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Aspects of Calculus for Preservice Teachers

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The purpose of this study was to compare the perspectives of faculty members who had experience teaching undergraduate calculus and preservice teachers who had recently completed student teaching in regards to a first semester undergraduate calculus course. An online survey was created and sent to recent student teachers and college mathematics faculty members who had experience teaching a first semester calculus course to help determine the aspects of calculus that they deemed most important in the teaching of calculus to pre-service mathematics teachers. Faculty members with experience teaching at the secondary level, faculty members without experience teaching at the secondary level, and recent student teachers' survey results were compared and there were some notable differences between the groups. The aspect that was ranked the highest among all groups was problem solving which is consistent with the views of major mathematical organizations, such as the Mathematical Association of America (MAA) and National Council of Teachers of Mathematics (NCTM). While all groups' views were similar and consistent with research, recent student teachers' responses suggest that when preparing future teachers in undergraduate calculus, more emphasis should be placed on connections to the secondary curriculum and applications in technology.

Since Calculus is an undergraduate entry-level course for many fields of study, instruction is generally not geared toward preservice mathematics teachers. This raises the question whether this type of learning environment is conducive to the preparation of a secondary mathematics teacher. Originally a doctoral dissertation (Fothergill, 2006), this study examines mathematics faculty and student teacher responses to a survey designed to obtain their perceptions of a theoretical first-semester undergraduate calculus course specifically designed for preservice secondary mathematics teachers. While many aspects of student understanding of calculus have been researched, this study examines the aspects to be emphasized in an undergraduate calculus course designed for preparing preservice mathematics teachers.

Background

According to the United States Department of Education (2000), the demand for certified mathematics teachers is growing at a quicker rate than the supply. Moreover, Brakke (2000) argued that to increase the interest in the mathematics field, higher education must help improve the quality of K-12 mathematics education programs. The National Research Council (NRC, 1989) stated, "No reform of mathematics education is possible unless it begins with the revitalization of undergraduate mathematics in both curriculum and teaching style" (p. 39). While reform in undergraduate mathematics has started, it has not gone far enough to incorporate the needs of preservice mathematics teachers.

As stated by Ferrini-Mundy and Findell (2001) and Clemens (2001) mathematics faculty ignored the needs of the preservice mathematics teachers who were becoming an increasing part of their department. Though mathematics faculty focus on mathematics content, Wu (2011) claimed that they should also focus on the professional development of future teachers. According to a RAND Corporation funded Mathematical Study Panel (Ball, 2003), preservice mathematics teachers should be prepared for teaching which is completely different from preparing students to conduct mathematical research. The report did not advocate less rigor; instead, it suggested that preservice teachers needed preparation for the specific mathematical demands they will face in the K-12 classroom.

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The Conference Board of Mathematical Sciences Report (2001) stated that the mathematics department is partially responsible for the education of mathematics teachers. Similarly, the NRC (2001) recommended that mathematics departments assume greater responsibility for offering courses that provide preservice mathematics teachers with appropriate content that is taught using the kinds of pedagogical approaches that preservice mathematics teachers should model in their own classrooms. Papick (2011) suggested the need for specialized courses for future teachers that address the connection of mathematical ideas to the topics that are taught in K-12 mathematics classrooms. According to Bell, Wilson, Higgins and McCoach (2011), professional development for inservice teachers has been shown to include illustrations of pedagogy and connections across mathematics concepts which lead to growth in mathematical knowledge for teaching; therefore undergraduate courses that reflect these qualities should be available for preservice teachers.

Rationale

With calculus being the capstone course for mathematics studied at the secondary level, it is important that preservice teachers have a strong mathematical teaching knowledge of calculus. Although not all preservice teachers will teach calculus at the high school level, it is still imperative that they understand how the content they are responsible for teaching relates to their students' further study in mathematics. The U.S. Department of Education (2000, 2002) stated that highly qualified teachers need to have a deep understanding of subject matter to be successful in the classroom. This requires developing teachers who are independent learners who can read, write, and communicate mathematics. It can be argued that a teacher with these qualities will be more confident in making curriculum decisions. Since calculus is often a preservice teacher's first college mathematics course, it is reasonable to study how we can improve the teaching of calculus to influence the preparation of preservice mathematics teachers.

