The Mathematics Educator
2013 Vol. 23, No. 1, 45–59

Elementary Teachers' Perspectives of Mathematics Problem Solving Strategies

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Participants in this study were asked to report what strategies were most often used in their attempts to foster their students' problem solving abilities. Participants included 70 second through fifth-grade elementary teachers from 42 schools in a large state of the south central region in the US. Data analyses of the interviews revealed that none of the educators utilized all problem solving strategies recommended by the National Council of Teachers of Mathematics. Further examination of the interviews indicated that having their students draw a picture and identify key information was the teachers' most taught word problem strategy. Here, we provide insight of the reported teachers' strategies and the need for more explicit instruction of problem solving strategies when educating mathematics teachers.

Researchers in mathematics education have argued that teaching the problem solving process is essential for student understanding of mathematics (e.g., National Council of Teachers of Mathematics, 2000; Common Core State Standards , 2010; National Research Council , 2001). Specific problem solving strategies recommended in National Council of Teachers of Mathematics' (NCTM) *Principles and Standards of School Mathematics* (2000) include using diagrams, looking for patterns , trying special values or cases, and require instructional attention. NCTM also suggested that teachers give students opportunities for the application of problem solving

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strategies across all mathematical content areas.

The Common Core State Standards Initiatives' (CCSSI) Preparing America's Students for College and Career (2010) include eight standards for mathematical practice. The first of these standards addresses problem solving by requesting that students should: "make sense of problems and persevere in solving them" (p.6). In the report by the National Research Council (NRC) *Adding it Up: Helping Children Learn Mathematics* (2001), researchers describe mathematical proficiency as interwoven strands involving both procedural and conceptual components. NCTM's *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (2006), reinforces that students should have a better understanding of procedural and conceptual knowledge when taught in a classroom context. This understanding "promotes problem solving, reasoning, communication making connections, and designing and analyzing representations" (p.15). Problem solving continues to be a key theme that enables students to learn the content in the context of a focused and cohesive curriculum that implements problem solving reason, and critical thinking (NCTM, 2006).

Researchers have investigated the relationship between solving word problems and student success in mathematics. Jitendra, Sczesniak, and Deatline-Buchman (2005) examined the validity of curriculum-based word problem solving measures as indicators of 77 third grade students' mathematical problem and computational abilities. The findings indicated that solving word problems was a useful indicator of third graders' mathematical proficiency. Griffin and Jiterndra's (2009) findings suggest "that high-quality word problem-solving instruction may be an effective instructional option in heterogeneous elementary classrooms to improve students' understanding of mathematics word problem solving and their computation accuracy" (p.199). Griffin and Jitendra argued that the process of problem solving is much more complex than solving an equation with numbers pulled from a word problem, concluding teachers should carefully design problem solving instruction.

Researchers have shown students have difficulty solving word problems (Bailey, 2002; Hart, 1996; Parker & Lepper,

1992). Reasons offered for this difficulty include: limited student experience with word problems (Bailey, 2002), lack of motivation to solve the word problems (Hart, 1996), and irrelevance of word problems to students' lives (Parker & Lepper, 1992). Parker and Lepper state "the need for techniques to enhance student interest in traditional educational materials may actually increase with age" (p.632). Fairbairn (1993) suggested that the terms *story problems* and *word problem*s can invoke uncomfortable memories for many people.

Historically, formal instruction in mathematics includes algorithms (i.e., a set of procedures to follow to solve a mathematical problem). Algorithms treat problem solving as unitary, deductive in nature, and systematic. Heuristic methods, which refer to experience-based techniques for problem solving, learning, and discovery, are used to speed up the process of finding a satisfactory solution by using strategies that are readily accessible. In his seminal book, *How to Solve It,* Polya (1945; 2009) treats the process of mathematics as experimental using four phases: (a) understand the problem; (b) obtain a plan of the solution; (c) carry out the plan; and (d) examine the solution obtained for reasonableness. Polya suggested there are many ways to solve problems and students should learn how to choose appropriate strategies, such as working backward, using a formula, and looking for a pattern. Sherman, Richardson, and Yard (2009) agreed saying, "The more heuristics with which students are familiar, the greater the likelihood of finding a solution" (p.208).

