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Preservice Teachers' Mathematical Task Posing: An Opportunity for Coordination of Perspectives

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This article provides detailed analysis, from a radical constructivist perspective, of a sequence of letter-writing exchanges between a preservice secondary mathematics teacher and a high school student. This analysis shows the ways in which the preservice teacher gained understanding of the high school student's mathematics and attempted to pose tasks accordingly, leading to a fruitful mathematical exchange. In addition, this article also considers the same exchange from what could be considered broadly as a situated perspective towards learning. We conclude by suggesting that these perspectives could be considered compatible within this study if a distinction is made between the student's point of view and the researcher's.

The purpose of this article is two-fold. The first is to provide a detailed analysis of one sequence of letter-writing exchanges between a preservice teacher (PST) and a high school algebra student. These exchanges, which were part of the methods course the PST took, involved posing mathematical tasks to high school students. The rationale for this project was to provide PSTs with an opportunity to learn the practice of posing tasks and assessing students' mathematics; this work builds upon research conducted by Crespo (2003) who analyzed the mathematical communication between elementary students and PSTs. In expanding on Crespo's work, we developed several measures for gauging cognitive activity and showed that in many of these measures the PSTs posed better tasks., We demonstrate with sample exchanges that the PSTs learned to amend a single task in order to make it more accessible to the student.

The second purpose of this article is to examine our previous work from a perspective that might broadly be considered socio-cultural or situated. Following the recommendations of other researchers (Cobb, 2007; Lester, 2005), the sample analysis included in this paper suggests a way to coordinate psychological and sociological perspectives on learning. In particular, we examine the various social contexts in which the letter-writing interactions were situated while considering cognitive activities that each participant brought to bear on those situations.

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We conducted our analysis of the entire body of data using a constructivist lens. Afterwards, we examined one example in detail and wanted to extend the discussion by considering what another theoretical perspective suggests. This post-hoc discussion does not have a clear method as it is meant to be suggestive of several methods available to the researcher. Any one of these methods could ultimately be used to examine these data in more detail from a situated perspective. However, despite the post-hoc nature of the situated analysis, we conclude that this extended discussion and coordination of cognitive and situated perspectives has enriched our understanding of the letter-writing interactions. To support this, we provide detailed analyses of a sequence of interactions in which a particular letter-writing pair maintained socio-cultural boundaries, a process in which the student's individual understanding played a central role.

Method and Theoretical Orientation

From a psychological perspective, we were concerned with the kinds of elicited cognitive activity that we could infer from the task exchanges. We relied on descriptions of cognitive processes described in three main sources: Bloom's taxonomy as described by Kastberg (2003), *Principles and Standards of School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000), and a chapter on "cognitively complex tasks" by Stein, Smith, Henningsen, and Silver (2000). From Bloom's taxonomy we borrowed the four highest levels of cognitive activity: *Application*, *Analysis*, *Synthesis*, and *Evaluation*. We borrowed the process standards—*Communication*, *Connections*, *Problem Solving*, *Reasoning and Proof*, and *Representations*—from the

NCTM document. Finally, we borrowed Stein et al.'s levels of cognitive demand—*Memorization, Procedures without Connections*, *Procedures with Connections*, and *Doing Mathematics*—and used all of these as descriptors of elicited cognitive activity (see Appendix).

Our use of Stein et al.'s four levels of cognitive demand do not differ significantly from the descriptions provided by Stein et al. with the exception of the category created to describe the highest level of cognitive demand: *Doing Mathematics*. First, we briefly review the other three categories. *Memorization* is precisely as it sounds. If someone asked a student to state the definition of an acute triangle and the student responded, then this mathematical task would be inferred by the researcher as *Memorization*. We provide a full discussion of these measures in Norton and Rutledge (2008).

On the other hand, *Procedures with* and *without Connections* both involve the use of a mathematical procedure to accomplish the task objective. For example, should a student be confronted with the task of solving a system of two linear equations, the student may readily solve for one variable in one equation and then substitute into the other, or perhaps the student would be more inclined to use a matrix and row reduction methods. In this example, we would likely infer from the student's behavior that a procedure was definitely used, but we would not be able to infer that conceptual understanding accompanied this activity. This does not mean that the student does not understand linear equations but rather we could not infer this understanding from the student's interaction with this particular activity.

Continuing this vein of thought, suppose the student were given the same task, but in addition to solving the task using a procedure, the student sketched the graphs of the two functions in order to verify the reasonability of the answer. In doing so, the student would be indicating that the correct answer is the point at which the two lines intersect. This would show, not only skill with the procedure, but a more robust understanding of what it really means to solve for two equations with two unknowns. We would likely infer from this behavior that the student had engaged in *Procedures with Connections*.