The purpose of this study is to explore the perspectives regarding aspects of calculus that mathematics faculty and student teachers deem important and, therefore, that should be emphasized in an undergraduate calculus course for preservice teachers. The study was based on the following questions:

1. What aspects of a calculus course do undergraduate professors deem most important when preparing preservice mathematics teachers?
2. What aspects of calculus do student teachers deem most important in preparing them to teach at the secondary level?

Methods

Both faculty and recent student teachers responded to a survey to rank aspects of calculus they deemed most important to the undergraduate mathematics preparation of preservice teachers. Faculty members who had experience teaching undergraduate calculus were chosen for the study. In addition, some had experience teaching secondary mathematics, but this was not one of the study selection criteria. Recent student teachers' perspectives are of interest because, with their fresh experience in the classroom, and not so distant experience in a calculus course, they can discern how their calculus course helped or did not help them in becoming a secondary mathematics teacher. Therefore, they can give insightful recommendations for a calculus course designed specifically for secondary education mathematics students.

Survey Development

Recommendations from major mathematical organizations were used to determine aspects of calculus that should be emphasized and included in the survey. The Mathematics Education of Teachers, a Conference Board of Mathematical Sciences (CBMS) report (2001), gives specific recommendations for the mathematical content and pedagogy for the preparation of secondary school mathematics teachers. It gives the most detailed outline of the college-level mathematics that secondary school teachers should be studying and recommends that preservice teachers' undergraduate study should develop:

1. Deep understanding of the fundamental mathematical ideas in grades 9-12 curricula and strong technical skills for application of those ideas.
2. Knowledge of the mathematical understandings and skills that students acquire in their elementary and middle school experiences, and how they affect learning in high school.
3. Knowledge of mathematics that students are likely to encounter when they leave high

school for collegiate study, vocational training, or employment.

4. Mathematical maturity and attitudes that will enable and encourage continued growth of knowledge in the subject and its teaching. (p. 122)

The report summarizes the benefits of the study of calculus for preservice secondary level mathematics teachers, recommending that first year mathematics education majors take calculus because:

> Calculus instructors can provide a useful perspective for future high school teachers by giving more explicit attention to the way that general formulations about functions are used to express and reason about key ideas throughout calculus. Its central concepts, the derivative and the integral, are conceptually rich functions. (p. 133)

More generally, the report suggests the following goals for the study of mathematics: developing mathematical maturity, understanding functions, and having a deep understanding of mathematical ideas and the skills needed to apply those ideas.

This CBMS report (2001) is aligned with the National Council of Teachers of Mathematics (NCTM) standards (2000) and the Undergraduate Programs and Courses in the Mathematical Sciences 2004 CUPM Curriculum Guide (Barker, Bressoud, Epp, Ganter, Haver, & Pollatsek). The NCTM process standards (2000) include problem solving, reasoning and proof, connections within and outside mathematics, and representations of functions. The CUPM curriculum guide, which helps mathematics departments in designing undergraduate curricula, recommends making connections, developing mathematical thinking, and using a variety of technological tools as goals for undergraduate calculus. These recommendations together with trends in calculus textbooks (Stewart, 2003; Strauss, 2002), informed the list of aspects that should be used when teaching calculus to preservice teachers. The survey included the following aspects:

- proof writing skills using formal definitions and theorems;
- mathematical reasoning and problem solving skills;
- strengthen the students' algebraic skills; visualization of functions and multiple representations of functions;
- mathematical maturity and prepares students for upper-level mathematics;
- mathematical-based technology skills (i.e. graphing calculator and calculus based software programs);
- connection between undergraduate mathematics and high school mathematics curriculum; and
- application to fields outside of mathematics

