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Guest Editorial…

Examining Mathematics Teachers' Disciplinary Thinking

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Shulman's (1986) seminal paper on subject-matter knowledge in teaching brought attention to different domains of teacher knowledge and how that knowledge might be cultivated. In particular, he described a "reflective awareness" (p. 13), developed from analysis of discipline-focused teaching and learning. This reflective awareness enables professionals to perform tasks in their particular disciplines but also enables them to communicate their thinking, rationales, and judgments as they do so. For mathematics teacher educators, being able to articulate our thinking, rationales, and judgments with respect to doing and teaching mathematics is extremely important as we attempt to help prospective teachers develop their own reflective awareness. In order to do so, we must have a well-defined sense of what the disciplinary thinking about teaching mathematics entails.

Although we have different focuses within mathematics education, with LouAnn teaching PreK–8 mathematics content courses and Kyle teaching middle grades and high school mathematics methods and practicum courses, we have found commonalities in the ways that our prospective teachers struggle to develop the disciplinary thinking processes that are integral to understanding mathematics and teaching it effectively. For example, prospective teachers in mathematics content courses often cannot make sense of their classmates' solutions when the method of solution differs greatly from their own. Similarly, prospective teachers in methods courses struggle when identifying and sequencing appropriate mathematical tasks for instruction. These skills are examples of *specialized content knowledge* (SCK), mathematical knowledge of particular importance to PreK–12 teachers (Ball, Thames, & Phelps, 2008). We have made our prospective teachers' development of SCK an important focus of our programs due to its positive correlations with student achievement (Hill, Rowan, & Ball, 2005). For example, we have attempted to situate activities, assignments, and assessment items in *mathematical tasks of teaching* (Ball, Thames, & Phelps, 2008)—everyday tasks of teaching that require the use of SCK. Such tasks include "choosing and developing usable definitions," "responding to students' 'why' questions," and "asking productive mathematical questions" (p. 400).

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As mathematics teacher educators, we have found it difficult to pin down and articulate in detail the disciplinary thinking used by mathematics teachers when enacting their SCK. The general nature of characterizations of critical thinking, such as focusing on the obscure notion of "concept" and practices such as brainstorming, making comparisons, and questioning, prompted us to seek a more discipline-specific solution. A program sponsored by our institution's Center for Faculty Innovation introduced us to a model aimed at decoding disciplinary thinking, that is, the thinking specifically used by experts in their discipline. Middendorf and Pace (2004) characterized this kind of thinking as something that is rarely presented to students explicitly.

Decoding the Disciplines Model

Middendorf and Pace (2004) presented a model based on seven questions (see Figure 1) that guides university faculty through a process to better understand the implicit ways of thinking exhibited within their disciplines and how to make those ways of thinking explicit to students. Rather than focusing on the general goal of critical thinking, the Decoding the Disciplines Model (DDM) targets specific bottlenecks to student learning, instances during the learning process where a significant number of students falter. Once a bottleneck is identified, the faculty member attempts to unpack how he or she might navigate through it. This results in a list of ideas and tasks used by the faculty member to work through the bottleneck. This list of ideas and tasks can serve as a heuristic guide for novices. The first six questions of this model form a cycle of inquiry, with the seventh question serving as an offshoot from the sixth. Through using the DDM, students are provided opportunities to practice and receive feedback on discipline-specific ways of reasoning or skills.

1. What is a bottleneck to learning in this class?
2. How does an expert do these things?
3. How can these tasks be specifically modelled?
4. How will students practice these skills and get feedback?
5. What will motivate the students?
6. How well are the students mastering these learning tasks?
7. How can the resulting knowledge about learning be shared?

Figure 1. The seven questions of the Decoding the Disciplines Model (Middendorf & Pace, 2004).

Our efforts to address the initial questions of the DDM were supported by a self-study methodology in which we acted as “critical friends” (Loughran, 2004, p. 157) by challenging each other’s claims and pushing for more explicit clarification of ideas. In addition, we shared the products of our work with a colleague outside of mathematics education but familiar with the DDM as a way to ensure we were “constantly asserting ideas and interrogating them, inviting alternative interpretations and seeking multiple perspectives” (Pinnegar & Hamilton,

2009, p. 165). To illustrate our use of the DDM, we will focus on a bottleneck for prospective teachers in the middle grades mathematics methods course, developing a sequence of tasks used to teach a new concept.

Identifying Bottlenecks

To identify bottlenecks, we examined prospective teachers’ work on assessments from their previous courses to determine specific instances where a majority demonstrated difficulty with key ideas of the course. For elementary and middle grades teacher candidates, we also considered data from a program-wide multiple-choice assessment of prospective teachers’ SCK of K–8 mathematics, which was modeled after the Learning Mathematics for Teaching assessment developed at The University of Michigan (Hill, Schilling, & Ball, 2004) as well as focus group interview data about the tasks on this assessment. Although it was easy to identify instances where our students struggled, it was often difficult to articulate precisely what that struggle entailed. To hone this precision, we strove to push each other for further clarification of our ideas by asking questions such as “How would you reason through that task?” and “What do you mean by that terminology?” For this process, we attempted to set aside our knowledge of familiar concepts and jargon-laden terms to clarify our own understanding of them. Repeating this process with our out-of-discipline colleague reinforced this push for a layman’s view, improving our ability to better articulate how one might navigate through a given bottleneck.

