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Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum

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Planning is an important phase of teaching, during which teachers make decisions about various aspects of instruction that ultimately shape students' opportunities to learn. Prior research on teacher planning, however, fails to adequately describe experienced teachers' planning decisions, and is unclear about the extent to which teachers use curriculum materials to inform their decisions. Using data from 6th grade mathematics teachers' use of curriculum materials, this study presents a discipline-specific model of experienced mathematics teachers' planning. The proposed model provides a lens for understanding the nature of teachers' planning decisions, and the conditions under which such decisions change over time.

Planning is an important and often underappreciated aspect of teaching practice, when teachers make decisions that ultimately impact students' opportunities to learn (Clark & Peterson, 1986; Floden, Porter, Schmidt, Freeman, & Schwille, 1980; Stigler & Hiebert, 1999). Planning commonly refers to the time teachers spend preparing and designing activities for students. From tasks and activities to instructional practices employed during lessons, teachers need to consider a variety of aspects of their instruction before students even enter the classroom. Teachers need to pay careful attention to designing their lessons; "effective teachers understand that teaching requires a considerable effort at design. Such design is often termed planning, which many teachers think of as a core routine of teaching." (Kilpatrick, Swafford, & Findell, 2001, p. 337). Reviews of teacher planning and decision-making further emphasize the centrality of planning processes in teachers' practice (Clark & Peterson, 1986; Clark & Yinger, 1977; Shavelson & Stern, 1981). Despite this general agreement about the importance of planning, few researchers have explicitly examined the precise ways in which teachers plan for mathematics instruction.

Prior research related to teacher planning presented a "linear" or "rational" model of teacher planning by delineating the various lesson elements teachers generally considered when planning their lessons (Popham & Baker, 1970; Taylor, 1970; Tyler, 1950). Under this model, teachers first consider the learning activities that take into account students' interests and abilities, then the learning goals and objectives of the lesson, and finally the evaluation procedures to be used during the lesson. Some researchers later argued that linear models of teacher planning do not adequately describe experienced teachers' planning processes and do not account for the complexities inherent in mathematics teaching. Rather, a variety of additional factors, such as teachers' experiences and conceptions of mathematics teaching and learning, also influence the ways in which teachers plan their lessons (Kilpatrick et al., 2001; Leinhardt & Greeno, 1986; Yinger, 1980).

More recent research on teachers' planning does not clearly indicate the extent to which teachers draw from curricular resources when making planning decisions. Moreover, there is even less research that focuses explicitly on teachers' planning in the context of the reform mathematics curricula that provide much of the instructional design for teachers (Kilpatrick et al., 2001; Trafton et al., 2001). Such reform curricula are increasingly prevalent in classrooms in the United States, embodying new modes of instruction (Reys, 2002). The challenges of planning lessons using such curricula may be somewhat different from the challenges of planning lessons with more conventional mathematics curricula. Thus, exploring how teachers plan in the particular context of reform curricula is critical if mathematics educators want to understand this important phase of teaching.

In order to explore the theoretical considerations presented in this article, the author has selected

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examples of teachers' planning routines taken from a larger study examining experienced 6th grade teachers' use of the Connected Mathematics Project (CMP) materials. CMP is a middle school reform curriculum developed in response to the National Council of Teachers of Mathematics (NCTM, 1989) *Curriculum and Evaluation Standards*. By emphasizing the discovery of mathematical ideas through tasks, CMP encourages students to make connections between topics and important mathematical ideas in order to help them apply their learning to real-world contexts. The larger study focused on how four teachers used the CMP teacher's guide in both the planning and enactment of their lessons. The teacher's guide includes summaries of the mathematical content, specific questions to ask students throughout a lesson, and examples of student errors.

Teacher Planning

Past research on teacher planning focused on the broad features and order of teachers' planning decisions and considerations, with minimal attention given to the particular ways that teachers considered engaging students with the content. Adhering to a linear model of teacher planning, Tyler (1950) and Popham and Baker (1970) found that teachers specified ordered objectives, selected learning activities, organized learning activities, and specified evaluation procedures. Similarly, Taylor (1970) found that teachers sequentially considered four aspects of a given lesson when planning: materials and resources, students' interests, the aims and purposes of teaching, and evaluation. Implicit in these studies is the notion that teachers create their own objectives and activities for students, which may reflect the design of the types of curriculum materials available to teachers at the time in which these studies were conducted.