Both the faculty and pre-service teacher survey obtained demographics such as professional backgrounds, gender, years of experience, and highest degree obtained, as well as opinions about what aspects of calculus they considered important when teaching calculus to preservice teachers. The survey student teachers asked them to rank the top three aspects of an undergraduate calculus course that would be most beneficial to pre-service mathematics teachers. In addition, the student teachers were asked open-ended questions about their experience in calculus and how it related to their first teaching experience. Faculty participants' survey asked them to rank in order of importance what they thought were the top three aspects of calculus that help preservice teachers become effective educators of secondary school mathematics. Both faculty and student teacher participants were asked to give any suggestions for the creation of a calculus course for preservice teachers.

Participants and Data Collection

The online survey was sent via e-mail to mathematics departments' faculty members from four-year colleges and universities in the United States that were randomly selected from a list maintained by University of Texas at Austin (2005). Colleges were chosen at random and then all faculty from the institution was emailed. The e-mail explicitly requested faculty members that had experience teaching undergraduate calculus to complete the online survey. However, since the survey was sent to all faculty members, it was inevitable that faculty members without experience teaching calculus were contacted.

Less than ten percent of the fifteen hundred faculty members responded, which can be partially attributed to the likelihood that many of the faculty members that were e-mailed did not fit the survey criteria. Although the low response rate could impact the validity and reliability of the study, the response rate is much higher if we disregard faculty members who were

invited to participate but did not meet survey criteria. Hence, these responses can provide useful information in regard to aspects of calculus that future and current educators deem important.

Former student teachers who had completed student teaching within the last year were sent an online survey. Using the University of Texas at Austin's (2005) website, the researcher chose schools at random and emailed college representatives from either mathematics or education departments at over 300 four-year colleges and universities in the United States. The college representative consisted of one of the following: a mathematics department chairperson, mathematics education chairperson, secondary education chairperson, or student teacher supervisor. In some instances, more than one representative was e-mailed from each school. The email asked the college representative to forward the online survey link to secondary mathematics education students who completed their student teaching practicum in the past year. The response rate cannot be determined because college representatives did not report how many recent student teacher received the survey link and each school has a different number of mathematics education students each year.

Data Analysis

The aspects of calculus that faculty and student teachers ranked the highest most often were identified as the aspects that should be emphasized when teaching calculus to future mathematics teachers. For each aspect the percentage of respondents ranking it first, second, or third most important was calculated. To investigate potential differences, responses from faculty with secondary teaching experience were compared against those without such experience. Lastly, responses from faculty with and without secondary teaching experience were compared with student teacher responses.

Results

The 114 faculty respondents consisted of 88 males and 26 females, with a mean of 20.1 years teaching experience. Eighty-five faculty members did not have experience teaching at the secondary level, while twenty-nine did have experience. Fifty-seven student teachers responded with 17 being male and 40 female.

Faculty Members

Figures 1 and 2 illustrate the overall results of the online survey given to faculty members. Overwhelmingly, *problem solving* received the highest number of responses with 68 out of 114, approximately 60%, of the faculty members choosing it as the most important aspect of calculus that should be emphasized in a calculus course designed for preservice mathematics teachers. *Visualization of functions* and *applications outside of mathematics* were also frequently selected. The aspects that received the least number of responses were *technology skills*, *proof writing skills, and connection to the HS curriculum*.

