

The Association between Teachers' Beliefs, Enacted Practices, and Student Learning in Mathematics

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Mathematics educators continue to explore ways to improve student learning. Of particular interest are the relationships between teachers' instructional practices, their beliefs towards mathematics teaching, and student learning outcomes. While some studies have found empirical links between teachers' enactment of specific instructional practices and gains in student learning, there is no conclusive connection between beliefs, instructional practices, and gains in student learning outcomes. This study examines a few critical relationships between: teachers' beliefs and instructional practices, teachers' beliefs and student learning outcomes, and teachers' instructional practices and student learning outcomes. Data from 35 teachers and 494 elementary school students indicated significant relationships between teacher beliefs and practices but not between teacher beliefs or instructional practice when related to student achievement in mathematics measured by curriculum-based tests. Implications for the design of professional development and for further research related to mathematics teachers' beliefs, their instructional practice and their student learning outcomes are also shared.

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Overview

Improving Student Learning in Mathematics

Mathematics educators and policy makers continue to examine how to best increase student learning outcomes in mathematics (Braswell, Daane, & Grigg, 2003; Gonzalez et al., 2004; Stigler & Hiebert, 1999; United States Department of Education [USDE], 2008; Wu, 2009). Despite mixed results, researchers have found empirical links between specific instructional practices and student learning outcomes (Carpenter, Fennema, Franke, Levi, & Empson, 2000; USDE, 2008; Wenglinsky, 1999). These instructional practices reflect a student-centered view on teaching mathematics, in which students engage in mathematically rich tasks and are supported by classroom teachers who pose questions and modify instruction based on students' mathematical thinking (Carpenter, Fennema, & Franke, 1996; National Council for Teachers of Mathematics, 2000).

In recent years, critics to this student-centered approach to teaching mathematics have emerged, citing a need to focus more on basic facts and mathematical algorithms (Marshall, 2006). The recently published report from the United States National Math Panel (USDE, 2008) found that evidence suggesting one specific approach being more effective than others was inconclusive. Some studies (e.g., Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Gonzalez et al., 2004; Polly, 2008) have found empirical links between student-centered approaches to teaching mathematics and statistically significant gains in student learning outcomes, but there is still a gap in the literature regarding the interplay between teachers' instructional practices and student learning.

Teachers' Beliefs in Mathematics

Teachers' beliefs towards mathematics and their impressions of effective mathematics teaching have been associated with teachers' enacted instructional practices (Fennema et al., 1996), their use of curricula (Remillard, 2005; Stein & Kim, 2008), and their willingness to enact student-centered pedagogies (Heck, Banilower, Weiss, & Rosenberg, 2008; McGee, Wang, & Polly,

2013; Remillard & Bryans, 2004). Discussion of the composition of teachers' mathematical beliefs has gone on for decades. Ernest (1991) argued that a mathematics teacher's belief system has three parts; the teacher's ideas of *mathematics as a subject for study*, the teacher's idea of *the nature of mathematics teaching*, and the teacher's idea of *the learning of mathematics*. Askew, Brown, Rhodes, Johnson and William (1997) characterized the orientations of teachers towards each of these components as *transmission* (T), *discovery* (D) or *connectionist* (C). Swan (2006) posited that an individual teacher's conception of mathematics teaching and learning might combine elements of each of them, even where they appear to be contradictory.

Swan (2006) explained Askew et al.'s (1997) categories in detail. Transmission-oriented teachers believe that mathematics is a set of factual information that must be conveyed or presented to students, and typically enact didactic, teacher-centered methods. Discovery-oriented teachers view mathematics as a set of knowledge best learned through student-guided exploration, and frequently tend to focus on designing effective classroom experiences that are appropriately sequenced. Lastly, connectionist-oriented teachers view mathematics as an intertwined set of concepts, and they rely heavily on experiences to help students learn about the connections between mathematical topics.

