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Data-Driven Intervention: Correcting Mathematics Students' Misconceptions, not Mistakes

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In an age when reform is based on standards and instruction is based on research, this article gives practical advice for how mathematics teachers can analyze errors in student problems to create interventions that aid not only the individual's development, but the entire class's as well. By learning how to correct mathematics students' misconceptions, rather than their mistakes, teachers have the potential to both target more students and increase those students' conceptual understanding of the topic at hand. From the post-test scores on the Common / Habitual Algebra Student Misconceptions – Function Families (CHASM) , a tool used to assess teachers' function family content knowledge and pedagogical content knowledge that was given after a three-day, overnight professional development workshop, , teachers averaged a 43% improvement in their ability to identify the common misconceptions present in students' scenario test example problems and in creating suitable interventions for that misconception. This article (a) highlights the results found from ten Algebra teachers' use of this pedagogical skill after a three-day overnight workshop entitled Teaching Algebra Concepts through Technology (TACT^2), (b) identifies and categorizes misconceptions, and (c) provides pedagogical intervention support for correcting misconceptions rather than errors. As one teacher commented, "A minor (pedagogical) tweak resulted in a major revelation."

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Background

In an age when reforms are standards-based and instruction is research-based, both professional education organizations such as the National Council of teachers of Mathematics (NCTM) and the National Education Association (NEA) and mathematics educators agree that a critical and beneficial outcome of classroom assessments is the utilization of gleaned information to improve classroom instruction (Ball, Sleep, Boerst, & Bass, 2009; Brown-Chidsey & Steege, 2005; Black & Wiliam, 1998; Edwards, 2004; Gersten, et.al, 2009; Johnson, Mellard, Fuchs, & McKnight, 2006; NCTM, 2007; NEA, 2005; Porter, 2002; Schoenfield, 2002; Stigler. & Hiebert, 1997). Since the No Child Left Behind Act of 2001 and the Department of Education's push to provide professional development opportunities for teachers that targeted the utilization of student data in classroom instruction (Cavanagh, 2004; Wright, 2010), teachers have been given the responsibility of analyzing both standardized summative and formative assessments for areas of weakness, and to adjust their instruction accordingly.

Despite the pressure, few teachers have been equipped to analyze assessment results and adapt their instruction. A recent experience of Mike Klavon, a mathematics consultant for the Ottawa Area Intermediate School District in Michigan, highlighted this knowledge gap, "I have not yet been able to find a data report that clearly informs a teacher how to modify instruction that can support targeted student needs." (Delta Math Meeting, April, 14, 2010). Mike discovered that teachers are presented with a myriad of assessments from state and national sources, such as the Michigan Educational Assessment Program/Michigan Merit Examination (MEAP), Michigan Merit exam (MME), Iowa test of basic skills, and Northwest Evaluation Association exam (NWEA). However, the formats of these data reports leave the teachers stymied; Most summative types of assessments or yearly data analyses report scores as some type of a Raush Unit scale score (usually referred to as a RIT score), identifying the range within which each child falls in each particular content area. For instance, a RIT score of 211-220 on the NWEA for the Numbers and

Operations represents the student's ability to do approximately 113 different learning objectives ranging from "Meaning, notation, place value and comparisons" to "Estimation." A child is assigned a RIT score for each mathematical strand assessed in the standardized test.

However, the data analysis from other tests differ. In the case of MEAP, scores are reported only by grade level and come in two general reports: (a) the percent of students that scored a 1, 2, 3 or 4 (1 and 2 meets adequate yearly progress) and (b) an item analysis identifying the percent of students in each grade level that answered each question correctly. MME offers similar building level reports but does not include an item analysis. Instead, it provides a percent break down by strand. In the final analysis of these assessments, teachers do not have the information to develop interventions effectively because standardized assessment topics are so broad and interventions so wide for each individual student that many teachers are baffled as to how they should effectively use the data (Brown-Chidley & Steel, 2005; Gersten, et al, 2009).