Doing Mathematics as defined by Stein et al. (2000) was too vague for our purposes; therefore we turned to Schifter (1996) who considered conjecturing as a part of doing mathematics. We adapted her definition by adding the additional requirement that the student had to give some evidence that they had made a conjecture and then tested the conjecture. If we saw evidence of both, then we classified that particular exchange as a case of *Doing Mathematics*.

Problem Solving was a difficult measure to define and operationalize. However, we ultimately found the definition as provided by Lester and Kehle (2003) to be of use.

> Successful problem solving involves coordinating previous experiences, knowledge, familiar representations and patterns of inference, and intuition in an effort to generate new representations and patterns of inference that resolve the tension or ambiguity (i.e., lack of meaningful representations and supporting inferential moves) that prompted the original problem solving activity. (p. 510)

Therefore, we only considered an exchange to have elicited *Problem Solving* if we found some degree of struggle from the student. It is important to note that this is specific to the student and the problem with which they are engaging, regardless of how difficult the problem seemed to a third-person observer.

With these definitions in mind, it is now critical to consider the ways we could identify these various processes in the interactions. To this end, we adopted a radical constructivist perspective that highlighted the researchers' inferences about students' mathematical activity (von Glasersfeld, 1995). This perspective places certain demands on the way we identified these measures in practice. For example, consider the measure *Analysis*. We had to keep in mind that students construct their own meanings for mathematical situations and analyze mathematical situations in a way that is different from our own analyses. Thus, to infer that a student had engaged in *Analysis*, we had to be able to imagine a reasonable and consistent way of operating in which the student broke down the situation into constituent mathematical parts to better understand it. We were conservative in making such inferences; we needed to be able to find clear indications of *Analysis* that fit with the totality of the student's written response.

In the next section, we describe the interactions of Ellen and Jacques (both pseudonyms) and our inferences about the cognitive activities that those interactions elicited from Jacques. As with all of the letter-writing pairs at the time, Ellen was a PST enrolled in the first of two mathematics methods courses, and Jacques was completing the final weeks of his second trimester of Algebra I. The elicited activities found in the following analyzed sequence between

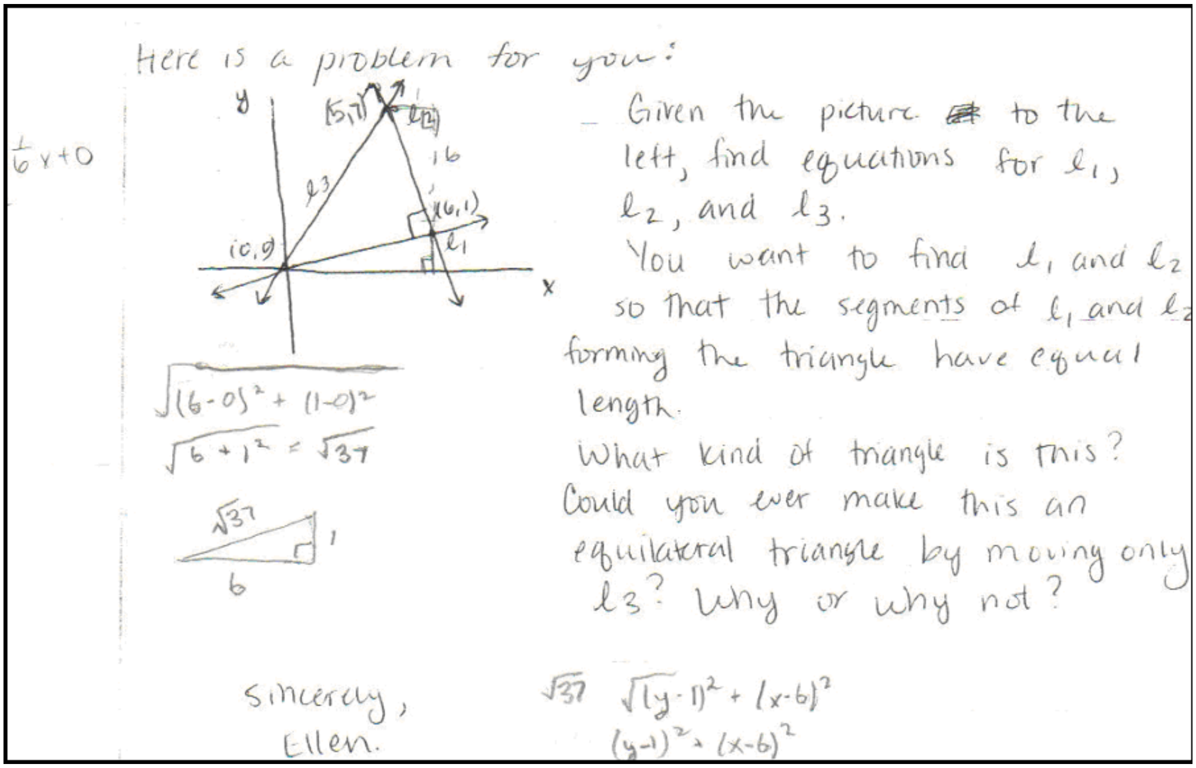


Figure 1. Ellen’s initial task to Jacques.