In a later study on teacher planning, Brown (1988) examined the extent to which 12 teachers adhered to a linear model of planning. Focusing on teachers' planning in different subject areas, Brown found that teachers tend to use curriculum materials and the objectives expressly stated in these resources as a starting point for their planning. She noted, "teachers operate as curriculum implementers and not curriculum planners as they consider objectives already written in curriculum guides" (p. 79). Yackel and Cobb (1996) noted that planning decisions about ways of facilitating students' activity in a history or English classroom are considerably different from those in a mathematics classroom. Nevertheless, Brown's (1988) study points to the integral role of curriculum materials in the process of teachers' planning, which was not clearly addressed by proponents of the linear planning model.

Some researchers have focused on the role of curriculum materials as a resource for teachers to draw upon when making planning decisions. For example, McCutcheon (1981) found that when planning for daily lessons, teachers tend to rely heavily on suggestions in the teacher's guide. In a study of one teacher's planning throughout the school year, Clark and Elmore (1981) found that curriculum materials are primary resources in the teacher's planning. Similarly, Smith and Sendelbach (1979) studied this issue at the level of teachers' unit planning. They found that teachers tend to construct a mental image or plan of the unit and then supplement their plan with notes based on the suggestions in the teacher's guide. Additional research has highlighted various factors that influence how teachers use these curricular resources in the planning and instructional processes (Ben-Peretz, 1990; Cohen et al., 1990).

Research points to experience as a potential factor that influences teachers' planning. Such research suggests that experienced teachers have more extensive and well-organized knowledge of both pedagogy and student learning, making them more flexible and attentive to the nature of the students' learning opportunities that they create (Borko & Shavelson, 1990; Leinhardt & Greeno, 1986; Livingston & Borko, 1989). For example, researchers found that when planning, experienced teachers make more extensive mental plans than written plans and rely less on curriculum materials than their less experienced counterparts (Bush, 1986; Leinhardt, 1983; Livingston & Borko, 1990).

As teachers' experience with a particular curriculum program increases, they become more familiar with the details, nuances, and presentation of the specific mathematics content in the curriculum. Thus, they may have developed daily routines in planning and engaging with the curriculum in particular ways. For the purposes of this study, experienced teachers are identified as having at least five years of teaching experience and at least three years experience using a certain curriculum program.

Other possible factors influencing teachers' planning decisions are the various conceptions teachers bring to bear on their practice. These conceptions contribute to what Remillard and Bryans (2004) refer to as teachers' orientation toward the curriculum. Experienced teachers have refined their conceptions of mathematics teaching, learning, and curricula because they have spent considerable time formulating and

applying these conceptions in the classroom. Teachers' conceptions of mathematics content are also important for understanding how teachers engage with the curriculum (Lloyd, 1999; Remillard, 1999; Remillard & Bryans, 2004). Moreover, it is important to understand the extent to which teachers' conceptions of mathematics teaching and learning align with the ideas about teaching and learning underlying the curriculum (Manouchehri & Goodman, 1998; Remillard & Bryans, 2004).

While linear models are useful for capturing certain basic elements of teacher planning, these models fail to account for an array of factors that have been identified as influencing teachers' planning processes, such as curriculum materials, teaching experience, and the various conceptions teachers have about teaching and learning. Although teachers have a variety of conceptions, this article will focus on teachers' conceptions of mathematics teaching, learning, and curricula for the purposes of this study.

The Work of Reform-Oriented Mathematics Teaching

Although the various factors highlighted by researchers as influencing teachers' planning are essential to consider when developing a new planning model, it is also important to consider the demands and characteristics of the particular discipline in which such planning occurs. Most of the research discussed previously does not explicitly focus on planning or instruction in the context of a specific discipline. Mathematics teaching, specifically in the context of reform mathematics curricula, involves particular demands and challenges that may shape teachers' planning processes.

The model developed in this study is grounded in a specific conception of mathematics teaching, drawing from the works of Stein, Smith, Henningsen, and Silver (2000), Lampert (1992, 2001), Clark and Elmore (1981), and Lampert and Ball (1998). These researchers describe in detail the nuances and complexities of mathematics teaching in a way typically embodied in reform curricula. Teachers need to consider the mathematical content and ways to engage students in discussion about the content, while simultaneously guiding students towards a particular goal. For example, during planning and instruction teachers modify tasks and ask high-level questions in order to promote students' understanding of the underlying ideas and concepts.