Hembree's (1992) meta-analysis of results included 487 reports on characteristics of problem solvers, conditions for harder and easier problems, and effects of different instructional methods on problem solving performance. Hembree found that heuristic methods were typically taught as a total process of problem solving saying "explicit training was found essential; without direct intervention and oversight, practice in using these sub skills did not result in better performance" (p.263). At all grade levels, extraneous data tended to make problems more difficult for students. Knowing how to eliminate extra information produced a large effect; more modest mean effects were found for guess-and-test and

composing original problems. Fuchs et al. (2003) argued that explicit instruction in self-regulated learning strategies is necessary to enhance mathematical problem solving performance in low, average, and high achieving third graders.

Hembree (1992) reported students who checked their work, used diagrams to represent problems, and used heuristics, had higher problem solving performance. When students were asked to adhere to step-by-step procedures, they performed lower than students using flexible approaches. No benefits were found to result from rereading or restating the problem or trying to find solutions purely by trial and error. Hembree showed "the only clear impact on student performance resulted from teachers especially trained in heuristics" (p.266).

Having students generate and develop their own mathematical problems that differ from particular situations or on the basis of their prior problem solving experiences may be an important activity that helps students develop as inquiry-based problem solvers (Cifarelli & Sheets, 2009). Problem posing has been reported in studies by English (1997) and Silver & Cai (1996). English designed a comprehensive framework for developing young children's mathematical problem posing and Silver and Cai (1996) conducted studies of problem posing that included activities of middle grade students.

The purpose of this study is to provide insight into the problem solving strategies teachers' reported using to teach solving of word problems with the aim to draw attention to instructional problem solving strategies. This research has the potential to help bring awareness to mathematics teacher educators and others in the field who provide instruction of problem solving strategies. Both in-service and pre-service teachers need to be able to teach the problem solving strategies recommended by the NCTM (2000) in order for their students to improve their problem solving skills.

Method

This research study involved a reexamination by the author of data from a previous study (Pearce, Bruun, Skinner, & Lopez-Mohler, 2013). The research was completed at a state

university where in-service and pre-service teachers attend classes, workshops, and meetings. The overarching research question that guided this re-examination was to identify what specific problem solving strategies do teachers report teaching to their students for word problems.

Participants

The participants in this study were 70 elementary teachers who taught mathematics in grades 2-5. The teachers represented 42 separate campuses which included 16 different public school districts, 2 charter schools, and 1 private school. The study was limited to second- through fifth-grade levels because students in kindergarten and first-grade were not as likely to be given written word problems to solve and students above the fifth-grade were more likely to be in middle schools with departmentalized mathematics classes. Displayed in Table 1 are the education levels of the teachers, years of teaching experience, and grade levels.

Table 1

Participant Demographical Data

Demographics	N	%
Level of Education		
Bachelors	43	62%
Masters	23	33%
Post graduate	1	1%
Doctorate	1	1%
Not reported	2	3%
Total Years of Teaching Experience		
1-3	13	19%
4-6	16	23%
7-9	10	14%
10-12	8	11%
13-15	5	7%
15+	18	26%
Grade Level		
Second	13	19%
Third	21	30%
Fourth	19	27%
Fifth	15	21%
Multi-grade	2	3%

The researchers recruited participants through three procedures: (1) those taking graduate course work at the university, (2) a general call to former graduate students, and (3) a general call to public school teachers. Individual teacher interviews were of those who taught mathematics in grades 2-5.

Procedure

The researchers gathered data through interviews that included the same open ended questions. In total, two university professors and two doctoral students individually interviewed 70 elementary teachers. Because the author arranged for current graduate students to be interviewed, the author did not conduct any of the interviews so as not to influence the responses. The interview guide included demographic questions of the teachers, and twelve open-ended questions. One of the open-ended questions, *what specific strategies do you teach the students to use to solve word problem?* was used as the basis for this article.