Recent research suggests that teachers are able to recognize general trends and topical weak spots, but lack the ability to translate this information into a suitable intervention that not only benefits individual student, but also allows the entire class to profit from instruction (Kadel, 2010; Ysseldyke, Burns, Scholin, & Parker, 2010). Teachers are given the data but are not given the proper training necessary to dissect and tailor this data into a viable way that effects their instruction (McMillan, 2004). This is true for both the standardized, large-scale assessments and the informal formative assessments given in the classroom.

After teachers give formative, classroom assessments (collecting data), they generally identify problems that were missed by the majority of students (analyzing data) and then do nothing more than just re-work these problems during the next class session (intervention). Commonly, teachers have not been shown how to detect the underlying misconception(s) in the error(s) (Ashlock, 2010; Franco, 2008). An error refers to a mistake made by a student, which can occur for a myriad of reasons ranging from a data entry / calculation error to a lack of conceptual understanding. When a student does not fully grasp a concept, they create a framework for the concept that is not accurate. Then, they answer problems through this framework (Bamberger, Oberdorf, Schultz-Ferrell & Leinwand, 2011; Davis & Keller, 2008; Hiebert & Carpenter, 1992). A misconception is a part of a student's framework that is not mathematically accurate which leads to him or her providing incorrect answers. As mentioned above, teachers are usually not adequately shown how to properly identify these misconceptions. Thus, the data received from individual students' work remains individualized and can be cumbersome when helping the class as a whole. The resulting interventions and instructions can not only be time-consuming, but also generally ineffective for long-term retention.

A meta-analysis of 30 studies discussing the effects of formative assessments on student improvement emphasized the importance of providing teachers with a set of skills that allow them to successfully link data to *suitable* interventions in order to improve mathematics instruction (Hattie, 2009). Responsiveness to intervention (RtI) models have begun to address the issue of providing teachers with the skills and information necessary for them to develop effective interventions. One of the eight suggestions from leading mathematics researchers in the Institute of Education Sciences report, Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools (2009), is to provide teachers with specific data that intentionally focus on grade level objectives; however, they conclude that "at the current time, this research is 'underdeveloped'" (p. 13).

The authors of a U.S. Department of Education, Office of Planning, Evaluation and Policy Development (2011) report entitled *Teachers' Ability to Use Data to Inform Instruction: Challenges and Supports* wrote that "Given the importance that federal education policy places on teachers' data literacy and ability to draw instructional implications from data, additional research and development in this area are warranted" (p. xii). Thus, although teachers have become adept to collecting and, to an extent, analyzing data, they are not analyzing this information for effective interventions that promote student success (Diamond & Cooper, 2007); Hamilton, Halverson, Jackson, Mandinach, Supovitz, & Wayman, 2009; Holmes, Finn, Blower & Chavarria, 2013; Popham, 2001). Hence, professional development programs are needed to provide the set of skills necessary for teachers to take raw data in varying forms and evaluate it in a manner suitable for correctly recognizing patterns with student difficulties and then appropriately developing the interventions that address them (Gold, 2005).

Assessing Misconception – Instruments and Participants

Instrument

The Common / Habitual Algebra Student Misconceptions–Family Functions (CHASM) tool is one instrument that assesses teachers' function family content and pedagogical content knowledge. Content knowledge refers to algebra function families of varying types from linear to exponential. Pedagogical content knowledge includes the *depth* at which teachers correct student misconceptions in those areas. CHASM's depth levels are defined in accordance with Webb's (2005) depth of knowledge (DOK) levels. A low level depth of pedagogical content knowledge (DOK1) equates to teachers who correct student errors, rather than misconceptions, and provide interventions that are algorithmic in nature. The second depth of pedagogical content knowledge (DOK2) is shown when a teacher corrects a student misconception in an algorithmic manner. The highest depth of knowledge level (DOK3) is displayed when teachers identify and target the misconception conceptually.