Jacques and Ellen represent the kinds of activities found across all the letter-writing pairs. We noticed an overall increase in cognitive level from purely procedural to *Procedures with Connection*s, and we determined that many of the PST’s tasks elicited *Communication*, *Application*, and *Analysis*. Across all exchanges between PSTs and high school students, we rarely identified instances of *Problem Solving* in our data. Therefore, the presence of this particular measure in Jacques and Ellen’s exchanges indicates the fruitfulness of their interaction. To clarify these statements, we explain how we inferred cognitive activity from Jacques’ written responses in each of four exchanges with Ellen.

Analysis

Figure 1 illustrates Ellen’s initial task[1] to Jacques, one that we might formally recognize as an analytic geometry problem. Note that Ellen made all of the algebraic manipulations after she received Jacques’ response.

Jacques’ response (Figure 2) indicates that he was unable to engage meaningfully in the task of finding equations for lines meeting the specified geometric conditions. However, he was able to assimilate the situation as one involving solutions to systems of equations. He had a lot of experience working with systems of equations in his algebra class, and the situation described in Figure 1 might appear to have many familiar features from such experiences (intersecting lines on a graph, coordinates, questions about linear equations, etc.). From his activity of manipulating two linear equations and their graph, we inferred that Jacques’ knowledge of solving systems of linear equations was procedural only. Therefore, we would expect that Jacques would be able to manipulate symbols (solve for ‘x’, substitute values, etc.), but he might not have a more connected understanding that would link the equations to graphical representation. Jacques might have had a more connected understanding of the concepts underlying the procedure, but there was no clear indication from which to infer this. Therefore, we coded the elicited activity as *Procedures without Connections*.

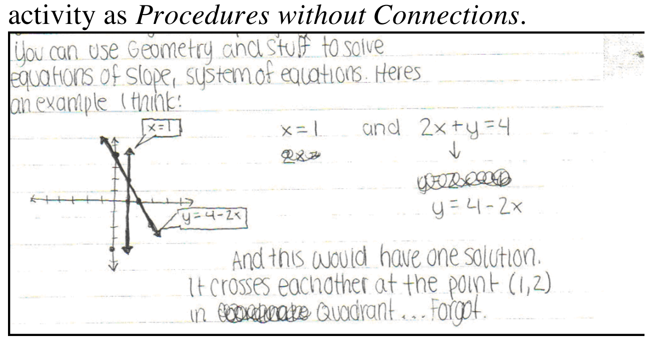


Figure 2. Jacques’ response to Task 1.

Other codes applied to Jacques' response included *Application* and *Communication*. The former was based on our inference that Jacques used existing ideas in a novel situation. He assimilated his knowledge of systems of equations to a situation involving finding equations of intersecting lines. He effectively applied an algebraic procedure to a new domain which observers might call analytic geometry. We based the latter code on our inference that Jacques' written language was intended to convey a mathematical idea that systems of equations can be used to find points of intersection.

In her next letter, Ellen affirmed Jacques' response but turned the conversation back to her original intent for the task. After restating the task, Ellen attempted to focus Jacques' attention on the angles, the lengths of the sides, and the type of triangle that she had drawn. Jacques responded (Figure 3) by pointing out that the marked angle was 90 degrees. In addition, he supplied three equations without work, thus rendering it more difficult to infer how he was operating. In writing the slopes as fractions, including the whole numbers, he was likely focusing on slope as rise over run. He accurately identified the slope of one line and the signs on all lines. We inferred that his procedure for computing slope as rise over run was connected with a graphical understanding of slope as the gradient of the line. Because there was evidence of cognitive activity that went beyond the application of procedures we identified this as *Procedures with Connections*.

In addition to coding *Application* and *Communication*, we also found indication of *Analysis* in Jacques' response because Jacques seemed to break down the graph to obtain specific mathematical information, such as the signs of the slopes of the lines. Finally, we inferred that Jacques had engaged in *Problem Solving* because he seemed to struggle (as indicated by his question, "is this correct?") and yet made progress in resolving the novel situation.

Ellen returned to the same problem situation again in posing Task 3. She attempted to focus Jacques' attention on the lower triangle in Figure 4. She wanted Jacques to connect the coordinate point (6,1) with distances on the triangle. She asked him if he could use those lengths to compute the length of l_1. (Note that Ellen defined l_1, l_2, and l_3 in her original letter to be the line and referred to the sides of the triangle in Figure 1 as the segments of those lines respectively. In Task 3, Ellen mixes the notation and uses l_1 to refer to the segment associated with the line l_1. We could not infer that this caused Jacques confusion.