To support students' understanding, teachers need a variety of pedagogical skills. For example, teachers need to be able to resist the urge to tell students how to work on the content so that they provide students with adequate time to think through what they are asked to do (Donovan, Bransford, & Pellegrino, 2000). Anticipating student responses and having an awareness of common errors can also help teachers effectively respond to and redirect students' discussions (Chazan & Ball, 1999; Fennema, Franke, Carpenter, & Carey, 1993). In addition, modifying tasks based on students' current knowledge and abilities may help teachers to be mindful of the cognitive activity in which students should be engaged (Stein et al., 2000). Employing such instructional practices to facilitate student learning in accordance with the principles of reform mathematics, however, requires extensive and demanding work on the part of teachers. Therefore, teachers face several challenges supporting students at such a high level of mathematical activity.

These challenges to teachers' work are considered the "problems" in mathematics teaching (Lampert, 1992, 2001). "Problems" in mathematics teaching refer to the work teachers do to further students' understanding of mathematics. This includes facilitating students' discussion around the content, continuously pressing students to explain their ideas and to communicate with each other, posing questions, and selecting solution strategies to present to the class. Teachers need to make important and often simultaneous decisions in ways that do not undermine students' thinking or the mathematical opportunities afforded by the content in reform curricula. Hereafter, "problems" will be used to refer to the challenges and decisions teachers face during mathematics teaching, as described in Lampert (1992, 2001).

Teaching includes not only the physical act of teaching, during which teachers interact with students, but also includes the time teachers spend preparing for these interactions (Jackson, 1966, 1968). Planning for the demands and challenges of mathematics instruction requires teachers to engage in a planning process that involves the development of skeletal frameworks rather than detailed scripts for teaching lessons (Rosebery, 2005). In particular, teachers must identify a particular mathematical topic to discuss and the means necessary to cover that topic, without necessarily delineating the precise steps needed to teach that topic. Therefore, planning for reform-oriented instruction requires teachers to select specific topics or concepts and to identify particular activities, instructional strategies, and suitable materials for discussing and engaging students with the topics or

concepts. In addition, during planning, teachers must anticipate potential problems that may arise during instruction and then make decisions regarding how to manage these problems. Planning problems refer to the considerations and decisions teachers face when both planning for and anticipating what will happen during a specific lesson.

A Model of Teacher Planning

There are several elements that a model of teacher planning in the specific context of reform-oriented mathematics instruction must capture. These elements include the approaches to mathematics teaching and learning embodied in reform curricula and teachers' various experiences and conceptions they bring to their planning decisions. Such a model must particularly capture how these elements interact with each other and ultimately influence teachers' planning decisions. The concept of planning problems is well suited for developing a conceptualization of planning because it incorporates the influence of these different elements. The model developed in this study draws heavily on the notion of planning problems and highlights the various elements that drive the emergence of such planning problems in teachers' practice.

As discussed previously, the work of reform-oriented mathematics teaching includes facilitating and supporting students' understanding in ways that will neither constrain students' opportunities to learn nor undermine students' thinking. Consequently, teachers need to plan for engaging in this sort of work during instruction. For example, teachers must plan questions they will ask students that will guide students' thinking about the content without giving them too much information, while also encouraging students to explain their ideas (Hiebert & Wearne, 1993; Maher & Martino, 1992; Moyer & Milewicz, 2002). Teachers need to anticipate different solutions students may offer, as well as alternative ways of thinking about a task, in order to facilitate students' learning and discussion of these strategies in ways that foster a shared understanding of the ideas (Kilpatrick, 2003; NCTM, 1991). Teachers also need to anticipate potential errors in order to respond appropriately and help students learn from incorrect solutions. Finally, teachers should be prepared to change or modify a task, in the case that students are struggling with a concept, in ways that both preserve the task's complexity and help students learn from working on the task (Stein et al., 2000). Though their intended plans often differ from their enacted plans, teachers need to carefully plan their lessons and anticipate how students will interact with the content during implementation in order to further students' understanding of different mathematical ideas. In this way, planning problems can be considered to be the anticipation of instructional problems.

Planning problems are inherently different for each teacher depending on the teacher's experiences, ideas, and conceptions, as well as the curriculum being used. For instance, asking higher-order questions that press students to justify and explain their thinking is also only a planning problem for teachers who view the use of such questions as contributing to student understanding. Planning problems also may be quite different for a teacher who adheres to a more conventional conception of mathematics teaching. Such a teacher may need to determine how to incorporate opportunities for students to practice the application of certain skills and procedures within a curriculum the teacher perceives as deficient. Furthermore, a teacher with experience implementing multiple curriculum programs must consider how to apply what they know of other mathematics curricula to their planning with a specific Standards-based curriculum. Therefore, planning problems appear to be a useful lens for understanding the relationship between teachers' experiences, conceptions of mathematics teaching and learning, and the curriculum used in the planning process.