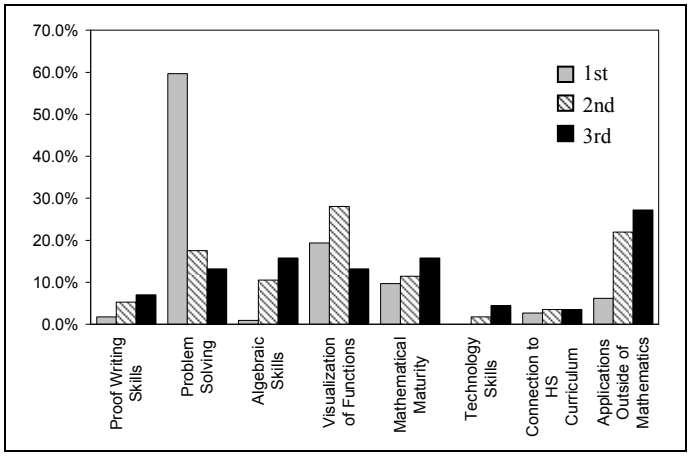

Figure 1. Faculty Members percentage of (n = 114)1st, 2nd, and 3rd ranked aspects.

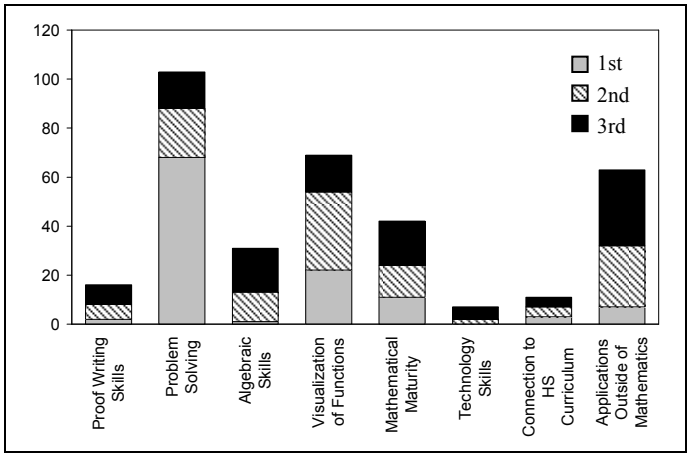

Figure 2. Faculty members (n = 114) 1st, 2nd, and 3rd ranked aspects.

The examiner combined all first, second, and third ranked responses selected for each aspect as shown in chart 2. For clarity, *problem solving* received 68, 20, and 15 responses respectively for first, second, and third ranking; therefore, *problem solving* received a combined response of 104 out of 114 faculty members. *Problem solving* had the most combined responses with approximately 91% of the faculty members choosing this aspect as one of their top three that they believe should be emphasized in a calculus course for

preservice mathematics teachers. *Visualization of functions* and *applications outside the mathematics curriculum* were other top combined responses, approximately 61% and 56% respectively.

Recent Student Teachers

Figures 3 and 4 illustrate their responses were similar to faculty with *problem solving*, *visualization of functions*, and *applications outside of mathematics* being the aspects of calculus they most often deemed important. A notable difference was that so few of the recent student teachers considered proof writing skills important; only four ranked it among their top three.

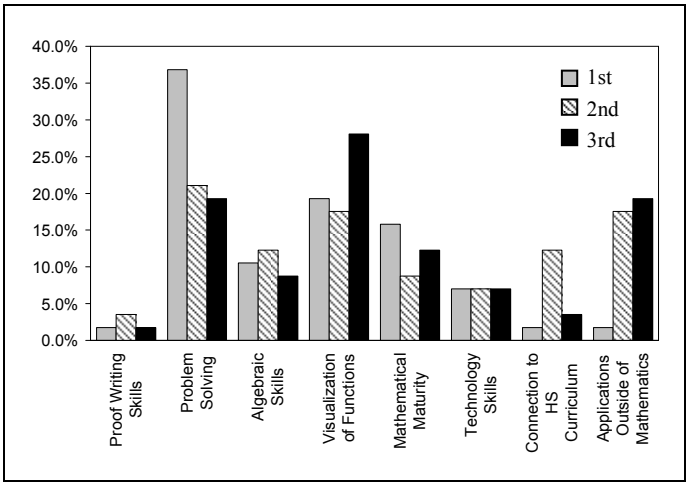

Figure 3. Student teachers percentage of (n = 57)1st, 2nd, and 3rd ranked aspects.