Jacques took advantage of Ellen's questions and applied the Pythagorean Theorem to the new situation (see Figure 5). Because he used the procedure to calculate a distance without clear prompting from Ellen, we coded this interaction as having elicited *Application* and *Procedures with Connections*. In other words, we inferred from Jacques' novel use of the Pythagorean Theorem that he understood it beyond rote computation and could use it flexibly in new situations. He had developed a kind of efficacy in using it, purposefully transforming information from the coordinate pair (6,1) into information about side lengths, *a* and *b*, of a right triangle. We also inferred from those actions that Jacques had attempted to *communicate* a mathematical idea (as indicated by his writing at the top of Figure 5) and that he had broken down (*analyzed*) the situation into constituent mathematical ideas (x=6 and y=1).

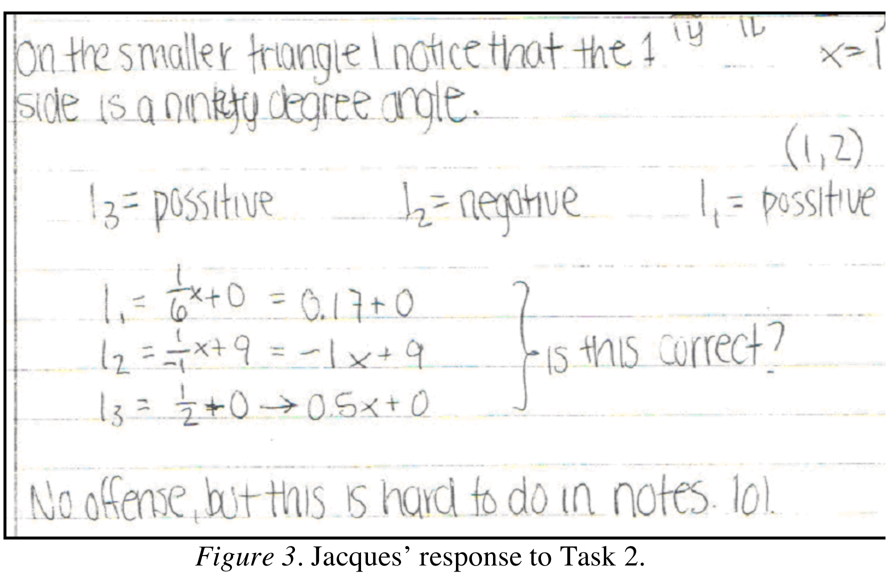
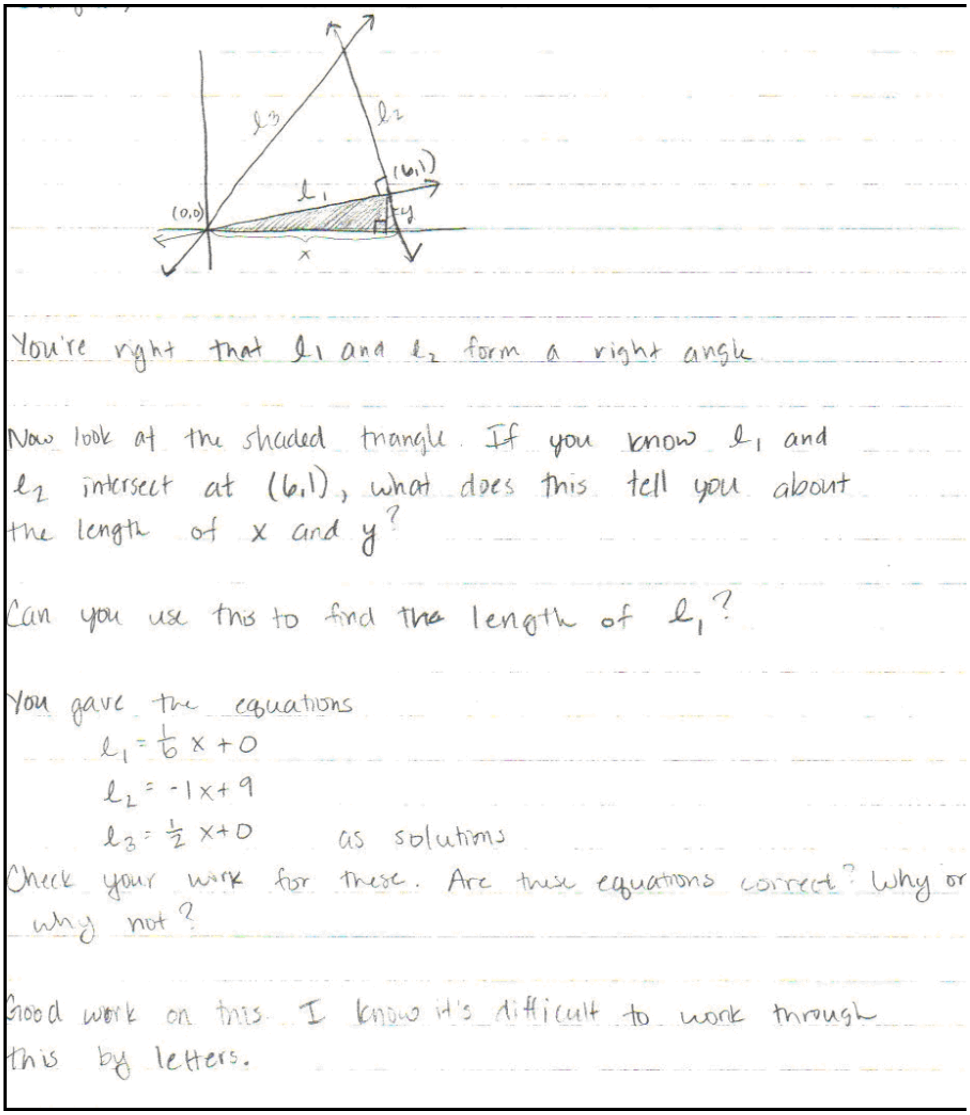