As the Planning for Mathematics Instruction (PMI) Model in Figure 1 illustrates, teachers' various conceptions influence their engagement with curriculum materials during planning. Additionally, the conceptions influence the type of planning problems teachers encounter in the course of their work. These planning decisions then influence teachers' lesson enactment and the types of learning opportunities they create with students during instruction. This lesson enactment then informs experiences and shapes the information teachers have to draw upon when they plan for and enact the lesson in subsequent classes or school years.

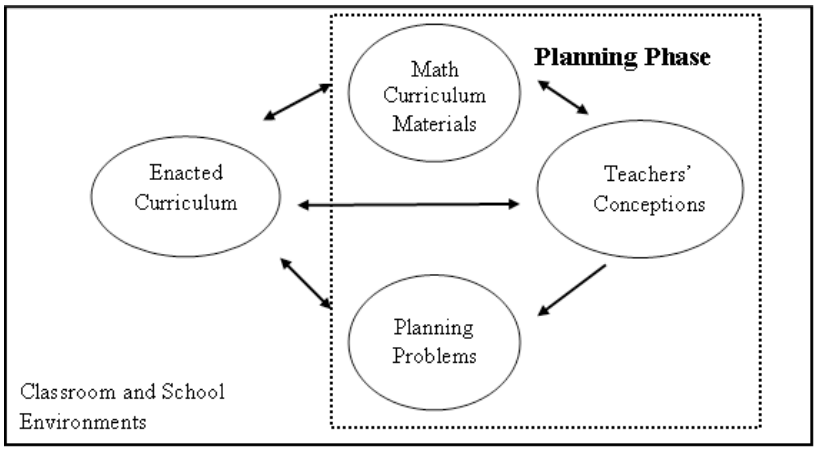

Figure 1. Planning for Mathematics Instruction Model.

As teachers' conceptions help to frame the planning problems they encounter, their various conceptions also serve as a resource for managing planning problems that arise in the course of their planning. When confronted with planning problems, teachers draw upon their previous experiences with the task and their ideas about what it means to learn and teach mathematics in order to make decisions about ways of managing these different planning problems. In some cases, teachers may also draw from the information and support provided within the actual mathematics curriculum materials. Notably, the CMP teacher guide provides the means for teachers to manage certain planning problems, such as anticipating solution methods students may generate, questions to ask students, and errors and misconceptions students may have in relation to a task. Though, the extent to which teachers use curriculum materials to inform their planning decisions is largely dependent on the nature of their conception of the curriculum.

Teachers' Planning Practices

The following teacher examples illustrate the various ways in which experienced teachers, with distinct conceptions towards the CMP curriculum, can engage with reform curriculum materials in the course of their planning and demonstrate how the PMI Model applies to actual teaching practices. Alicia, Richard, and Susan were selected for the present study because their planning decisions and considerations captured the range of variation in planning routines and problems encountered.[1] All three teachers were teaching sixth grade at the same middle school at the time the study took place. As this study uses teaching and curricular experience to define experience, Alicia has been teaching 16 years, using CMP for 3 years; Richard has been teaching 17 years, using CMP for 10 years; Susan has been teaching 6 years, using CMP for 3 years. These teachers were observed planning and enacting the same unit, Bits and Pieces III, which focuses on operations with rational numbers (Lappan et al., 2006). The data collected includes interviews with teachers prior to and immediately following classroom observations to understand teachers' lesson plans and their reflections on their lesson enactments. Although post-hoc examinations of teachers' planning were conducted, teachers were interviewed the same day of the observation in order to increase the accuracy of teachers' responses. Field notes from classroom observations and artifacts from teachers' lesson planning are additional data sources.

Prior to using CMP, Alicia used a more conventional mathematics curriculum for 13 years and claims to strongly believe that CMP does not provide students with sufficient opportunities to practice skills and procedures. She views her role as a teacher as that of an intervener, providing direct guidance and explicit instructions for students, which is evident in her planning decisions. Alicia first reads through the entire lesson in the student book and solves the task: "And I do the whole [lesson] myself, you know without looking at the teacher's guide or anything…And then I have an idea about what might be tricky and what might not be." Alicia says she then looks through the accompanying teacher's guide and decides the most appropriate course of action, taking into consideration both who her students are as learners and the constraints of class time. "And I kind of pick and choose what I think will work best with my students… And most times I won't use all of it." In general, Alicia claims to regularly modify the suggested content of the lesson and the suggestions for how students should engage with the content during the lesson. Her view of CMP, and ultimately her conception of what students

should learn, is readily apparent in the nature of her lesson additions and deletions: "I'm usually thinking about what I need to add…Like word problems I usually skip…and substitute other practice problems that I feel like need to be emphasized more."