The enactment of student-centered and standards-based pedagogies requires teachers to embrace both a discovery and a connectionist stance (Swan, 2007). Teachers are charged with the role of designing learning environments and facilitating students' exploration of concepts through a variety of hands-on activities and games (Mokros, 2003). Following these activities, teachers guide students' discussions of the activities and help them make explicit the mathematical concepts that were embedded in the tasks. In order for the implementation of standards-based instruction to be effective, teachers must facilitate both the activities and the discussion of the mathematics. Although discovery and connectionist dispositions are related to the philosophical underpinning of standards-based mathematics curricula, the transmission view is contradictory. Transmission-oriented teachers relate best to traditional curricula in which information is presented and followed by substantial practice

opportunities. Therefore, it is reasonable to expect that teachers who embrace standards-based mathematics curricula are oriented towards either the discovery or connectionist views.

Teachers' Mathematics Instruction

Mathematics education researchers have classified teachers' instruction in numerous ways. Qualitative studies (e.g., Cohen, 2005; Henningsen & Stein, 1997; Peterson, 1990; Schifter & Fosnot, 1993), typically using case study or ethnographic methodologies, provide intensive and longitudinal data about teachers' instruction. Some studies (e.g., Fennema et al., 1996; Hufferd-Ackles, Fuson, & Sherin, 2004; Polly & Hannafin, 2011; Schifter & Simon, 1992) have embraced a multi-methods approach, in which qualitative observation data are quantified using a rubric or scale. These reports then provide numerical data for teachers' instruction as well as descriptions of their enacted pedagogies. Lastly, survey studies (e.g., Heck, Banilower, Weiss, & Rosenberg, 2008) collect self-reported data from teachers on their instructional practices. These survey studies sometimes are done in isolation, or coupled with classroom observations to increase the reliability of the self-reported data.

Researchers have attempted to classify teachers' instructional practices, such as teacher or student centered, in a variety of ways (Garet, Porter, Desimone, Birman, & Yoon, 2001; Heck et al., 2008; Swan, 2006; Tarr, Reys, Reys, Chavez, Shih, & Osterlind, 2008). These researchers have observed that teachers' instructional practices may vary based on the concept they are teaching, and the types of curricula resources utilized. Further, although teachers' practices may shift slightly, their self-report of their instructional practices typically aligns to observed instructional practices (Desimone, Porter, Garet, Yoon, & Birman, 2002; Swan, 2006).

Gaps in the Research

Student-centered mathematics instruction and beliefs that are standards-based (discovery and connectionist) have potential to lead to greater student learning outcomes than those pedagogies and beliefs that are more teacher-centered (Fennema et al., 1996; Wenglinsky, 1999). However, there is a lack of empirical studies

that link both teachers' beliefs about mathematics teaching and learning and their instructional practices to student learning outcomes. In this study we aim to examine the links between (a) teachers' beliefs and student learning outcomes, (b) teachers' instructional practices and student learning outcomes, and (c) teachers' beliefs, instructional practices, and student learning outcomes.

This study was guided by the following research questions:

1. How are teachers' beliefs regarding mathematics teaching and learning associated with their teaching practices in mathematics?
2. Are there significant differences between grade levels and school districts with respect to student gains in mathematics achievement following the intervention?
3. How are teachers' beliefs regarding mathematics teaching and learning associated with their students' learning of mathematics?
4. How are teachers' beliefs regarding mathematics teaching and learning associated with their students' learning of mathematics?

Methods

Participants

Participants in this study included 53 elementary school teachers (grades K through 5) that were involved in a mathematics professional development program focused on standards-based instruction. All teachers were certified to teach elementary school and taught in two school districts near a large city in the southeastern United States. They were identified as teacher-leaders from their respective schools as a requirement to participate in the professional development. Thirty-two teachers were from a large urban school district and the remaining 21 teachers were from a neighboring suburban school district. Thirty-seven percent ($n = 20$) hold only a bachelor's degree, 30% ($n = 16$) hold a master's degree, and one teacher holds a bachelor's degree and certification specific to their content area. The rest did not report their highest degree held. Eighty-seven percent ($n = 46$) identified as Caucasian while 13% ($n = 7$) identified as African American.