Figure 3. Jacques' response to Task 2.



You're right that l_1 and l_2 form a right angle

Now look at the shaded triangle. If you know l_1 and l_2 intersect at (6,1), what does this tell you about the length of x and y?

Can you use this to find the length of l_1?

You gave the equations

$l_1 = \frac{1}{6}x + 0$

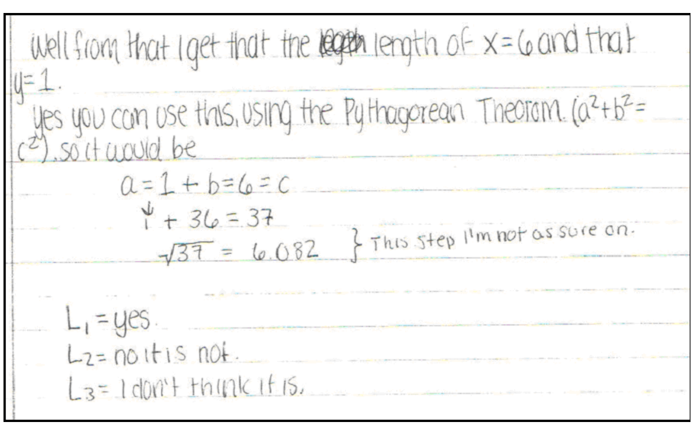
$l_2 = -1x + 9$

$l_3 = \frac{1}{2}x + 0$ as solutions

Check your work for these. Are these equations correct? Why or why not?

Good work on this. I know it's difficult to work through this by letters.

Figure 4. Ellen's third task for Jacques.


Well from that I get that the ~~legth~~ length of x=6 and that y=1.

Yes you can use this, using the Pythagorean Theorem. ($a^2+b^2=c^2$), so it would be

a=1 + b=6 = c

1 + 36 = 37

$\sqrt{37}$ = 6.082 } This step I'm not as sure on.

L_1 = yes.

L_2 = no it is not.

L_3 = I don't think it is.

Figure 5. Jacques' response to Task 3.

Discussion: Summary of Exchanges

Although not part of this particular report, Ellen used the same problem stem in the next task with similar results, in terms of elicited cognitive activity (we inferred from his response *Procedures with Connections*, *Application*, and *Communication*). Jacques seemed to respond well affectively; he commented in his response that, "this is really fun doing this, u [*sic*] are making it very understandable." We inferred from the pair's interactions that Jacques had constructed ways of using procedures, like the Pythagorean Theorem, that were connected to meaningful concepts. He had also constructed a tenuous grasp of formalized linear equations; that is, he seemed able to generate only linear equations that went through the origin. From such inferences, we argue that Ellen had successfully engaged Jacques in a variety of high-level processes such as *Problem Solving* and *Analysis*. Across all four tasks in the sequence, Ellen consistently elicited *Application* and mathematical *Communication* from him. In addition, she and Jacques engaged in activities that started as procedural, but quickly progressed to and remained at *Procedures with Connections*.

Extended Discussion from a Situated Perspective

Broadly speaking, a situated view of learning would include what Wenger (1998) refers to as apprenticeship forms of learning or ideas about learning in communities of practice. These forms of learning, as Lave (1997) states, "are likely to be based on assumptions that knowing, thinking, and understanding are generated in practice, in situations whose specific characteristics are part of practice as it unfolds" (p. 19). In other words, learning mathematics is about learning the social practices of school mathematics, often including the establishment of norms about what constitutes appropriate mathematical activity and mathematical learning.