Richard, on the other hand, claims to be a strong proponent of the instructional approach embodied by the program. Having taught CMP for over 10 years, he believes that CMP is a desirable alternative to a conventional mathematics curriculum because it allows him as a teacher to facilitate student discussion and to play less of a central role in the classroom discussion: "I would characterize it less as teacher-driven…and more kid-driven…I think the focus becomes--to me I'm giving up being the center of attention." When planning, to make sure that he understands the content for himself, Richard says he first reads through the lesson and solves the task that he will use with students. He then tries to ascertain, from reading the student book, exactly what the lesson is about, the "big idea" students are to come away with, and also places where students may struggle or misconceptions students may have while working on the lesson: "…[J]ust so I get a sense…of what problems they're going to struggle with. Just looking at the answer doesn't help me out. If I actually work through the problem, that gives me a sense of where they're going to struggle." Richard says he uses the teacher's guide during planning only when an idea or concept or even the wording of a particular question in the student book is unclear. The student book provides him with the necessary information he needs for enacting the task during instruction.

> I think if I would read through the [teacher] manual and not read through the kid edition, I'd just feel like I'd be at a disadvantage to know what to expect from the kids…It tells me as a teacher what to know conceptually…but it doesn't help me understand quite how I think the kids are going to react to what I'm asking them to do.

In general, Richard claims to plan for using most, if not all, of the lesson elements as described in the curriculum.

Although both Alicia and Richard use the curriculum materials for the content features of the lessons (albeit to a limited degree in Alicia's case), both teachers appear to have different views of the curriculum, and thus use the materials in very different ways when planning. While Alicia "picks and chooses" from the lesson suggestions what she considers relevant and important for her students, Richard tends to plan for enacting the lesson as described in the materials. Notably, both of these teachers do not follow the lesson suggestions in a prescriptive fashion, including Richard who seems to only rely on the materials for content features of the lessons and not for suggestions as to how to engage students with the content. By engaging with the materials in such an adaptive, or even modified fashion, Richard and Alicia leave the lesson open to interpretation, making room for their conceptions of mathematics teaching and learning to inform their lesson planning.

In contrast to the other two teachers, Susan, having taught for 6 years and used CMP for 3 years, appears to adhere closely to the lesson and corresponding suggestions in the curriculum. She believes the instructional approach espoused in CMP is an ideal way to support students' mathematical development and their ability to communicate their understanding. She views her role as that of a referee, mediating students' discussions of their proposed solution strategies during instruction:

> And so sort of mediating that discussion is sort of the biggest part because…learning of the strategies is supposed to take part amongst themselves. So it's like I have very little, this is how it works, you know, it's more, ok what are your ideas? Let's put them together.

When planning for a lesson, Susan follows the suggestions in the teacher's guide almost as a script for the lesson. She will first read through and solve the task in order to think about how students will approach and solve the task, as well as to consider potential solution strategies that may arise. After working through the content, Susan says she reads through the longer lesson summary in the teacher's guide to understand the overall direction and purpose of the lesson. In addition to reading the detailed teaching notes, she reads both the suggested questions and answers to the task, and then includes these elements in a slideshow presentation she uses during instruction. Though, she only includes suggested questions in her lesson if she decides they are worthwhile and appropriate for her students: "So…whenever there are suggested questions, I see ok, is this a meaningful question for my students?" Susan says she reads through the lesson suggestions to ensure she does not extend the discussion of a concept further than what is expected or stated in the materials, and also to gain an overall sense of the lesson and the mathematical ideas embedded in the lesson: "So, it's good to read that sometimes just to see ok, this is only where they need to get to at the end."

Table 1

Summary of planning problems teachers encountered during unit

Planning Problem	Alicia	Richard	Susan
Anticipating students' work on task	Recalled previous experience with lesson	Read student book, solved task himself	Planned for more teacher-direction of task
Treatment of content in curriculum	Read teacher guide to clarify content, but focused only on "important" aspects	Read teacher guide to clarify content, but focused only on "important" aspects	Read teacher guide to clarify content, and planned to follow lesson suggestions

Susan also says the teacher's guide provides her with an image of how students will engage with the task: "…it gets me ready for what they might say. What's the book going to be after? You know, what's sort of the big idea that they want to come away with." She states that her purpose for using the student book is to understand the task for herself. She uses the teacher's guide, on the other hand, to understand how students may approach and solve the task, including potential misconceptions students may have in relation to the task.