Participants also included 688 students who were in the participating teachers' classrooms. Gender and ethnicity were reported by teachers for their aggregate classrooms. Fifty percent ($n = 344$) of the students were female and 50% ($n = 344$) were male. Thirty-nine percent ($n = 268$) of the students were Caucasian, 34% ($n = 234$) were African American, 20% ($n = 138$) were Hispanic, 4% ($n = 28$) were Asian, and 3% ($n = 21$) were identified by their teachers as "Other." Fourteen percent ($n = 96$) were identified as Limited English Proficient (LEP) and 10% ($n = 69$) were identified as having Individualized Education Plans (IEP).

Instruments

Teacher's beliefs

The teachers' beliefs questionnaire (Appendix A) was developed by Swan (2007) to examine teachers' espoused beliefs about mathematics, mathematics teaching, and mathematical learning. For each of those three dimensions, teachers report the percentage to which their views align to each of the transmission, discovery, and connectionist views. Participants were instructed that the sum of the three percentages in each section should total 100.

Swan (2007) noted a clear distinction between the transmission orientation and the remaining two orientations but not a very clear distinction between the discovery and connectionist orientations. Further, discovery and connectionist categories both aligned with standards-based orientations to teaching and learning of mathematics (McGee et al., in press). Therefore, we coded teachers into two categories: transmission and discovery/connectionist. Teachers were coded as discovery/connectionist if they indicated at least 50% in either discovery or connectionist category. Due to the alignment of both the discovery and connectionist categories with standards-based orientations to mathematics teaching, the data on teachers' beliefs were analyzed as a dichotomous variable; "1" represented teachers with transmission views toward teaching mathematics and "0" stood for teachers with a discovery/connectionist views toward teaching mathematics.

Teachers' practices

The teachers' practices questionnaire (Appendix B) examines participants' self-report about instructional practices related to their mathematics teaching (Swan, 2007). Each of the 25 items reflects either student-centered or teacher-centered pedagogies, and teachers identified the extent of their use of those instructional practices. For each item, participants rated themselves on a 5 point Likert scale where 1 represents "none of the time" and 5 represents "all of the time." Following the same procedure as Swan (2007), a practice scale was constructed by reverse coding student-centered statements (Items 5, 6, 7, 11, 12, 15, 16, 17, 20, 21, 24, and 25) and summing the ratings obtained. The scores ranged between 25 and 125 with higher scores reflecting more student-centered behaviors. The Cronbach's alpha reliability coefficient of these 25 items was .79 (Swan, 2007).

Teachers' mathematical knowledge for teaching

All teacher-participants completed the Mathematical Knowledge for Teaching assessment. The Content Knowledge for Teaching Test (see sample in Appendix C) measure teachers' knowledge of mathematics content and knowledge of students and content (Hill, Rowan, & Ball, 2005).

Student achievement measures

The student achievement measures used in this study were end-of-unit assessments from the *Investigations in Number, Data, and Space* elementary mathematics curricula (TERC, 2008). Teachers administered the same assessment before teaching the unit (pre-tests) and immediately after completing the unit (post-tests). Each assessment was scored by project evaluators using a rubric that had been co-developed by teacher-participants and professional development facilitators. One of the professional development facilitators trained the evaluators how to score the assessments, and inter-rater reliability was found using 10 work samples for each of the six grades. Of the 60 work samples, there was agreement on 58 samples (96.67%). For the two work samples that there was not agreement, the professional development

facilitator and the evaluator discussed and reconciled to reach consensus. After each assessment was scored using the rubric, scores were converted to a percentage. Gain scores were used in the analysis to examine student growth from the pre-test to the post-test.