The CHASM assessment tool is a derivative of the University of Louisville's Diagnostic Teacher Assessment for Mathematics and Science, the only quantitative assessment for secondary teachers' pedagogical content knowledge (Bush, 2009; Holmes, 2012). In contrast, Deborah Loewenberg Ball's Learning Mathematics for Teaching Project assessment is geared towards elementary and middle school teachers (Hill et al, 2008; Hill, Dean, & Schilling, 2007; Loewenberg-Ball, Bass, Hill, 2011). Content validity was confirmed by a team of 7 experts, made of up mathematicians, mathematics educators (including researchers), and algebra teachers; these experts established the algebra content misconceptions and alignment to the common core standards. (Wilson et al., 2007). Two statistical methods were used to confirm assessment reliability: Cronbach's alpha for internal consistency (.85) and intra-class reliability coefficients for scorer reliability (.97).

Participants

In this workshop, ten certified secondary school mathematics teachers from a school district in Michigan voluntarily participated in a three-day, overnight workshop entitled Teaching Algebra Concepts through Technology (TACT^2). These teachers stated they had numerous reasons for volunteering, including: (a) learning *practical* tools and examples to help excite their students, (b) learning about an user friendly inquiry-based program that could enhance increase learning and cut down on their work load and (c) obtaining some free interactive software and student accounts. What became apparent through discussions during the workshop was that these teachers wanted help in reaching a population of students whom had become more disengaged and who were lacking in requisite skill sets.

Training on conceptual understanding of pedagogical concepts, how to help students understand more conceptually, and pedagogical "best practices" were delivered primarily through HeyMath! which is an interactive online E-learning program from Singapore (Sankaran & Rajan, 2013). These teachers took the CHASM pretest prior to the professional development and a posttest immediately following. It is these results that are highlighted herein.

Correcting Misconceptions, not Mistakes

Many studies have been undertaken in order to investigate the types of misconceptions that students encounter in Algebra (An, 2004; Clement, Narode & Rosnick, 1981; Fuchs & Menil, 2009 Jurkovic, 2001) and on the importance of reviewing student errors for increased student conceptual understanding (Durkin 2009; Grobe & Renkle, 2007; NCTM, 2000, 2007).

However, few studies focus on the *skills* necessary to identify and correct these misconceptions. Even fewer explain how to practically apply the skills to data-driven instruction. The emphasis has been on student conceptual development during instruction. However, while conducting a three-day, overnight workshop on function families, we found that by equipping teachers with the skill to diagnose student misconceptions on exams, it aided in the correcting of student work for future instruction. Misconceptions are derived from problems due to conceptual misunderstandings. Mistakes are derived from computational or minor mishaps. Errors are either; teachers must discern if errors are misconceptions or mistakes A minor tweak in the way teachers looked at student work produced a major revelation in how they approached student "mistakes."

During the workshop, these ten teachers wrote lesson plans based upon the diagnosed misconceptions. First, however, they had to become proficient at diagnosing misconceptions. In order to do this, the teachers were given a series of sample student problems that contained common Function Family misconceptions. These misconceptions were those that (a) research showed students habitually made when working with linear and quadratic equations, or when interpreting exponential and polynomial graphs and (b) aligned with the National Common Core Standards. Teachers were told to identify the error and explain how they would correct the misconception. The errors were either computational or conceptual in nature. Computational errors, such as accidently stating $3x^2 * 2 = 6x$ or $4 * 5x = 21x$ instead of $6x^2$ and $20x$, respectively, require teachers to direct students' attention to their mistake. However, conceptual errors, such as consistently adding instead of multiplying (e.g., stating $3x * 2 = 5x$ or $4 * 5x = 9x$) require teachers to determine the underlying cause or misconception involved and address the misconception when communicating with students. For conceptual errors, teachers direct – through their written or verbal comments -- students' attention to correcting the misconception.

In order to correctly and efficiently correct misconceptions, however, teachers have to discover the underlying pattern (systematic errors) that unites every mistake in a number of different student-scenario examples. The test problem below is a sample problem worked during the workshop (Figure 1); in it, the students in the problem gave incorrect responses to “write a possible equation for the polynomial shown below.” Although each made only a few errors, the underlying misconception was their inability to realize that an even degree of the leading term produces a graph with the same end behavior in both directions.