In contrast to Alicia, Susan appears to agree with the principles underlying CMP, and accordingly plans for enacting lessons in the unit largely as described in the materials. Moreover, unlike Richard, Susan uses the curriculum not only for the content features of the lesson, but also she uses the suggestions for how to engage students with the content during the lesson. Thus, Susan, the teacher with less teaching experience as compared to Alicia and Richard, draws heavily from the suggestions in the curriculum when planning, and plans to enact the lesson largely as described in the materials. By adhering to the lesson suggestions in an almost prescriptive fashion, Susan leaves little room for her own interpretation of the lesson.

As these examples illustrate, the nature of teachers' engagement with curriculum materials during planning is determined by a variety of factors. Although Richard agrees with CMP's overall approach to teaching and learning, he largely relies on the curriculum materials solely for its content. It appears that he does not require much pedagogical support when planning; instead, he typically limits himself to reading and working through the student book. Susan, on the other hand, seems to rely on the materials for both content and pedagogical purposes during her planning for the unit, closely following the lesson suggestions. In contrast to Richard and Susan, Alicia does not seem to believe that CMP provides students with sufficient opportunities to practice basic skills and procedures, and therefore modifies the lesson suggestions as needed when planning. In fact, all three teachers held varied conceptions of the curriculum – curriculum as a guide to varying extents (Alicia and Richard) and curriculum as a script (Susan). As these examples illustrate, teachers' various conceptions influence the types of planning problems these teachers encounter and the ways in which teachers manage these problems as they arise during planning for the unit.

Planning Problems

As discussed previously, planning problems constitute a fundamental structural component of the PMI Model because they highlight the relationships between teachers' experience, conceptions of mathematics teaching and learning, and the actual curriculum program used by teachers. Applying this model to teachers' practices requires close analysis of the planning problems experienced by these teachers and the factors underlying the emergence of these planning problems. The teachers in this study primarily encountered two different planning problems – (1) anticipating potential errors and misconceptions students may have in relation to a task and (2) treatment of content in the curriculum. Although teachers encountered several planning problems throughout the unit, these two problems were selected for analysis because they illustrate how the PMI Model depicts teachers' planning practices. Though the two types of planning problems teachers primarily encountered during their planning were quite similar, teachers varied considerably in the ways in which they managed these two planning problems. Table 1 summarizes the planning problems these three teachers encountered during their planning for the unit, and briefly describes the ways in which the teachers managed these different problems. Table 1 does not reflect the frequency in which participating teachers encountered planning problems during the unit.

Anticipating Students' Work on Tasks

A specific planning problem all three teachers encountered during their planning for the unit was anticipating how students would work on the content of the lesson. Alicia, for example, relied on her experience from previous classes to anticipate how students would engage with the content of several lessons in the unit. Based on experiences with classes in previous years, Alicia anticipated that students would struggle with lining up the decimal points when adding and subtracting decimals in a certain lesson. She therefore planned to enact the entire lesson as a whole-class activity to help students through the lesson, as opposed to providing opportunities for students to work collaboratively in groups during the lesson, as was the suggested organization. Similarly, she anticipated students would struggle with another task involving computing discounts by drawing upon her previous experience with that lesson, and again planned the lesson as a whole class activity. As she described in her planning, Alicia believed that by implementing lessons as either whole-class or individual activities, she was better able to address student difficulties and "guide them in the right direction." Richard regularly encountered this same planning problem but managed it quite differently than Alicia. Rather than relying on his previous experiences, Richard anticipated how students would work on the different tasks by working through the lessons himself in the unit – he read the student book and solved the task while thinking about how students would approach the task and what potential aspects might confuse students. In doing so, Richard attempted to forecast the various ways in which students could engage with the content, which reflected his more non-conventional conception of mathematics learning.