Data Analysis

Independent sample *t*-tests with pooled variance were employed to see if differences exist between teaching practices of teachers with beliefs of transmission and discovery/connectionist. Analysis of Variance (ANOVA) was used to examine school district differences as well as grade-level differences on gain scores between pre-test and post-test of mathematics achievement. Cohen's *d* was used as a measure of effect size for *t*-tests whereas partial η^2 was used as a measure of effect size for ANCOVA. Hierarchical linear model (HLM) was used to examine the impact of teacher beliefs and practices on student mathematics achievement.

Results

Teachers with transmission orientation in teaching ($n = 23$) had significantly higher frequency of teacher-centered practices ($M = 72.09$, $SD = 10.09$) than teachers with discovery/connectionist orientation in teaching ($n = 29$, $M = 62.83$, $SD = 9.34$), $t(50) = 3.43$, $p = .001$, $d = 0.95$. Similar results were found for teacher beliefs in learning. Teachers with transmission orientation in learning ($n = 11$) had significantly higher frequency of teacher-centered practices ($M = 73.09$, $SD = 8.01$) than teachers with discovery/connectionist orientation in learning ($n = 41$, $M = 65.27$, $SD = 10.74$), $t(50) = 2.25$, $p = .03$, $d = 0.83$. However, teachers with transmission orientation in mathematics ($n = 14$) did not report significantly higher frequency of teacher-centered practices ($M = 68.93$, $SD = 9.99$) than teachers with discovery/connectionist orientation in mathematics ($n = 38$, $M = 66.18$, $SD = 10.92$), $t(50) = 0.82$, $p = .42$, $d = 0.26$. The effect sizes associated with statistically significant differences were large whereas that associated with statistically insignificant differences was small (Cohen, 1988). Descriptive statistics of student gain

scores in mathematics achievement by school districts and grade level are presented in Table 1.

Table 1

Means and Standard Deviations of Student Gain Scores in Mathematics Achievement Tests by School Districts and Grade Level.

		Grade K	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5
District	<i>M</i>	16.92	41.81	10.26	36.76	18.60	5.57
A	<i>SD</i>	(27.51)	(35.55)	(22.02)	(23.76)	(27.40)	(18.31)
	<i>n</i>	67	58	137	19	43	106
B	<i>M</i>	17.12	34.71	7.41	29.55	17.39	20.13
	<i>SD</i>	(31.05)	(40.99)	(25.33)	(26.88)	(20.87)	(29.85)
	<i>n</i>	37	94	12	13	23	79

ANOVA revealed a statistically significant interaction effect of school district and grade level for student gain scores, $F(5, 676) = 2.76, p = .02$, partial $\eta^2 = .02$. Although the effect size for this difference is small, it is very likely that the grade-level differences vary across the two school districts. Therefore, we decided to look at the grade-level differences in each school district separately. Statistically significant grade-level differences were noticed in School District A, $F(5, 424) = 20.22, p < .001$, partial $\eta^2 = .19$. This is a large effect size (Cohen, 1988). Post-Hoc tests with Tukey's HSD (Honestly Significant Difference) method suggested that students in the first grade made statistically significantly more gains than students in other grades except in Grade 3 at a significance level of .05. Students in Grade 3 made statistically significantly more gains than students in Grades K, 2, and 5. It is interesting to note that Grade 5 students made significantly less gains than students in all other grade levels except Grade 2. Although all of the assessments focused on number sense, the second and fifth grade assessments covered skills that students had worked with in the previous grade. No statistically significant differences were found between students in Grades 2 and 4 or between students in Grades 3 and 4. In School District B, however, no grade-level differences were noticed using the same post-hoc tests at the same significance level although ANOVA did show a statistically significant grade-level effect, $F(5, 252) = 3.23, p = .01$, partial $\eta^2 = .06$.