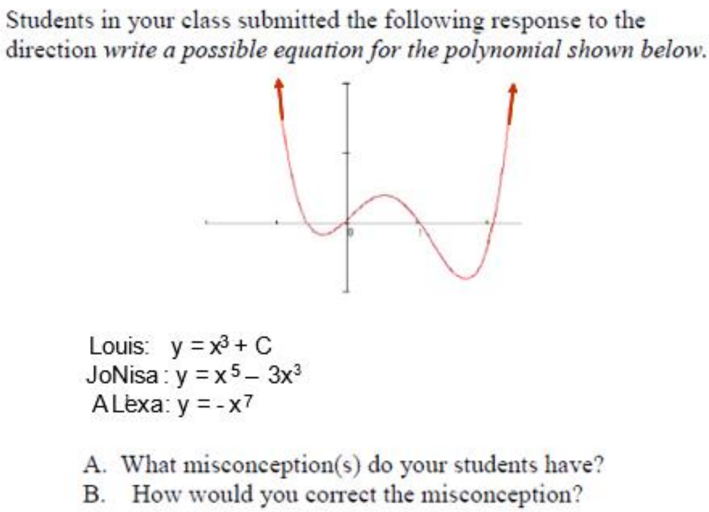

Figure 1. Chasm 4.3, Item 10

At first glance, a teacher may think that the students did not understand leading terms. However, interpreting the data shows that this isn’t the case. In each student’s response, the degree of the leading term was bigger than the preceding terms: x^3 is greater than a constant; x^5 is greater than x^3, x^7 is greater than nothing. Hence, as a teacher of these students, reviewing leading terms would be an ineffective intervention or pedagogical strategy. Although JoNisa’s response seems as if she doesn’t understand characteristics of even and odd functions, her mistake is actually in reversing these characteristics; according to CHASM notes, she simply attributed the end behaviors of even functions to an odd function. An intervention that would broadly sweep the classroom and possibly help as many students as possible would be to target the characteristics of even / odd exponents --

staying away from symmetry. The underlying conceptual problem is relatively straight forward; these students simply reversed the concepts but were correct otherwise in their interpretation of the polynomial.

A secondary misconception lies in understanding the concept of a polynomial. Because a polynomial is written as the sum of terms of varying degree, a polynomial with mixed parentage would have both even and odd degree terms included. Although Louis understood that the graph was represented algebraically by more than one term neither he nor A'Lexa understood the role of odd and even exponents. However, we should caution that by doing so, we are looking at these student errors in a vacuum. It may be that Louisa and A'Lexa have different misconceptions, such as mistakenly writing an equation to identify the leading term of the polynomial. Identifying student misconceptions from their work is a matter of identifying and categorizing *patterns*. Thus, it is extremely important when developing assessments to have multiple questions targeting the same concept in order to better classify misconceptions. However, if the class as a whole is systematically making the same mistake, this constitutes a class misconception and should be addressed accordingly. It is important to note that as a teacher learns to identify major misconceptions and not each individual student error, he/she will need to make judgment calls as to what gets addressed in the class setting and what becomes an individual student intervention. In the example above, addressing A'Lexa's negative coefficient – computational error – individually intervening would be appropriate.

Research supports the pedagogy of identifying misconceptions rather than student mistakes. "Borasi argued that(conceptual) errors could be used to foster a more complete understanding of mathematical content, as a means to encourage critical thinking about mathematical concepts, as springboards for problem solving, and as a way to motivate reflection and inquiry about the nature of mathematics" (Tirosh & Tsamir, 2005, p. 30). Unfortunately, many teachers still correct mistakes rather than misconceptions. Figure 2 below from the morning of Day 2 of the workshop is an example of this. Notice how not detecting the underlying misconception can cause teachers to be scattered, inconsistent, or erratic when implementing a reasonable intervention. Each student would need to be re-taught the entire lesson.

