Given the eight possible categories previously described in this study, the chi-square test indicated whether the percentage of students, say, who *did well* in each category of course level taken was significantly different from that in any of the other categories. The use of the chi-square test assumes that a random sample, representative of the population, was taken. In this study, we used the entire population of freshmen students who took mathematics in their entering fall semester at our college for the years from 1997 to 2001 ($n = 1710$). The null hypothesis stated that there was no association between the two variables. The alternative hypothesis stated that the grade depended on whether or not the student followed the placement test result recommendation.

Results

We wanted to know whether there was an association between the level of the course taken and the grade earned. Table 1 shows the number and percentage of students who did well in the class (C– or higher) and the number and percentage of students who did poorly (D+ or lower, or withdrew from the class) from 1997 through 2001. It was generally accepted that grades of D and F were unsatisfactory, as evidenced by the fact that almost all comparable studies used cut-off grades of C or C–. Those that used the C cut-off often had a minimum grade requirement of C for a student to move on to a subsequent course. Our department does not have such a requirement, and thus the choice of C– for this study was somewhat arbitrary. We felt that our professors might be less likely to slightly inflate the grade of C– to C than would those at schools with the minimum requirement. As described above, students were classified according to the level of the course taken: easier than the one recommended, the one recommended, or a higher-level course than the one recommended. A fourth category was used in order to include students who did not take the placement exam, but took a mathematics course.

The data were analyzed using the chi-square test (see Table 2). We found that there was a relationship between the two variables for first year students who took a mathematics course in the fall of 1997 [χ^2 (3, $n = 369$) = 24.66, $p < 0.0005$]. The same conclusion held for the data corresponding to the fall of 1998

Table 1

Number and Percentage of Students Each Year Disaggregated by Grade and Course Category

Course	1997		1998		1999		2000		2001	
	$\leq$ D+	$\geq$ C–	$\leq$ D+	$\geq$ C–	$\leq$ D+	$\geq$ C–	$\leq$ D+	$\geq$ C–	$\leq$ D+	$\geq$ C–
Easier	6	23	2	17	1	12	5	21	2	8
	21%	79%	11%	89%	8%	92%	19%	81%	20%	80%
Recom.	47	168	57	168	55	220	57	205	50	200
	22%	78%	25%	75%	20%	80%	22%	78%	20%	80%
Higher-Level	30	30	33	34	6	16	4	12	7	15
	50%	50%	49%	51%	27%	73%	25%	75%	32%	68%
No Exam	28	37	22	28	4	18	12	16	18	16
	43%	57%	44%	56%	18%	82%	43%	57%	53%	47%
n	111	258	114	247	66	266	78	254	77	239

Table 2
Contingency Table and Chi-Square Test (Expected counts are printed below observed counts; shaded cells indicate expected counts less than 5.)

Course	1997		1998		1999		2000		2001	
	≤ D+	≥ C−	≤ D+	≥ C−	≤ D+	≥ C−	≤ D+	≥ C−	≤ D+	≥ C−
Easier	6	23	2	17	1	12	5	21	2	8
	8.72	20.28	6.00	13.00	2.58	10.42	6.11	19.89	2.44	7.56
Recom.	47	168	57	168	55	220	57	205	50	200
	64.67	150.33	71.05	153.95	54.67	220.33	61.55	200.45	60.92	189.08
Higher-Level	30	30	33	34	6	16	4	12	7	15
	18.05	41.95	21.16	45.84	4.37	17.63	3.76	12.24	5.36	16.64
No Exam	28	37	22	28	4	18	12	16	18	16
	19.55	45.45	15.79	34.21	4.37	17.63	6.58	21.42	8.28	25.72
n	111	258	114	247	66	266	78	254	77	239
Chi-Square										
test	24.66		21.22		N/A		6.56		18.42	
p value	<0.0005		<0.0005		N/A		0.087		<0.0005	

$[\chi^2 (3, n = 361) = 21.22, p < 0.0005]$. We did not perform a chi-square test for the data corresponding to 1999 because there were 3 cells with expected counts less than 5 (see shaded cells in Table 2) and Moore (2001) does not recommend the use of the chi-square test when more than 20% of the expected counts are less than 5. The relationship was not significant for the fall of 2000 at an alpha level of 0.05 $[\chi^2 (3, n = 332) = 6.56, p = 0.087]$ when taking $p < 0.05$ to be statistically significant. The relationship was again significant for the data corresponding to the fall of 2001 $[\chi^2 (3, n = 316) = 18.42, p < 0.0005]$. In sum, the percentage of students who did well in their first undergraduate mathematics course was higher for those students who followed the advice or took an easier course than the one recommended based on their placement test score.

The chi-square test only showed evidence of some association between the variables. We then looked at the tables to determine the nature of the relationship or association (Moore 2001). Table 3 shows the number of students who took the recommended course, an easier one, a higher-level course, as well as the number of students who did not take the placement exam, and the mean and median grades on a 4.0 scale for each group from 1997 to 2001. The following correspondence between letter-grades and number-grades was used at Merrimack:

A	4.0	B	3.0	C	2.0	D	1.0
A−	3.7	B−	2.7	C−	1.7	D−	0.7
B+	3.3	C+	2.3	D+	1.3	F	0.0

In addition, students who withdrew from a course were counted and were assigned a number grade of 0 for this study.