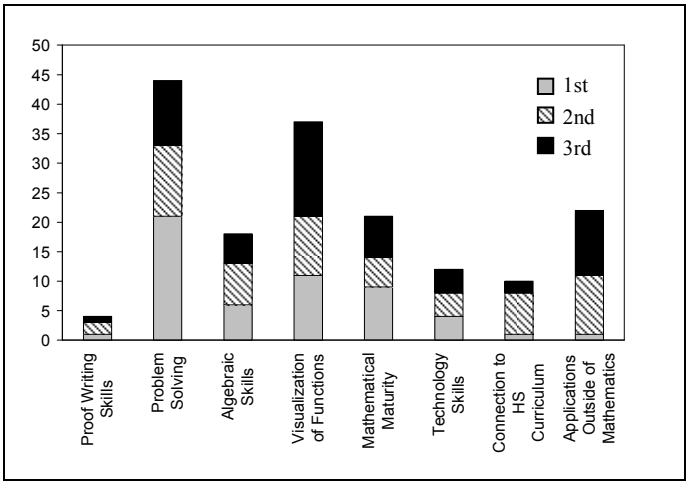

Figure 4. Student teachers (n = 57)1st, 2nd, and 3rd ranked aspects.

Faculty Members With and Without Experience Teaching at the Secondary Level

To investigate possible differences in their perspectives, the author then divided the faculty members into two categories: faculty members with experience teaching at the secondary level and without experience teaching at the secondary level, hereafter referred to as faculty with experience and faculty without experience. The faculty member with no experience teaching at the secondary level consisted of 68 males and 17 females and had a mean of 20.8 years experience teaching calculus. The faculty members that had experience teaching at the secondary level consisted of 20 males and 9 females, with a mean of 18.1 years experience teaching calculus.

P*roblem solving* was chosen by both groups as an important aspect to emphasize when teaching calculus to preservice mathematics teachers (see Figure 5). The chart demonstrates that 92.9% of the faculty without experience teaching at the secondary level and 86.2% of the faculty with experience teaching at the secondary level had selected *problem solving* as one of their top three aspects of calculus. Faculty members with experience had a higher percentage of responses in *visualization of functions, algebra skills, technology skills, connections to the high school curriculum,* and *mathematical maturity* as compared to faculty members without experience. The greatest difference occurred in the category of *visualization of function;* 72.4% of faculty with experience had this aspect in their top three, but only 56.5% faculty without experience listed it in their top three. It should also be noted that 10.3% of faculty with experience thought that *connection to high school curriculum* was the most important aspect, whereas not one faculty member without experience chose that as the most important aspect.

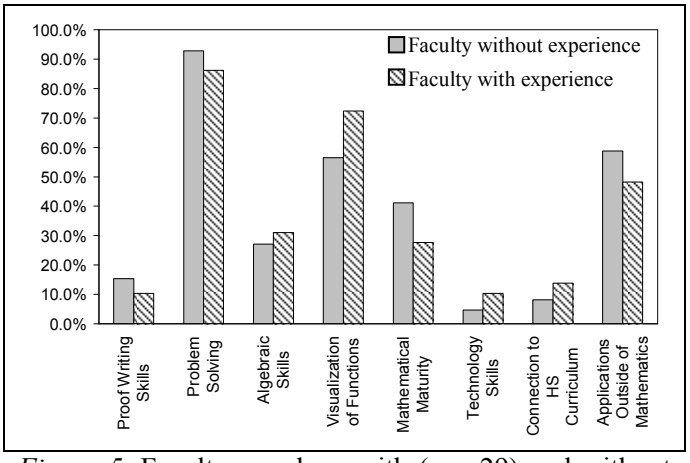

Figure 5. Faculty members with (n = 29) and without (n = 85) teaching experience at the secondary level, combined 1st, 2nd, and 3rd rankings.