Results

The author and two researchers independently conducted a content analysis of the 70 teacher interview transcriptions to determine commonalities and trends. The analysis included pre-set or a priori categories to determine the presence of specific information, as well as specific categories taught to students for problem solving. The pre-set categories included nine strategies recommended by the NCTM in *Principals and Standards of School Mathematics* (2000) as shown in Table 2. These nine include the recommended seven from the state standards where the research was conducted. The seven strategies from the state standards include: (1) drawing a picture, (2) looking for a pattern, (3) systematic guessing and checking, (4) acting it out, (5) making a table, (6) working a simpler problem, or (7) working backwards to solve a problem (Texas Essential Knowledge and Skills, 2006). The author re-examined the interviews to determine the extent to which the

teachers' responses related to the nine pre-set NCTM categories.

Table 2

NCTM Strategies Taught to Students for Problem Solving

Strategy	2nd	3rd	4th	5th	Multi	Total	%
Draw a picture	6	11	12	6	1	36	40%
Choose an operation	3	7	6	2	0	18	20%
Make a table or graph	1	4	6	3	1	15	17%
Act it out	1	6	1	0	0	8	9%
Work backwards	0	2	1	2	0	5	6%
Guess, test, revise	1	0	3	0	0	4	4%
Work a simpler problem	0	0	0	2	0	2	2%
Make an organized list	0	0	1	0	0	1	1%
Find a pattern	0	0	0	0	1	1	1%

The findings from the analysis of teachers' responses to the aforementioned question that relate to the pre-set NCTM categories can be found in Table 2. Listed in Table 3 are the other categories mentioned by teachers from the analysis of teachers' responses to the same question. Table 3 strategies were not among those recommended by the NCTM (2000) but cited in the research literature (Barlow & Cates, 2007, Cifarelli & Sheets, 2009, Hart, 1996, Hembree, 1992, Jonasssen, 2003). Teachers' responses according to grade level or other (multi-grade), and the total number of responses that fit into the category are given. The number exceeds the participant total since teachers explained using more than one strategy.

Discussion

The data on problem solving strategies reported by the teachers revealed that none of the 70 teachers interviewed reported teaching all the problem solving strategies expected to be taught at their respective grade level. Two of the teachers taught four NCTM strategies, 50 of the teachers taught at least

one NCTM strategy, and 18 reported teaching no NCTM strategy. One teacher taught seven strategies from Table 2 and Table 3

Specific Strategies Taught to Students for Problem Solving

Strategy	2nd	3rd	4th	5th	Multi	Total	%
Identifying key information	5	6	8	11	1	31	36%
Clue words	3	6	6	3	0	18	21%
Eliminate extra information	2	5	2	3	1	13	15%
Reread	4	4	1	0	0	9	10%
Writing own problems	0	0	4	1	0	5	6%
Understand, Plan, Solve, and Evaluate (UPSE)	2	1	2	0	0	5	6%
Reword	1	1	0	1	0	3	3%
Replace names	1	0	1	0	1	3	3%

3, and nine teachers reported teaching one strategy. Although the author expected to find some individual variance in teacher problem solving strategy knowledge and application among those interviewed, a wider range of application of strategies was anticipated because the NCTM's *Principles and Standards of School Mathematics* (NCTM, 2000) stressed the need for students to practice a wide range of strategies.

Drawing a picture is the most common NCTM strategy alluded to by the teachers in the study (Table 2). In Hembree's (1992) meta-analysis of problem solving studies, drawing a diagram related to better performance on solving word problems, as well as training for skill in representation provided the largest performance improvement. Students who used diagrams often and moved easily back and forth between words and mathematical concepts were better at problem solving. The IES Practice Guide on problem solving by Woodward et al (2012) makes the following recommendation: "Teach students how to use visual representations" (p. 23). Drawing a picture is a NCTM recommended strategy and should continue to be stressed by teachers. Students should learn that there are many ways to solve problems and be able to utilize more than one strategy so that they can experiment in

order to choose appropriate strategies. The following discussion includes some of the other strategies named by participants.

Identifying key information vs. clue words

In Table 3, the most popular strategy reported by teachers was to identify key information in the text, by circling, underlining, or highlighting this information. A third-grade teacher said, "the main strategies are actual techniques that come from reading comprehension, such as to underline, circle things in the word problems, find the question, and underline it." A fourth-grade teacher taught students to, "underline, circle, somehow highlight; bring them to not lose focus to the question so that they don't lose track as to the purpose as they are trying to sort through the information." A fifth-grade teacher warned, "I like for students to circle the most important words. It's a flag for me if I see a student highlighting the entire word problem and I know I have to retrain how they look for what is important in the word problem."