Discussion and Conclusion

From Table 1 we saw that students who took the recommended course or an easier one did much better than those who took a higher-level course or did not take the placement exam. The same conclusion can be drawn from Table 3, with the exception of the year 2000. In 2000 there was no significant difference among the average grades received by those students who took the recommended course and those who took a higher-level course. An explanation for this may be that the proportion of students who took a higher-level course dropped from 19% in 1998 to 7% in 1999, and to 5% in 2000. The few students who took a higher-level course seemed to know that they would be able to succeed. In addition, the percentage of students placed into our developmental course, Math I, has been decreasing. Twenty five percent of the 521 who took the mathematics placement exam in 1998 were recommended to take Math I. Twenty percent of the 532 who took the mathematics placement exam in 1999 were in that category. Those figures went down to 14% out of 525 students in 2000 and 16% out of 517 in 2001. One of the reasons for this decrease was that students were allowed to retake the placement exam and to place out of our developmental mathematics course. Even though that possibility was available to students before, more effort has been made in the last

Table 3

Mean and Median Grade (on a 4.0 Scale) by Year and Course Category

Course	1997			1998			1999			2000			2001		
	n (%)	Mean	Median	n (%)	Mean	Median	n (%)	Mean	Median	n (%)	Mean	Median	n (%)	Mean	Median
Easier	29 8%	2.8	3.0	19 5%	2.7	2.7	13 4%	3.1	3.3	26 8%	2.4	2.5	10 3%	2.4	2.7
Recom.	215 58%	2.4	2.7	225 62%	2.4	3.0	275 83%	2.5	2.7	262 79%	2.5	2.7	250 79%	2.6	3.0
Higher-Level	60 16%	1.7	1.5	67 19%	1.6	1.7	22 7%	2.2	2.0	16 5%	2.4	2.5	22 7%	1.9	1.7
No Exam	65 18%	1.8	2.0	50 14%	1.7	2.0	22 7%	2.4	2.2	28 8%	1.8	2.2	34 11%	1.6	1.2
n	369			361			332			332			316		

few years to avoid improperly placing students in a non-credit course (Math I).

With the exception of 1999, there is no significant difference, using z tests, between the proportions of students who did well or poorly among those students who did not take the mathematics placement exam.

It is not surprising that students who enrolled in the recommended course or an easier course performed better than did students who enrolled in a higher-level course than the one recommended. What is important here is that approximately 80% of these students who took the recommended or easier course succeeded with a grade of C– or higher.

We have found the mathematics placement exam to be a useful tool to place students in the appropriate mathematics course, and we have been successful in convincing most of our students to follow our advice with respect to which courses to take. A challenge for us has been persuading students to take Math I, the non-credit class, when they are not ready for a college level course. While there is no perfect placement method, we have found that our test is better than SAT scores in placing students into the appropriate course. In addition, our multiple-choice test is easier to administer than methods used at other schools mentioned in this paper.

From our experience, a well-designed in-house placement test geared towards our curriculum is a simple and powerful tool for placing incoming students in an appropriate mathematics course. Keeping

adequate records and analyzing them with regard to the placement test's effectiveness, as we have done in this study, is a key component in maintaining the validity and reliability of the test itself. A number of years ago, the placement test score was viewed as the basis for a "recommended" mathematics course for each student, to be followed or not, as the student chose. Today, the entire Merrimack community appears to view the test score with increased respect because of the results presented in this paper. These ongoing statistical validations of the connections between proper placement and successful achievement have served to legitimize the placement test as part of a larger effort to increase retention on our campus.

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Appendix A: Sample Problems

The following problems are similar to the ones given on the actual exam, but the format is different. These sample problems are free response. The actual exam has a multiple choice format, in which several answers are provided to each problem, and only one of them is correct.

1. Without a calculator, evaluate:

    a. $|5 - 9|$ b. $\sqrt[3]{-8}$ c. $\dfrac{3.42}{.02}$ d. $\dfrac{8}{9} - \dfrac{5}{12}$

2. Simplify the following:

    a. $\dfrac{12x^2 + 3x}{3x}$ b. $\dfrac{-2x^3}{(-2x)^3}$ c. $\dfrac{(25x^4 y^8)^{-1/2}}{(3x^3)^2}$ d. $\dfrac{a}{a+2} + \dfrac{4}{a-3}$

    e. $5x + 18 - 4(x + 7)$ f. $\log(x^2 - 1) - \log(x + 2) + \log x$

3. Solve the following for x:

    a. $ax - b = 5$ b. $|2x + 1| = 5$ c. $x = \sqrt{2x + 15}$

    d. $\dfrac{1}{4^x} = 64$ e. $\begin{aligned} x + 2y &= 5 \\ x + 4y &= 7 \end{aligned}$ f. $x^2 + x < 6$

4. Solve, then simplify the radical: $x^2 - 2 = -4x$

5. Find an equation for the line through the points $(-1, -2)$ and $(1, 4)$. Give the slope, m, and the y-intercept, b.

The following problems are representative of the additional section of the Placement Test for Science and Engineering majors.

6. Let $f(x) = \dfrac{1}{\sqrt{x-1}}$. Find the domain of f.

7. Find the zeros of the function $f(x) = \dfrac{2x - 3}{x + 1}$.

8. Find the inverse function, $f^{-1}(x)$, if $f(x) = \sqrt[3]{x} + 2$.

9. Which of the following points is not on the graph of $y = e^{x^2 - 1}$?

    $(0, e^{-1})$, $(1, 0)$, $(2, e^3)$, $(3, e^8)$

10. Convert $135°$ to radians.

11. Simplify in terms of $\sin\theta$: $1 - \cos^2\theta = ?$

12. Which of the following is greatest? $\sin 30°$, $\sin 45°$, $\sin 90°$, $\sin 180°$