One bottleneck was identified using a methods course assessment on lesson planning. In this assessment, many prospective teachers struggled to use and sequence tasks within the targeted students’ zones of proximal development. For example, in an introductory lesson about fraction division, one prospective teacher began his lesson by asking students to solve the task $\frac{5}{3} \div \frac{1}{2}$ using manipulatives and, from this solution, independently develop an algorithm to divide any two fractions. Although this task has the desired goal of students understanding the underlying mechanics of the division algorithm, it uses a relatively difficult quotient, provides only one concrete example, and does not provide a context for the quotient, focus on the meaning of fraction, or connect to previously learned computation strategies (recommendations offered by Van de Walle, Karp, & Bay-Williams, 2010). Other prospective teachers provided multiple contextual tasks to develop the concept, but struggled to sequence them in an order that would build understanding. In each of these cases, the prospective teachers lacked the SCK needed to identify the subtle mathematical differences between similar tasks and distinguish between the relative complexities caused by these differences. For example, some began their progressions using non-unit-fraction divisors before those with unit fractions. Therefore, we identified the development of a sequence of tasks used to teach a new concept as a bottleneck for the prospective teachers.

An Expert's View

For each identified bottleneck for prospective teachers, we sought to write a detailed description of what we, as expert mathematics teachers, would do to navigate through it. Because some of these processes were automatic or almost instinctual for us, we found it difficult to articulate our thinking without glossing over subtle nuances that might be crucial for a novice teacher. Using the discourse strategy previously described, we challenged each other to define and clarify our own disciplinary thinking.

To identify the thinking one might use to create a sequence of tasks used to introduce a new mathematical concept, Kyle looked to recreate the experience of a novice by working with a mathematical concept with which he was familiar as a learner, but not as a teacher (mirroring the situation faced by prospective teachers). Because he had never taught calculus, he focused on the steps he would undertake to design a sequence of tasks to teach the concept of related rates. This process involved unpacking the mathematics found in textbook examples, identifying the relationships between them, and using these relationships as a foundation for developing student understanding. From this work, the disciplinary thinking was generalized into a set of small incremental steps (see Figure 2) that could *guide* prospective teachers during their initial attempts to navigate the bottleneck.

Bottleneck: Developing a sequence of tasks used to teach a new concept.

1. Examine the curriculum framework goal(s) to be addressed.
2. Determine the *big idea(s)* (Charles, 2005) associated with these goals.
3. Write learning objectives for the lesson that relate back to the *big ideas*.
4. Work each example task in the book. In this process, note:
   a. Different representations that might be productively used in a solution
   b. Connections or common themes between the tasks, objectives, and big mathematical idea(s)
   c. Prerequisite knowledge needed to engage in each task
   d. Non-contextual differences between the tasks (changes in mathematical complexity or required level or type of thinking)
5. Identify stages of development needed to understand the concept and perform related skills.
6. Identify existing tasks corresponding to these stages. For example, could the provided textbook examples serve this purpose? Would additional tasks be needed?
7. Brainstorm possible student strategies or solutions for these tasks.
8. Evaluate and modify the identified tasks to optimize student strategies and misconceptions.

Figure 2. A list of the small incremental steps for navigating the bottleneck of developing a sequence of tasks used to teach a new concept.

Modeling and Practice

Once we had achieved a sense of the disciplinary thinking needed to navigate a particular bottleneck, our attention shifted to designing course activities that would enable prospective teachers to learn and practice that thinking themselves. Examining the prospective teachers' work during these activities has helped us to identify additional bottlenecks and provided further insight into our view of disciplinary thinking. For example, Kyle's prospective teachers struggled with identifying big mathematical ideas, the second step in the process shown in Figure 2. Given the struggles of his prospective teachers, Kyle returned to the literature and found evidence that might support his observations in class:

> Some mathematical understandings for Big Ideas can be identified through a careful content analysis, but many must be identified by "listening to students, recognizing common areas of confusion, and analyzing issues that underlie that confusion" (Schifter, Russell, and Bastable 1999, p. 25).
>
> Research and classroom experience are important vehicles for the continuing search for mathematical understandings. (Charles, 2005, p. 10)

The possibility that his prospective teachers' difficulties with big ideas may stem from a lack of teaching experience has prompted Kyle to plan experiences for his class using classroom data (video, written cases, vignettes, etc.) to provide his prospective teachers with opportunities to listen to students, to recognize common misconceptions, and to analyze issues that help to create these misconceptions.

Looking Ahead

This work is an iterative process. As we continue working with our prospective teachers, we further refine our bottleneck articulations, descriptions of our unpacked disciplinary thinking, and the associated classroom activities whose purpose is to help our learners navigate through the identified bottlenecks. As we implement our work in our classrooms, assessment plays a key role in shaping future iterations in two ways. First, using pre- and post-assessments will quantify prospective teachers' gains in mastering disciplinary thinking. Second, qualitatively examining their responses may enable us to identify other bottlenecks (Kurz & Banta, 2004).

As discussed, we have found that some of the steps we have identified to illuminate our disciplinary thinking for prospective teachers are in fact bottlenecks themselves, requiring further unpacking and clarification. For example, determining big mathematical ideas and brainstorming possible student strategies or solutions for a task, two processes identified as key steps for developing a sequence of tasks to teach a new concept, are not trivial. As a result, we have labeled these skills as bottlenecks as well and have undertaken defining the disciplinary thinking needed for each. In this way, focusing on bottlenecks as a fundamental idea has enabled us to better define our course objectives and hone our instruction and assessment, with the goal of ultimately improving our prospective teachers' performance in their future classrooms.

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