Scenario:	Teacher responses
Students in your class submitted the following responses to the question find the equation to describe the following situation: Ted's comic book collection, which was worth \$1300 five years ago, has been increasing in value by 12% per year since then. What is the current value for the collection? i. 1300(1.12)(5) ii. $1300(.12)^5$ A. What misconception(s) do your students have? B. How would you correct the misconception?	A) The student forgot the 1 before the decimal percent. Although he multiplied by 5 to express the repeated number A) Students are substituting values into the equation incorrectly $A=P(1+r)^n$ A) Students do not understand exponential growth A) I know these aren't right, but I'm not sure how… A) Students don't understand exponents A) Students don't understand percentage interest. A) They don't know how to use the formula correctly

Figure 2. Algebra teacher responses: correcting mistakes rather than misconceptions.

Minor tweak resulted in a major revelation

The good news is that in this workshop, the change from correcting student mistakes to correcting student misconceptions was a minor tweak. Once pointed out, teachers quickly become adept at discovering underlying misconceptions in student work and creating effective interventions / instructional lesson plans. From the post-test scores on the Common / Habitual Algebra Student Misconceptions– Family Functions (CHASM), teachers significantly ($p < .01$) increased from 188 to 268, a percent of increase of (43%), in their ability to identify the common misconception in several examples of students' function family problems and the ability to create suitable, conceptual-based interventions. As a group, the ten teachers improved on 75% of the test (12 of the 16) items. In only three days, all of the teachers increased their ability to identify misconceptions from 36% to 61% ($p < .01$). Once misconception were identified, their ability to correct misconceptions conceptually rose from 47% to 63% ($p < .01$). It is evident that more attention is needed in focusing on providing teachers with effective interventions; nevertheless, the skill was easily adopted in a short amount of time. As one teacher commented, “A minor tweak resulted in a major revelation.”

Pedagogical Interventions – Correcting Misconceptions

As teachers correct their students’ papers, they need to think about what errors are present. Are they conceptual, thereby misconceptions? Or are they computational and thereby mistakes? Teachers should ask the following: Why are they occurring? Is the problem a vocabulary/definition problem? Could it be a calculation/computational error? Or is it a true conceptual problem? In creating the most effective interventions, a teacher should make certain that s/he is covering errors that the majority of students are making – discerning the underlying misconception of those errors, and addressing them accordingly. The next section defines and identifies three different types of misconceptions and how to match appropriate responses to interventions with each different type.

Matching Interventions to Types of Errors

After analyzing over 300 responses from teachers in the workshop correcting student errors, we found that causes of student errors could be broken into three distinct categories: vocabulary misconceptions, computational errors, and erroneous belief misconceptions. A vocabulary misconception centers on mistaken terminology or language; a computational error centers around calculation mistakes; and erroneous belief misconceptions are inaccuracies in mathematical thinking. Below are examples of each type of error/misconception and appropriate teacher interventions for each category are exhibited. The suggested interventions come primarily from two sources, a compilation of teacher tested interventions from the last 4 years of the TACT[2] workshop and misconception intervention best practices (Ashlock, 2010; Franko, 2008).

Vocabulary misconceptions

A vocabulary / definition error would be addressed by focusing on the part of the language that is misunderstood, not by teaching an entire lesson on the concept under investigation. For instance, if a teacher asks students to identify the zeros in the functions below (Figure 3), what could they conclude?