Despite the caution of many researchers in using clue words as a problem solving strategy, it was the second most popular strategy used by teachers in this study. Highlighting key information and eliminating extra information as strategies is in contrast to the method of teaching clue words in light of Jonassen's (2003) research on problem solving strategies. He found that the "search for key words" strategy (e.g. context clues) was common in classrooms where students had little problem solving success, and teachers taught and re-taught students struggling with mathematics this strategy year after year in spite of their lack of success. Another third grade teacher said, "We have been teaching them to read for key words without really reading the problem and they just assume that if it says, 'how many more' it's going to be subtraction."

Cathy Seeley, former president of NCTM, gave the following example of the problem of teaching "clue words" where she highlights the strategy of elementary students who look for the words "in all" or "all together". Students believe these words to mean that they should add up the numbers in the

problem, but the words might be used differently in a problem such as this:

> A truck just delivered enough desks to be placed **in all** the classrooms in the new school building. They unloaded 80 palettes of 6 desks each. There are 16 classrooms in the building. How many desks will be placed in each classroom? There is nothing wrong with students knowing that the words "in all," when used to describe a collection of things, often means to put groups of objects together. But this is based on the meaning of the words and the meaning of the operations, and it is very different from asking them to memorize tricks like "when you see the words 'in all,' you should add" (Seeley, 2006, para. 15-16).

Van de Walle (2010) cautions to avoid relying on clue words such as "altogether" meaning to add, "left" and "fewer" to subtract, and "each" to multiply. Three arguments against teaching clue words are: (a) clue words can be misleading (as seen in the example above); (b) many problems have no clue words; and (c) clue words encourages looking for an easy way of solving the problem. (p.163)

Hembree's meta-analysis reported that using key words produced a mean with borderline significance. A second-grade teacher described the strategies she used, "We have little chants where if they see specific words like 'sum' then they know to add." A fourth-grade teacher warned, "We always circle key words and I try to make a really big deal about the words that lead you to understand what the problem is. For instance total means addition, and difference is used for subtraction. We talk about that the key words can mean more than one thing and can be distracters."

Polya's Problem Solving Method

Teachers reported they had used the procedure UPSE, which is an acronym for Understand, Plan, Solve, and Evaluate, derived from Polya's (1945; 2009) four-step model described above. This method required the students think about what they are looking for, what they have to work with, the operation needed to solve the problem, and making sure the answer makes sense. Polya's model was given as a process

standard for problem solving in , the state in which the research was conducted and is a general instructional heuristic that is typically found in mathematics textbooks and includes multiple representational and solution strategies (e.g., use manipulatives, use a table, act it out, draw a diagram, choose an operation and write a number sentence). Several of the local school districts in this study used a purchased curriculum and a fourth-grade teacher reported, "We had a program we used for math for four years where the trainer used the model where you have four quadrants: UPSE Understand, Plan, Solve, and Evaluate." The method was mentioned by five of the seventy teachers (less than 10% of the total); and even so, one participant was not sure of the steps. A second grade teacher said, "The book has them read the problem. There are some steps they need to follow. I don't really remember the steps but they are in the book."

Writing as a Strategy

The findings suggest that the majority of elementary teachers are not utilizing practices that have been researched and found to have positive effects, such as students' writing their own problem (Barlow & Cates, 2007; Cifarelli & Sheets, 2009). Only five of the seventy teachers stated that they had the students write their own problems and no second or third grade teachers reported using this strategy. One fourth-grade teacher said, "I allow the children to write their own word problems. I think it's important for developing their critical thinking skills." Another said, "I have them create their own problem which I think is important because they can understand it more once their create their own and then we even have each other solve them." A fifth-grade teacher who used this strategy said, "They create word problems together in groups; they talk about how to solve them, and present them in class. That is where sharing the strategies come into play."