When building the HLM model, we lost 18 teachers and 12 students due to attrition and student absences. A two-level HLM model (students nested within teachers) was used. At Level 1, a non-conditional model was used because the gain scores in mathematics achievement tests was the only student-level variable available. At Level 2, three teacher-level variables (mathematics content knowledge, beliefs towards teaching mathematics, and teacher practices with regard to frequent use of teacher-centered activities) were used to predict the parameters at Level 1 (intercept and slope). The intercept represent the adjusted mean gain scores of all students as the predictors with continuous scales were grand-mean centered whereas the predictor with dichotomous scale was uncentered. The slope is the expected change in gain scores associated with a unit increase in the predictors. The error term at both the student and teacher levels were treated random because we were interested making specific statements about the teacher variables of our interest (Raudenbush & Bryk, 2002). HLM analyses suggested a statistically significant and negative association between teacher’s use of teacher-centered activities and student gain scores in mathematics achievement, $t(30) = -2.15$, $p = .04$. This means that one unit increase in the sum of teacher-centered activities reported on the teacher practice survey is associated with a loss of 0.64 in the student gain scores. Students taught by teachers with transmission orientation in teaching ($n = 281$) are likely to see less gains than students taught by teachers with discovery/connectionist orientation ($n = 210$), $t(30) = -3.44$, $p = .002$. The association between teacher content knowledge and student gains on the mathematics achievement test was not statistically significant, $t(30) = 0.39$, $p = .70$ (See Table 2 for estimates of parameters).

Table 2

Estimation of Teacher Effects on Student Achievement Gains in Mathematics

	Coefficient	SE	t	df	P
Adjusted Mean Gain Score	29.34	4.28	6.85	30	<.001
Teacher Content	0.11	0.29	0.39	30	.700
Teacher Belief	-16.90	4.91	-3.44	30	.002
Teacher Practice	-0.64	0.30	-2.15	30	.039

Discussion

Our results showed teachers with transmission orientation to both mathematics teaching and mathematics learning reported using more teacher-centered practices in the classroom. Further, students in transmission-oriented classrooms had statistically significantly smaller gain scores on the curriculum-based assessments. The findings shared warrant further discussion.

Interaction between Beliefs and Practices

Swan's (2006) beliefs instrument examines teachers' dispositions towards mathematics teaching, mathematics learning and mathematics content. Consistent with prior research on the interaction between teachers' beliefs and their instructional practices (Phillipp, 2007; Swan, 2007), teachers in this study who reported a transmission (teacher-centered) orientation to mathematics teaching also reported more teacher-centered practices in the classroom. Teachers with discovery/connectionist orientation to mathematics teaching reported more frequent uses of student-centered pedagogies. Similar results were found in relation to teachers' disposition towards mathematics learning; discovery/connectionist orientations towards learning were empirically associated with more frequent report of enacting student-centered pedagogies.

Although teachers' orientations towards the teaching and learning of mathematics were related to their use of student-centered instructional practices, no statistical link existed between teachers' orientation towards mathematics as a subject and their instructional practices. This suggests teachers' views of mathematics as a subject, whether a set of procedures, a creative set of ideas, or an interconnected body of concepts, does not influence how they teach.

Implications for professional development

When these findings are considered in the context of professional development and teacher change, this data supports the ideas that teachers' instructional practices are more apt to shift to a more student-centered approaches if teachers engage in

activities that influence teachers' beliefs about mathematics teaching and how students learn mathematics towards more discovery/connectionist approaches. Approaches such as looking at student work and examining the interplay between teaching and learning has led to shifts towards more learner-centered beliefs and practices (Carpenter et al., 1996; Carpenter et al., 2000; Polly, 2006). However, these researchers were unable to empirically conclude which occurred first. Some teachers attempted standards-based pedagogies in their classroom that then shifted their beliefs to become more student-centered. However, in other cases teachers' beliefs shifted during workshops before enacting standards-based pedagogies in their classroom.