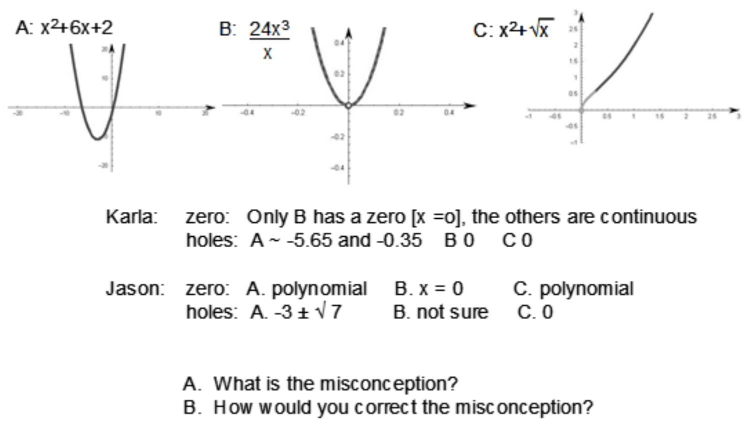

Figure 3. CHASM 4.3, Item 13

In interpreting the above data, one would be correct in concluding that Karla and Jason are dealing with a vocabulary problem of defining ‘zeros’ vs. ‘holes.’ There is no need to design an elaborate intervention or revisit the entire concept of ‘zeros’ again. This is an example of a minor intervention, and a common student vocabulary error. As before, we note that Jason is uncertain about how to explain a ‘zero’ in a ‘hole’ environment. Depending upon if other students are similarly stymied, this conceptual error would be addressed individually. In the workshop, a teacher commented that a similar situation happened in her class. However, she went the route of completely teaching the ideas behind roots and zeros again. As she did, she had students working in pairs, and overheard one student tell his partner that he wished she’d stop repeating the same topics when he had just confused the definition. When the partner agreed, it made her think that a complete reintroduction to the concepts was unnecessary. During the workshop, as we discussed this particular item and other similar ones such as percent of increase /decrease and interpreting slope, it became clear that vocabulary misconceptions was a category by itself.

Computation errors

Computational errors are calculation errors. Though seemingly simple to identify and address, they are not because a teacher has to decide if they are actual calculational inaccuracies or signs of an erroneous belief. In the workshop, teachers consistently got these problems wrong, finding them difficult to identify. The discussions in the workshop helped to the participating teachers highlight computational errors in the given problems. Additionally, the teachers described actual examples from their own classrooms which added even more to the discussions. These instances that participants drew from real teaching practice helped to solidify the veracity of the concept of computational errors being a major classification of student error. Figure 4 is an example of a true computational error.

Students in your class submitted the following solve the equation $5x^2 - 6x + 3 = 0$.

Jared: $a = 5, b = -6, c = 3$

$$\frac{-6 \pm \sqrt{36 - 4 * 5 * 3}}{10} = \frac{-6 \pm \sqrt{-4}}{10}$$

Chelsea: $a = 5, b = 6, C = 3$

$$\frac{6 \pm \sqrt{36 - 4 * 5 * 3}}{10} = \frac{6 \pm \sqrt{-24}}{10}$$

A. What is the misconception?
B. How would you correct the misconception?

Figure 4. CHASM 4.3, Item 1.

In this case, Jared and Chelsea have only miscalculated. There is no true misconception in evidence. Thus, no intervention is necessary unless one discovers that their students constantly repeat the same computational error. Remember, in identifying misconceptions, teachers need to think about systematic errors or patterns.

Erroneous beliefs

Still, the computational errors must be scrutinized in case they are erroneous beliefs (conceptual errors) in masquerade. For instance, if students did not realize that there was no real number solution to the square root of a negative number, and presented the answers above as real solutions, then a different instructional intervention is needed. Erroneous beliefs are the crux behind common student misconceptions. This was born out in the workshop. From examining student work and the workshop teacher's responses, it was discovered that there is usually a common theme that runs throughout students' errors. Figure 5 is an example of the erroneous belief misconception.

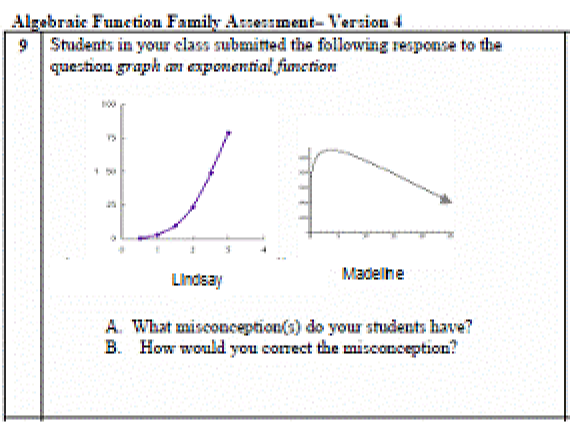

Figure 5. Chasm 4.2, Item 9.