Susan also anticipated how students would work on the task, primarily drawing upon what she knew of her students' previous work throughout the unit, but also the information included in the teacher's guide. She became aware of this planning problem by not only reading the teacher's guide, but also from her previous experience with a particular lesson involving decimal division in which students seemed to struggle with long division. Although the materials alerted her to this potential source of confusion for students, she did not seem to use the suggestions in the teacher's guide to support students' understanding of long division. Instead, she used her view of how students should learn in order to address students' difficulty with the content and planned to enact the lesson in particular ways to lessen the likelihood that students would struggle. This was also the case in Susan's planning for a lesson involving computing discounts. In both situations, Susan's previous experience, her view of how struggling students should learn, and her proclivity to follow the curriculum suggestions closely, influenced how she managed the problem of anticipating how students would work on the lesson. She often reduced the complexity of the tasks by telling students how to solve them, taking away students' opportunities to wrestle with the central ideas, but still enacted the lesson largely as written.

Treatment of Content in Curriculum

The treatment of the content within two lessons dealing with long division also emerged as a planning problem for these teachers. All three teachers considered long division as a particularly important concept for students to know and to be able to do. However, the long division algorithm was not explicitly presented in the unit; it was presented as a set of two interrelated lessons in order for students to understand the underlying rationale of the algorithm and the role of place value when dividing decimals. All three teachers had to consider how to enact these two lessons in light of their conceptions towards the content and the curriculum. Alicia and Richard modified the lesson to focus on the procedural aspects of decimal division in these two lessons. This modification reflected their conceptions about what they considered to be the most important aspects of the content. Moreover, this modification comported with their conceptions of the curriculum as a guide rather than a script for their lesson planning. This particular conception of the curriculum left room for the teachers' conceptions toward the content to dictate how teachers planned to enact the lessons.

While Susan also encountered this planning problem, she planned to enact the lesson largely as written in the curriculum despite her reluctance to do so. Her conceptions toward both the curriculum and the content influenced how she framed and managed this problem. Susan felt inclined to change the treatment of the content because her conception of the content clashed with the treatment of the content in the curriculum. However, her desire to plan her lessons largely in accordance with the lesson suggestions provided a push in the opposite direction to teach the content as written: "I don't know about this lesson because students have always struggled with division….Though [the lesson] helps students understand, so I just have to be patient." Susan had to consider what content to enact with students in light of

these conflicting conceptions. Her conception to plan for lessons in accordance with the lesson suggestions ultimately outweighed her conception of the content. The shape of Susan's planning problem contrasts with that of Alicia's and Richard's in that they did not negotiate conflicting conceptions. In summary, the three teachers in this study encountered different problems in the course of their planning for the unit. Despite the fact that the CMP materials provided the means to manage some potential planning problems, teachers seemed to rely largely on their previous experiences and particular conceptions to manage their planning problems. Therefore, in the case of all three teachers, the PMI Model highlights how teachers with diverse conceptions and experiences frame and manage particular planning problems.

Discussion and Conclusion

The previous discussion highlights the interrelationship among curriculum materials, teachers' various conceptions, and the types of and ways in which teachers frame and manage planning problems that arise in the course of their work. During planning, teachers often use curriculum materials as a starting point for their lesson planning. The nature and extent of teachers' engagement with the curriculum materials, however, is determined primarily by their various conceptions. Teachers' various conceptions then influence the type of planning problems they encounter, and also how teachers manage these planning problems.

Teachers' lesson enactment also contributes to the pool of knowledge and information they have to draw from in subsequent years, thereby influencing their conceptions. As Figure 1 illustrates, the PMI Model represents an iterative process that is continuously shaped by teachers' experiences over the course of their careers. With every lesson, teachers potentially encounter unanticipated questions or new strategies that contribute to the knowledge they can draw from when planning the same or related lessons in subsequent years. The ways in which teachers' enact lessons with students over time can also inform how teachers conceive of what it means to teach and learn school mathematics. The proposed model provides a way to understand how teachers' planning practices change, or fail to change, over the course of their careers.

The PMI model suggests a possible cause for the "experience problem," perhaps one of the most significant problems teachers face as they advance through their careers. Unlike their less experienced colleagues, experienced teachers have to consider how to make use of their prior knowledge and experience. With regard to teachers' various conceptions, the "experience problem" consists of how to utilize experienced teachers' assumptions about and prior knowledge of mathematics curricula, and their ideas about what it means to learn and teach mathematics.

The planning routines of Alicia, Richard, and Susan reflect how the experience problem plays out in actual teaching practice. Experienced teachers do not face significantly fewer or different planning problems as compared to less experienced teachers. On the contrary, all three teachers anticipated that a group of students would struggle with a particular aspect of a lesson, or even struggled themselves with certain aspects of the mathematics content. Yet, these teachers encountered these planning problems differently. The differences among these teachers seem to be in their conceptions of the curriculum and content, the prevalence of their conceptions in their planning decisions, and ultimately their instructional decisions.