Beliefs, Instructional Practices, and Student Achievement

This study supports prior research linking student-centered pedagogies to statistically significant gains on student learning outcomes (Fennema et al., 1996; Heck et al., 2007; National Research Council, 2001). The findings indicated a significant association between teacher beliefs and student learning outcomes. Based on the HLM analysis, students in classrooms of teachers with a discover/connectionist-orientation to teaching mathematics are expected to gain 16.90 percent points more between the pre-test to the post-test than their peers who have transmission-oriented teachers. This finding is consistent with the findings of Fennema et al. (1996) in that students in classrooms whose teachers reported more student-centered beliefs to teaching mathematics had statistically significant higher gains on a problem solving assessment.

Further, the relationship between teacher practices and student achievement is statistically significant. Since teachers' practices are a ratio score, for an increase of 1% towards teacher-centered practices, students would be expected to score 0.64% less gain on the curriculum-based assessments. Whereas there is no clear conclusion between specific instructional practices and student achievement (USDE, 2008), some studies support the link between student-centered pedagogies and student learning outcomes (National Research Council, 2001, 2004). Although the 0.64% decrease for a one unit shift towards teacher-centered practices seems small, it potentially has substantial influence on student

learning.

Differences between Grade Levels and School Districts

Another finding from our study suggested grade-level differences in student gains in one school district but not in another school district. One plausible explanation is the teachers' enactment of the curricula. School District A began implementing the Investigations curriculum during the year of the study, while teachers in District B ranged from beginners to having two years of experience teaching with the curriculum. Curriculum implementation research found that teachers' instruction improved as they became more experienced with the curricular materials (Stein & Kim, 2008).

The differences between gain scores across grade levels were most likely influenced by two factors. First, the assessments varied based on content. As stated earlier, the second and fifth grade assessments covered skills that students had worked with in the previous grade. The concepts assessed in the other grade levels were new to students. Second, the assessments had a varying number of items on them, so when students' scores were converted to percentages an item in one grade was worth more percentage points than an item from a different grade.

Conclusions

This study examined multiple data sources from teachers and their students to examine the relationship between teachers' beliefs towards the teaching and learning of mathematics, their instructional practices, and their students' achievement on curriculum-based assessments. Significant associations were found between teachers' dispositions to mathematics teaching and learning and their enacted instructional practices. Further, students whose teachers had reported teacher-centered beliefs and teacher-centered practices had significantly lower gain scores on curriculum-based assessments.

The findings in this study shared warrant further research. Specifically, there is a need to further explicate how these findings should influence subsequent research studies about the interplay between teachers' mathematical beliefs, instructional practices and

student learning outcomes. When teacher-participants attend professional development, researchers should examine the impact of these learning opportunities on beliefs, enacted practices, student achievement, and the continued relationship of these data sources.

References

- Askew, M., Brown, M., Rhodes, V., Johnson, D., and William, D. (1997). *Effective teachers of numeracy: Final report*. London: King's College.
- Braswell, J., Daane, M., Grigg, W. (2003). *The Nation's Report Card: Mathematics Highlights 2003*. Washington, DC: National Center for Educational Statistics.
- Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, 97(1), 3–20.
- Carpenter, T. P., Fennema, E., Franke, M.L., Levi, L., & Empson, S. B. (2000). *Cognitively Guided Instruction: A Research-Based Professional Development Program for Elementary School Mathematics*. Report No. 003. Wisconsin Center for Educational Research: Madison, WI.
- Cohen, S. (2005). *Teachers' professional development and the elementary mathematics classroom: Bringing understandings to light*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*. Hillsdale, NJ: Lawrence Erlbaum Associates.
- Desimone, L. M., Porter, A. C., Garet, M. S., Yoon, K. S., & Birman, B. F. (2002). Effects of professional development on teachers' instruction: Results from a three-year longitudinal study. *Educational Evaluation and Policy Analysis*, 24(3), 81–112.
- Ernest, P. (1991). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics Teaching: The State of the Art* (pp. 249–54). London: Falmer.
- Fennema, L., Carpenter, T., Franke, M., Levi, M., Jacobs, V., & Empson, S. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, 27, 403–434.