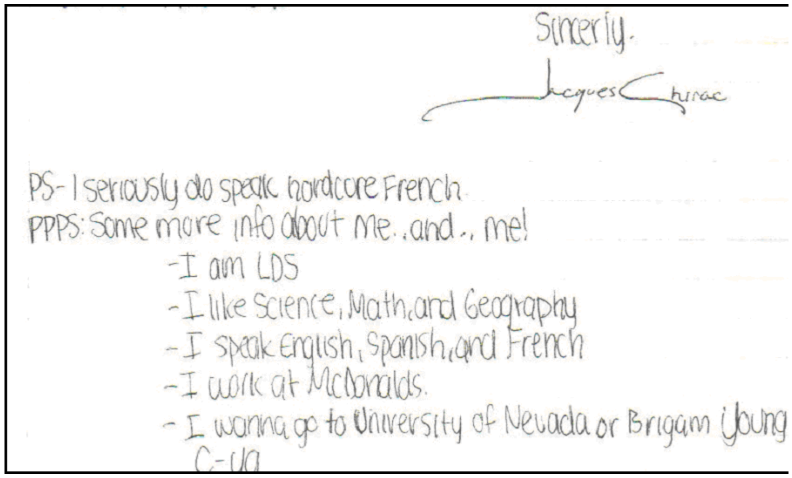
Reconsidering the teacher-student interactions from this point of view, we argue that two main issues emerged during the task iterations. The first issue is that the PST and student possessed compatible understandings about their roles in relation to one another concerning mathematical activity. In addition, and non-trivially, they both agreed to take up these roles. We will support this idea by showing that, although the tasks seemed formal and lacking in real-life relevance, the student readily engaged with them. The second issue is that that the way in which these two took up their respective roles could be associated with what it traditionally means to engage in mathematical activity in the classroom. The PST re-constituted this in more and less obvious ways throughout the exchanges. One example of how we support this second idea is by showing that the PST never incorporated any of the student's personal interests into the mathematics.

Considering the first issue, Lave (1997) contends that practices in school can remove ownership of mathematics from students. In other words, practices in school mathematics classrooms encourage students to learn the practices of schooling, which may not be the same as the practices of mathematics. For example, students may be inducted into the practice of completing pre-designed steps in a problem from the teacher. The practice then becomes the generation of the steps on the part of the teacher and the working of each step independently on the part of the students. The overall mathematical meaning or goal may be lost or, as Lave would contend, ownership of the problem is taken from the students.

It is interesting to consider the task-posing sequence between Ellen and Jacques at this level. Using a situated perspective, we can see that Jacques and Ellen were "on the same page". Expectations about what constitutes mathematical activity seemed agreed upon by both participants. Specifically, the interactions could be viewed, as Lave (1997) described above, as a series of often procedural steps designed by the PST to guide the student through this pre-conceived task, and this constituted the agreed upon mathematical activity in which the two engaged.

Furthermore, this kind of agreement could be interpreted in several ways. For example, some researchers refer to scripts and would argue that both participants were using a dominant script for communication (Gutierrez, Rymes, & Larson, 1995) where the participants can be viewed as using the standard way of talking and acting in certain situations. In this case, it could be argued that Ellen adopted the standard way of interacting with the student and likewise that Jacques took up a typical way of interacting with someone in an instructional role. Similarly, others might argue that they were both participating in the same or similar (big-D) Discourse (Gee, 1999). In either case, this tacit agreement between the members of the pair could be a strong factor in determining the kinds of psychological activities that we inferred from the exchanges. In particular, had the student not agreed, it possibly would have impacted the mathematical interactions, perhaps leading to disengagement from either party.

As indicated earlier, the nature of the mathematics was one that was likely disconnected from the student's personal experience in many ways. It was an "abstract" situation that provided little intrinsic motivation. In other words, the student could question why there would ever be lines moving around to create an isosceles triangle, like in Figure 1. This kind of question from the student is important as it highlights the nature of the mathematics in which we expect our students to engage. It is not to say that such an activity is necessarily "bad" or "good." With this in mind, the power of the teacher script is palpable as the student, in good humor, engages with this task despite the lack of introduction to the purpose behind the task and despite the PST not providing any sort of motivation for it.


Sincerly.
Jacques Chirac

PS- I seriously do speak hardcore French
PPPS: Some more info about me..and.. me!
-I am LDS
-I like Science, Math, and Geography
-I speak English, Spanish, and French
-I work at McDonalds.
-I wanna go to University of Nevada or Brigam Young
C-Ya

Figure 6. Jacques' introductory letter to Ellen.