Faculty Members vs. Student Teachers

Figure 6 compares the results of all three groups. While some aspects seem to have similar results, one aspect that demonstrated a difference in perceptions between faculty members with and without experience and the student teachers was *connection to the high school curriculum*. *Connection to high school curriculum* was chosen by 8.2% of faculty members without experience teaching at the secondary level as one of their top three aspects. Student teachers had more than doubled the percentage of faculty members without experience with 17.5% of them choosing the *connection to the high school curriculum* as an important aspect.

Technology skills were chosen by 4.7% of the faculty members without experience at the secondary level as one of their top three aspects. In contrast, 10.3% of faculty members with experience put *technology skills* as one of their top three aspects more than doubling that of faculty members without experience. Moreover, 21.1% of student teachers put *technology skills* into their top three aspects making this percentage four times higher than that of faculty members without experience.

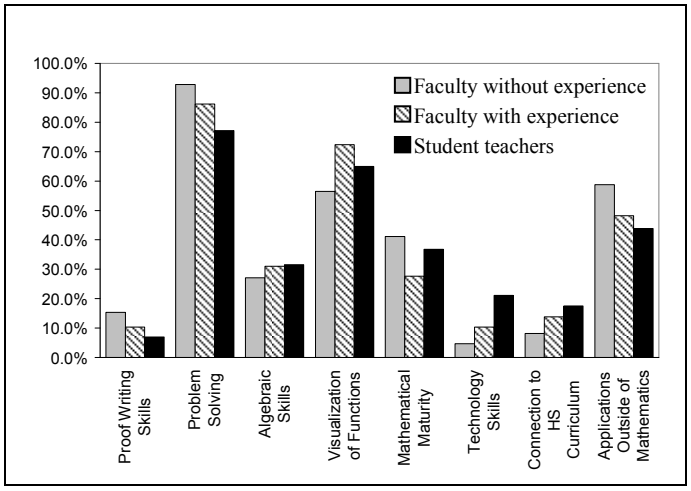

Figure 6. Faculty members with (n = 29) and without (n = 85) teaching experience at the secondary level and student teachers combined 1st, 2nd, and 3rd rankings.

Discussion

It is interesting to note that faculty and student teachers agreed with the highest five aspects to be emphasized in a calculus course designed for preservice secondary mathematics teachers (see Table 1). With major mathematics affiliations such as the MAA and NCTM promoting problem solving, it is no surprise that problem solving was ranked by both the faculty members and student teachers as the most important aspect to be emphasized in a first semester undergraduate calculus course designed for preservice mathematics teachers.

However, an argument can be made that undergraduate calculus is not meeting all the needs of prospective secondary mathematics teachers. While student teacher perceptions agreed with the faculty's in most aspects, student teachers ranked technology skills and connections to secondary curriculum higher than did faculty. Since faculty perceptions differ from the student teachers in these aspects, faculty members may not be meeting these needs. These results indicate that preservice teachers value making connections to the mathematics they will be teaching and that to better meet their needs college should put greater emphasis on making connections to the secondary curriculum and technology in their coursework for preservice teachers.

Table 1
Comparison of Faculty and Student Teacher Top Three Responses Combined

	Faculty	Student teachers
1. Problem solving	91.2%	77.2%
2. Visualization of functions	60.5%	65.0%
3. Applications outside of mathematics	56.1%	43.9%
4. Mathematical maturity	37.7%	36.8%
5. Algebraic skills	28.1%	31.6%
6. and 7. Proof writing skills	14.0%	7.0%
7. Connections to HS curriculum	9.6%	17.5%
7 and 6. Technology skills	6.1%	21.1%

Recommendations

A first calculus course can provide an initial training ground for preservice teachers. It may benefit colleges with a large secondary mathematics education population to develop a calculus course designed specifically for preservice mathematics teachers, so that vertical connections can be made between high school and college level mathematics. This can provide prospective teachers with content knowledge, as well

as pedagogical knowledge that can be used in their future secondary teaching.