- Garet, M., Porter, A., Desimone, L., Briman, B., & Yoon, K. (2001). What makes professional development effective? Analysis of a national sample of teachers. *American Educational Research Journal*, 38(4), 915–945.
- Gonzalez, P., Guzman, J. C., Partelow, L., Pahlke, E., Jocelyn, L., Kastberg, D., & Williams, T. (2004). *Highlights from the Trends in International Mathematics and Science Study (TIMSS) 2003*. Washington, DC: National Center for Educational Statistics.
- Heck, D. J., Banilower, E. R., Weiss, I. R., & Rosenberg, S. L. (2008). Studying the effects of professional development: The case of the NSF's local systemic change through teacher enhancement initiative. *Journal for Research in Mathematics Education*, 39, 113–152.
- Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28, 524–549.
- Hill, H.C., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42, 371–406.
- Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, 35, 81–116.
- Marshall, J. (2006). Math wars 2: It's the teaching, stupid! *Phi Delta Kappan*, 87(5), 356–363.
- McGee, J. R., Wang, C., & Polly, D. (2013). Guiding teachers in the use of a standards-based mathematics curriculum: Perceptions and subsequent instructional practices after an intensive professional development program. *School Science and Mathematics*, 113(1), 16–28.
- Mokros, J. (2003). Learning to reason numerically: The impact of Investigations. In S. Senk & D. Thompson (Eds.), *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* (pp. 109–131). Mahwah, NJ: Lawrence Erlbaum Associates.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Research Council. (2001). *Adding it up: Helping children learn mathematics*. In J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

- National Research Council (2004). On evaluating curricular effects: Judging the quality of K-12 mathematics evaluations. J. Confrey & V. Stohl (Eds.). Committee for a Review of the Evaluation Data on the Effectiveness of NSF-Supported and Commercially Generated Mathematics Curricular Materials. Mathematical Sciences Education Board, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.
- Peterson, P. L. (1990). Doing more of the same: Cathy Swift. *Educational Evaluation and Policy Analysis*, 12(3), 261–280.
- Phillipp, R. (2007). Mathematics teachers' beliefs and affect. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 257–315). Reston, VA: NCTM.
- Polly, D. (2006). Participants' focus in a learner-centered technology-rich mathematics professional development program. *The Mathematics Educator*, 16(1), 14–21.
- Polly, D. (2008). Modeling the influence of calculator use and teacher effects on first grade students' mathematics achievement. *Journal of Technology in Mathematics and Science Teaching*, 27(3), 245–263.
- Polly, D. & Hannafin, M. J. (2011). Examining how learner-centered professional development influences teachers' espoused and enacted practices. *Journal of Educational Research*, 104, 120–130.
- Raudenbush, S., & Bryk, A. (2002). *Hierarchical linear models: Applications and data analysis methods*. Thousand Oaks, CA: Sage.
- Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research*, 75(2), 211–246.
- Remillard, J. T. & Bryans, M. B. (2004). Teachers' orientations toward mathematics curriculum materials: Implications for teacher learning. *Journal of Research in Mathematics Education* 35, 352–388.
- Schifter, D. & Fosnot, C. (1993). *Reconstructing mathematics education: Stories of teachers meeting the challenge of reform*. New York: Teachers College, Columbia University.
- Schifter, D. & Simon, M. A. (1992). Assessing teachers' development of a constructivist view of mathematics learning. *Teaching and Teacher Education*, 8(2), 187–197.
- Stein, M. K. & Kim, G. (2008). The role of mathematics curriculum materials in large-scale urban reform: An analysis of demands and opportunities for teacher learning. In J. Remillard (Ed.), *Teachers' use of mathematics*

curriculum materials: Research perspectives on relationships between teachers and curriculum (pp. 37-55). New York: Routledge.