It would appear that students did not understand asymptotic limits of exponential functions and that this is the key conceptual error. Endpoints, while still an issue, are not a major priority. The idea of 'changing direction' also is not a critical matter. From past workshops, we've discovered that teacher notes on individual student papers are able to address these secondary concerns appropriately. To find the key misconceptions, we suggest that teachers look at all of their students' work on a single concept, and then determine what errors they have in common.

A good way to discover what students may be thinking when examining their answer for a particular problem is simply to ask them. Research suggest having secondary students explain the "why" of a problem increases not only their own understanding, but the resulting data increases teachers' effectiveness in plugging gaps in student misconceptions (Garofalo & Lester, 1985; Martinez 2006; Van der Stel & Veenman, 2010). Thus, including the "What was your rationale?" or "Explain why you responded as you did" to classroom problems might be a tremendous benefit for data driven instruction. These answers would provide insight into students' thought process, thus helping to pinpoint underlying misconceptions. This next section offers suggestions for collecting data that supports finding student misconceptions.

Uncovering Misconceptions

From both the workshop and best practices, useful tips for diagnosing student misconceptions follow. Teachers can make assessments (whether a graphic organizer, class worksheet, or quiz/test) thematic. All problems that address one idea should be clumped together; questions that address another topic are similarly grouped. For example, suppose a class was working on function family graphs. The assignments would be created around the unit / lesson standards or the "take away" topics taught in that unit / lesson. A "take away" topic is the one or two concepts that students need to understand at the end of the lesson. A teacher should ask: What do students *really* need to "take away" from this lesson? Once the lesson has been distilled down, what are the "take-away" points? Whether a teachers creates their assignment around the standards or "take away" points, our research suggests that they should group all items in the assignment or of an individual section (multiple choice, open ended, word problems) around that topic.

Figure 6 below is an example of a delineated topical two part assignment. Items are centered on specific "take away" topics. Notice that the metacognitive "Why" questions are asked in this illustration.

Quadratics – Quiz (multiple choice or open ended / word problems)

Part I: Identifying quadratic expressions
parabola form
three term expression

Part II: Solve by
factoring
finding roots
Quadratic equation
Completing the square
discriminate to understand roots

Part I: Example Questions
Which of the following are parabolas? Why?
a) $y = x^2$
b) $y = 2x^2 - 4x - 5$
c) $y = -\frac{3}{4}x^2 - x + 6$

Write the coefficient of the second term in each of the above functions

Part II:
Jared was asked to solve the equation $3y^2 - 9y = 0$
(a) Can you spot the mistake in his work? (b) Why would factoring have helped him?

$$3y^2 = 9y$$
$$3y = 9$$
$$y = 3 \mid \text{the solution is } 3.$$

A) Solve each of the equations below by any method you choose (factoring, completing the square, quadratic equation. B) Explain why you chose a particular method.

$9x^2 + 1 = 6x$ $\quad 5S^2 - 19S + 10 = 0$ $\quad x^2 - 6x + 6 = 0$
$w^2 - 3w = 18$ $\quad 20T = 13 - 2T2$ $\quad 2z^2 + 7z - 4 = 0$

Figure 6. Example from HeyMath! of a typical question with metacognitive "Why?" questions. Adapted from **http://www.heymath.com/heymath/**

Conclusions

This study identified and categorized misconceptions and provided pedagogical intervention support with examples for correcting misconceptions rather than errors, in an attempt to provide teachers with the knowledge necessary for implementation. As a response to intervention and a data-analysis pedagogical tool, we believe it will be invaluable in the classroom for increasing student understanding of mathematical problems. However, further study is needed to examine the longitudinal effects of this pedagogical practice on student academic success. The intent of this article is to enlighten educators for action. The contents herein can be used as a springboard for professional development, workshops, and pre-service teacher education on the how to of correcting misconceptions rather than mistakes in student work. Detecting patterns of misconceptions is not always an intuitive process, but it can be acquired through ideas described above in a short amount of time. Teachers were able to discern between mistakes and misconceptions with up to 43% improvement at the end of just one three-day workshop. Our hope is that other teachers will benefit from using this pedagogical approach in their classrooms. All in all, identifying and correcting misconceptions, not mistakes, is a skill well worth developing for teachers.

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