Considering the second issue, however, we see something a little different. Although we hypothesize a certain level of agreement between the two at the Discourse level (or similarly, we could suggest that they both are adhering to a dominant script about school mathematics), we also hypothesize that the PST *demanded* that the student speak the Discourse of school mathematics. To support this claim, we consider the initial introductory letter where Jacques stated "I want to be both president of the United States and the owner of my own fast food franchise chain." He then closes the letter by providing a great deal more information about himself (see Figure 6). Here Jacques shares that, among other things, he is interested in science and works at a fast-food restaurant. However, Ellen, for purposes of task selection, ignored these facts. She picked a task that was what some may consider "de-contextualized." Yet, researchers such as Lave (1996) would consider these kinds of tasks highly contextualized in certain "socially, especially politically, situated practices" (p. 155).

Further, by ignoring where the student was "coming from" in this way, the PST potentially lost the opportunity to establish a Third Space (Gutierrez et al., 1995). It is in this space where student backgrounds and interests can meet with teachers' learning objectives and provide for fruitful collaboration. In this series of tasks, we could ask, "Could Ellen have incorporated the student's interest in business?" or, "Could she have embedded the task in a political context?"

Before concluding this section, it is important to note that a situated perspective does not demand that teachers include students' personal information into tasks. As cited above, some authors associated with the situated perspective have questioned the value of teaching what would traditionally be considered purely "abstract" mathematics. By considering the example of Ellen and Jacques, we show the degree to which Ellen maintains her dedication to the "abstract" task. By considering Jacques' personal information (his home life, cultural background, and interests), we show that Ellen did have at least one other option for a type of task besides an "abstract" one.

In her final letter, Ellen explicated a certain set of values and ways of looking at mathematics, stating to Jacques that he should be pleased with his "perseverance" and that she hoped he had learned "to approach a complicated problem as a series of smaller, easier problems." This view of mathematics exemplifies what could be called the dominant Discourse of traditional mathematics teaching. It is what Lave (1997) might consider to be the kind of practice that can remove ownership of the subject from the student. Situated theorists such as Lave may argue that it perpetuates a way of teaching mathematics that can limit student agency and the role of the student in generating new ideas—a practice that is not consistent with the actual practice of research mathematicians (Boaler & Greeno, 2000).

Coordination and Conclusion

This article has described two perspectives on the same set of interactions between a PST and a high school student. One details the individual at work, and the other gives a "bigger picture" of the world in which the individual operates. However, we encourage a more careful consideration of how these two tracts of analysis are related. For example, when Jacques applied the Pythagorean Theorem to Task 3, we explained his actions as an assimilation of the situation into his conceptual understanding of the procedure (von Glasersfeld, 1995). Alternatively, we might have explained his actions as resulting from an identification of common attributes across the new situation and previous situations in which Jacques had used the Pythagorean Theorem (Greeno, 1997). Both explanations seem valid, and even compatible, as long as we clarify issues involving the observer and points of view.

We suggest that the two explanations are indeed compatible if we attempt to adopt the student's point of view in both cases. Although there may be many commonalities between the situation described in Task 1 and our observations of a student's previous experiences with the Pythagorean Theorem, our observations are a poor substitute for the student's lived experience. Otherwise, we should have expected Jacques to apply the Pythagorean Theorem in his response to Task 1, as Ellen clearly expected him to do (as indicated by her markings in Figure 1). This necessitates the kind of inference we made about Jacques' actions; we have no access to students' lived experiences, and so we must make inferences based on our observations, knowing full well that students' points of view and, thus, students' mathematics may be quite different from our own. So, if we take "common attributes" to describe commonalities from the student's point of view of the new situation and previous experience, the situated perspective complements a cognitive perspective.

Consider our operationalization of *Problem Solving* as another example in which we might reconcile perspectives. *Problem Solving* required that the student lack a readily defined way of resolving the task; we had to be able to infer that he experienced some threshold of cognitive struggle. From a situated perspective, we were not simply measuring whether the student was able to deal with novel situations, but we were also measuring the *degree* to which the student had experienced these kinds of situations before. In particular, if a student had experienced the same situation many times before, then we would be unlikely to assess this as *Problem Solving* (as the solution/procedure would likely be generated effortlessly by the student); however, we would still be unlikely to assess *Problem Solving* if the student had little experience with similar problems, as the student would likely be unable to engage. The likely cases where we would assess *Problem Solving* would be between these two. It would have to be a case where, from his point of view, the student had *relatively* similar experiences, but yet different enough that we would detect cognitive struggle. For example, with Ellen's second task, Jacques showed some familiarity with the set-up (the part that called for linear equations), yet he struggled with the novel parts of it (the parts that required him to deduce the missing points so that he could use a point-slope formula, for instance).