Stigler, J. & Hiebert, J. (1999). *The teaching gap*. New York: The Free Press.

Swan, M. (2007). The impact of task-based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education*, 10(4-6), 217–237.

Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education*, 75, 58–70.

Tarr, J. E., Reys, R., Reys, B., Chávez, Ó., Shih, J., & Osterlind, S. (2008). The impact of middle school mathematics curricula and the classroom learning environment on student achievement. *Journal for Research in Mathematics Education*, 39, 247–280.

TERC (2008). *Investigations in Number, Data, and Space* (2nd edition). New York: Pearson.

United States Department of Education (2008). *The final report of the National Mathematics Advisory Panel*. Washington, DC: Author.

Wenglinsky, H. (1999). Does it compute? The relationship between educational technology and student achievement in mathematics. *Educational Testing Service Policy Information Center*. Retrieved September 10, 2002, from www.mff.org/pubs/ME161.pdf.

Wu, M. (2009). A comparison of PISA and TIMSS 2003 achievement results in mathematics. *Prospects*, 39(1), 33–46.

Appendix A

Teacher Beliefs Questionnaire

Teacher name: _____ Grade(s) taught: _____

Indicate the degree to which you agree with each statement below by giving each statement a percentage so that the sum of the three percentages in each section is 100.

A. Mathematics is:

Percents

1. A given body of knowledge and standard procedures; a set of universal truths and rules which need to be conveyed to students: _____
2. A creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods: _____
3. An interconnected body of ideas which the teacher and the student create together through discussion: _____

B. Learning is:

Percents

1. An individual activity based on watching, listening and imitating until fluency is attained: _____
2. An individual activity based on practical exploration and reflection: _____
3. An interpersonal activity in which students are challenged and arrive at understanding through discussion: _____

C. Teaching is:

Percents

1. Structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to grasp what is taught: _____
2. Assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences: _____
3. A non-linear dialogue between teacher and students in which meanings and connections are explored verbally where misunderstandings are made explicit and worked on: _____

This questionnaire was adapted from Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education*, 75, 58-70. Permit for use was obtained on May 29, 2009.

Appendix B

Teacher Practices Questionnaire

Teacher name: _____

Grade(s) taught: _____

Indicate the frequency with which you utilize each of the following practices in your teaching by placing an 'X' in the appropriate column.

#	Practice	Almost Never	Some times	Half the time	Most of the time	Almost Always
1.	Students learn through doing exercises.					
2.	Students work on their own, consulting a neighbor from time to time.					
3.	Students use only the methods I teach them.					
4.	Students start with easy questions and work up to harder questions.					
*5.	Students choose which questions they tackle.					
*6.	I encourage students to work more slowly.					
*7.	Students compare different methods for doing questions.					
8.	I teach each topic from the beginning, assuming they don't have any prior knowledge of the topic.					
9.	I teach the whole class at once.					
10.	I try to cover everything in a topic.					
*11.	I draw links between topics and move back and forth between topics.					
*12.	I am surprised by the ideas that come up in a lesson.					
13.	I avoid students making mistakes by explaining things carefully first.					
14.	I tend to follow the textbook or worksheets closely.					
*15.	Students learn through discussing their ideas.					
*16.	Students work collaboratively in pairs or small groups.					
*17.	Students invent their own methods.					
18.	I tell students which questions to tackle.					
19.	I only go through one method for doing each question.					
*20.	I find out which parts students already understand and don't teach those parts.					
*21.	I teach each student differently according to individual needs.					
22.	I tend to teach each topic separately.					
23.	I know exactly which topics each lesson will contain.					
*24.	I encourage students to make and discuss mistakes.					
*25.	I jump between topics as the need arises.					

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*denotes reverse coded items that are more student-centered rather than teacher-centered.

Appendix C

Sample of Content Knowledge for Teaching Mathematics (CKT-M)

Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought. Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3