There are many connections that can be made while teaching calculus to preservice teachers and these connections need to be explicit. When taught at the secondary level, logarithmic functions may seem an abstract concept with limited application. Hence, when teaching logarithmic differentiation to preservice teachers, the instructor can make explicit reference to logarithmic functions and rules of logarithmic expressions taught at the secondary level. The process of finding the *n*-th derivative of the sine function is similar to finding the value of *i* to the *n*-th power, a common part of algebra in secondary mathematics curriculum. The instructor can use the derivative to connect the concept of finding a relative minimum or maximum value of a function to the concept of finding the derivation of the formula for the axis of symmetry of a parabola.

Calculus is the culminating course of high school mathematics; therefore, preservice teachers should have a deep understanding of this content. As the instructors for this course, mathematics faculty members have a responsibility for preparing future teachers. Mathematics faculty members teaching calculus to future teachers should be teaching in a way that meets the needs of their students and helps them develop as professional educators.

Limitations & Further Research

While this study suggests that there are differences in perspectives on calculus between faculty members and future teachers, further research is still needed. One might argue that student teachers may not have enough experience to connect what they learned in a calculus course to the high school curriculum. Student teachers have a somewhat limited experience at the secondary level and their student teaching experiences can vary greatly. Some may say it is too early in their teaching career to make judgments about what is needed in a calculus course for preservice teachers. On the other hand, the student teachers' responses mostly matched the faculty responses and established research, lending credence to their perceptions of their learning needs. In future studies one might include more experienced inservice teachers who are more familiar with what makes teachers successful and who are better able to reflect on their learning of calculus. Further research could also include how other undergraduate courses, required for preservice teachers, such as linear algebra, abstract algebra, and geometry could be modified to benefit them.

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APPENDIX

Faculty Member Survey

Gender: M or F Years teaching Calculus: _____

Do you have experience teaching at the secondary level? : _____

Highest Degree Earned: _____

Please rank the following statements about aspects of calculus that you believe helps pre-service teachers become effective educators of secondary school mathematics. Please put 1 next to the most important, 2 next to the second most important, and 3 next to the third most important.

____ Calculus helps to develop proof writing skills using formal definitions and theorems.

____ Calculus helps to develop mathematical reasoning and problem solving skills.

____ Calculus strengthens the students' algebraic skills.

____ Calculus helps develop an understanding and visualization of functions and multiple representations of functions.

____ Calculus builds mathematical maturity and prepares students for upper-level mathematics.

____ Calculus facilitates the development of mathematical-based technology skills (i.e. graphing calculator and calculus based software programs).

____ Calculus demonstrates a connection between undergraduate mathematics and high school mathematics curriculum.

____ Calculus provides insight into its application to fields outside of mathematics.

Please indicate any other aspect that you believe help pre-service teachers.

Do you feel your answers would differ, if asked about non-mathematics education majors?

Student Teachers Survey

Gender: _____

Please rank the following statements about aspects of calculus that you believe helps pre-service teachers become effective educators of secondary school mathematics. Please put 1 next to the most important, 2 next to the second most important, and 3 next to the third most important.

____ Calculus helps to develop proof writing skills using formal definitions and theorems.

____ Calculus helps to develop mathematical reasoning and problem solving skills.

____ Calculus strengthens the students' algebraic skills.

____ Calculus helps develop an understanding and visualization of functions and multiple representations of functions.

____ Calculus builds mathematical maturity and prepares students for upper-level mathematics.

____ Calculus facilitates the development of mathematical-based technology skills (i.e. graphing calculator and calculus based software programs).

____ Calculus demonstrates a connection between undergraduate mathematics and high school mathematics curriculum.

Please indicate any other aspect that you believe help pre-service teachers.