Regardless of their conceptions, teachers' conceptions of curriculum and mathematics teaching and learning can become calcified over time (Leinhardt, 1983; Leinhardt & Greeno, 1986). As a result, teachers may become inattentive to how their planning decisions influence students' opportunities to learn and they may become resistant to external influences such as new curriculum programs or professional development experiences. Consider Alicia, who seemed to adhere to a more conventional conception of mathematics teaching and learning during her planning. Because Alicia has quite extensive teaching and curricular experience in using more conventional mathematics curricula, she planned to focus students' work on practicing computations and procedures and planned to modify lessons as whole-class discussions rather than collaborative work groups. It appears that Alicia's conceptions of teaching and learning have become somewhat cemented throughout her teaching career and seemed to have hindered her from planning for enacting CMP lessons in this unit in accordance with the curriculum's underlying principles. The PMI Model also captures this common aspect of teachers' practice – as this model represents an iterative process, allowing for teachers' conceptions to become reinforced as teachers amass an increasing amount of knowledge and experience.

The PMI Model has the potential for even broader utility because the planning problems and teacher conceptions discussed here constitute only a handful of

the planning problems and conceptions that might influence teachers' lesson planning. For example, teachers' perceptions of limited time during a lesson may prove problematic for some teachers when deciding how much of the lesson to cover in the time allotted. Another planning problem may arise when teachers have to anticipate how to orchestrate the use of multiple solution strategies during a given lesson, thinking carefully about the order in which to present certain strategies and the mathematical affordances of discussing different strategies. In addition to accounting for a broad range of planning problems, the proposed model also can account for a broad range of teachers' conceptions. For example, teachers' conceptions of their role as teachers may influence how they engage with the curriculum materials during planning. This engagement, in turn, will give rise to new planning problems and ways of managing these problems. Regardless of the precise planning problems and conceptions that may influence teachers' work, these key elements of the PMI Model help to explain teachers' considerations and decisions made during the planning process.

Given the PMI Model and the notion of planning problems that provides its underlying structure, the question for future research becomes how reformers can work to improve teachers' practice, and ultimately student learning. Viewed by many as a driving force of reform, mathematics curriculum materials have the potential to boost educational achievement while embodying new modes of instruction. However, teachers can hold diverse conceptions that stand in contrast to the conceptions of teaching and learning underlying the curriculum, which can hinder teachers from planning in accordance with the curriculum. Teachers who have more experience with conventional curricula and exhibit a more conventional conception of teaching are desensitized to the modes of instruction entailed in implementing reform curricula. Researchers have found that for teachers with extensive experience teaching with more conventional methods and curricula questioned the value and relevance of reform curricula (Manouchehri & Goodman, 1998; Preston & Lambdin, 1995). Still, reformers can target teachers' conceptions directly by designing professional development experiences that are aimed at helping teachers shift their views of what it means to know, learn, and teach mathematics. This is not to say that teachers should participate in the equivalent of a philosophy course, but rather reformers can situate teachers' learning in the actual practice of teaching, wherein teachers can experience what learning and teaching mathematics in reform oriented ways entails. At the very least, the PMI Model underscores teachers' conceptions as a target for reform efforts because they structure and provide a major resource for managing planning problems that arise in the course of teachers' work.

In summary, the PMI Model highlights how teachers' various conceptions frame and influence how teachers' manage planning problems that arise when preparing for mathematics instruction. The model is structured around Lampert's (2001) notion of teaching problems, which can illuminate processes teachers engage in during their daily planning, thus providing a useful lens to understand the nature of teachers' planning routines and reasons underlying their decisions during this phase of teaching. The concept of teaching problems is useful for understanding teachers' practice because it captures the interactions among teachers' various conceptions, their engagement with actual curriculum materials, and their previous experiences. Although the discussion of the PMI Model is specific to teacher planning in a reform mathematics context, such a model of teacher planning is applicable to the planning that occurs in the context of conventional curricula as well, though the nature of teachers' planning problems may be different. As the examples presented in this article illustrate, despite the principles of teaching and learning underlying a curriculum, teachers' various conceptions heavily influence teachers' engagement with the materials during planning, thereby influencing the ways in which teachers manage problems, and the types of planning problems teachers encounter. Applying the PMI Model to understand planning problems, how these problems change over time, and under what conditions they change highlights important elements in mathematics teachers' planning processes.

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