This discussion of *Problem Solving* does offer some instance of how these two perspectives could be compatible; however, while analyzing this measure along side the others, it became apparent to us that this coordination also presented productive criticisms of the two perspectives. For example, as mentioned above, Lave (1997) states that certain practices in mathematics can remove student agency—practices such as breaking down problems into multiple steps. This practice is a common practice amongst mathematicians as described by Polya (1973). It is likely that Lave indicated something more subtle than just the mere act of breaking down a problem into steps; yet, it is not clear how to interpret this statement in the situation with Jacques. Was Ellen supporting the kind of mathematics to which Lave was referring or was she moving the student towards something that resembled the practice of mathematicians like Polya?

On the other hand, the constructivist perspective has a heritage of recursive model building with subsequent refinements of these models (Steffe & Thompson, 2000). Unfortunately, given the constraints of our study as well as its purpose, we did not revise our models through recursion. In other words, we formed models of the students' cognitive processes as indicated by their written responses to tasks, but we did not have opportunities to test and revise purposefully those models through continuing interaction with the students. We could have emulated recursive model building by looking back through previous responses from the student in an attempt to identify a consistent way of operating across the tasks, but our methods dictated that we assess each week's responses separately. Therefore we did not fully utilize the tools available to the radical constructivist. This is a weakness to our approach and one that became clear to us as we compared other perspectives to our own. In particular, our analysis of *Problem Solving* could have looked substantially different if we had built a stable model of Jacques' mathematics. This would have been a model we could likely have used to interpret his response to the task more insightfully. Moreover, it is not clear to us what kinds of model building heuristics are available to researchers using such a perspective, leading to a further divergence from the situated perspective.

In conclusion, we have shown how one PST elicited cognitive activity from a student over the course of letter-writing exchanges. We have also indicated how our measures could be viewed as indications of prior experiences and dependent upon the student's comfort with certain norms associated with traditional mathematics teaching. In addition, considering mathematics as situated in larger socio-cultural structures, we have been able to critique our own analysis and ultimately suggest paths for further exploration.

More generally, we have given a suggestive way in which two competing lenses can be used to consider data and create a conversation between two competing paradigms. This conversation provided alternative ways to view the same data, but also generated fruitful criticisms of the approaches. These alternative ways of viewing the data hinged largely on the perspective adopted by the researchers in considering their data. For example, if a situated perspective focuses upon the opportunities presented in a certain task, then it becomes an important issue as to who is identifying these opportunities. If the opportunities are considered to be from the learner's perspective, then such a perspective may have many pragmatic commonalities with radical constructivism.

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[1] In the case of the exchanges described here, the PST built all subsequent tasks as modification of an original problem (what we call "Task 1"), so in many ways, there was only one primary task in these exchanges. This did not always have to be the case however, and the PSTs were free to change tasks completely between exchanges. For the sake of coding and analysis, we called any mathematical request from the PST a "task" regardless as to whether it was a brand new task or a modification of a previous task. So, in this paper, "Task X" means "the mathematical task request made by the PST during week X."

Appendix

Table A1

Descriptions of Cognitive Processes Described in Bloom's Taxonomy

Cognitive Process	Short Definition
Application	Using previously learned information in new and concrete situations to solve problems
Analysis	Breaking down informational materials into their component parts so that the hierarchy of ideas is clear
Synthesis	Putting together elements and parts to form a whole
Evaluation	Judging the value of material and methods for given purposes

Note. From "Using Bloom's Taxonomy as a Framework for Classroom Assessment," by S. K. Kastberg, 2003, *Mathematics Teacher*, *96* (6), p. 403. Adapted with permission of the author.

Table A2

Description of Cognitive Processes Described by the NCTM Process Standards

Cognitive Process	Short Definition
Communication	Expressing mathematical ideas in words to clarify and share them, so that "ideas become objects of reflection" (p. 60)
Connections	Relating mathematical ideas to each other, and to previous experiences in other domains, such as science
Problem Solving	"Engaging in a task for which the solution method is not known in advance" (p. 52), which involves the use of strategies in struggling toward a solution.
Reasoning and Proof	Making analytical arguments, including informal explanations and conjectures
Representation	The "process and product" (p. 67) of modeling mathematical ideas and information in some form, in order to organize, record, and communicate.

Note. Summarized from *Principles and Standards for School Mathematics* (NCTM, 2000).

Table A3

Cognitive Processes Described by Smith, Stein, Henningsen, and Silver

Cognitive Process	Short Definition
Memorization	Memorizing or reproducing "facts, rules, formulae, or definitions" (2000, p. 16) without any apparent connection to underlying concepts
Procedures without Connections	Using a procedure or algorithm that is implicitly or explicitly called for by the task, without any apparent connection to underlying concepts
Procedures with Connections	Using procedures to deepen understanding of underlying concepts
Doing Mathematics	Investigating complex relationships within the task, its solution, and related concepts, often involving metacognition, analysis, and problem solving

Note. These definitions are summarized from *Implementing Standards-Based Mathematics Instruction* (Stein et al., 2000).