Limitations of the Study

One limitation was in data gathering. This was completed through interviews and not direct observation. Although

interviews and self-reported data are not always an accurate reflection of classroom practices, the authors' experiences suggest that the data gathered were representative of classroom instructional practices. The second limitation was that the teachers interviewed were volunteers, the majority of whom (63%) had been recruited through graduate courses. However, the 70 teachers interviewed came from 42 separate campuses and represented four different grade levels and the responses of those teachers recruited from graduate classes were not noticeably different from those not in graduate classes.

Implications for Further Research

The present work generated a number of new research questions that may be addressed through further investigations. One such project would be to study teachers to see how much instructional time for word problem solving instruction is distributed across the week or during the school year. Furthermore, examining the differential effects of instruction on students performing at various achievement levels (e.g., low, average, and high) or the effects of students of different grade levels (e.g., fourth-to eighth-grade students) may help to determine whether different strategies increase problem solving success.

Implications of the findings also provide insight into the lack of explicit strategy use and the need for direct instruction during teacher education for in-service and pre-service teachers. Hembree's meta-analysis of problem solving studies showed a positive impact on students' performance was the result of teachers especially trained in heuristic methods (1992). In response to a question, how would you rate your success in teaching students to solve word problems, one teacher summed this up by saying, "I would feel a lot better if I had a better method to teach them." The overwhelming majority of teachers interviewed for this study expressed an interest in learning additional information on how to teach their students to solve word problems.

Conclusion

The researcher's goal was to provide insight into the problem solving strategies teachers' reported using to teach solving of word problems with the aim to draw attention to instructional problem solving strategies. Drawing a picture is the most common NCTM (2000) strategy alluded to by the teachers in the study. Data analyses of the interviews revealed that none of the educators utilized teaching all nine problem solving strategies recommended by the NCTM.

The second most popular strategy reported by teachers was to identify key information in the text, by circling, underlining, or highlighting information. This is not one the NCTM strategies but it is similar to a reading strategy to help students to understand text. Identifying key information is in contrast to teaching students to look for clue words, which was the third most used strategy that teachers reported. Merely looking for clue words without understanding is not successful according to the research (Jonassen, 2003).

This research has the potential to help bring awareness to mathematics teacher educators and others in the field providing instruction of problem solving strategies. Students taught to apply a heuristic will have a higher problem solving performance than those taught there is only one way to solve a problem. The findings from this study suggest that both in-service and pre-service teachers should teach the problem solving strategies recommended by the NCTM in order for their students to improve their problem solving skills and thus need to be provided with the skills in how do to so.

References

English, L. D. (1997). Development of fifth grade children's problem posing abilities. *Educational Studies in Mathematics, 34*, 183–217.

Fairbairn, D. M. (1993). Creating story problems. *Arithmetic Teacher, 41*(3), 1401–42.

Fuchs, L. S., Fuchs, D. Prentice, K., Burch, M., Hamlett, C. L., Owen, R., … Schroeter, K. (2003). Enhancing third-grade students' mathematical

problem solving with self-regulated learning strategies. *Journal of Educational Psychology, 95*, 306–315.

Griffin, C. C., & Jitendra, A. K. (2009). Word problem-solving instruction in inclusive third grade mathematics classrooms. *Journal of Educational Research, 102*, 187–202.

Hart, J. (1996). Effects of personalized word problems. *Teaching Children Mathematics,*

Hembree, R. (1992). Experiments and relational studies in problem solving: A meta-analysis. *Journal for Research in Mathematics Education, 23*, 242–273.

Jitendra, A. K., Sczesniak, E., & Deatline-Buchman, A. (2005). An exploratory validation of curriculum-based mathematical word problem-solving tasks as indicators of mathematics proficiency for third graders. *School Psychology Review, 34*, 358–371.

Jonassen, D. H. (2003). Designing research-based instruction for story problems. *Educational Psychology Review, 15*, 267–296.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics.* Reston, VA.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), A report from the NRC. Washington, DC: National Academies Press.

Parker, L. E., & Lepper, M. R. (1992). Effects of fantasy contexts on children's learning and motivation: Making learning more fun. *Journal of Personality and Social Psychology, 62*, 625–633.

Pearce, D., Bruun, F., Skinner, K., & Lopez-Mohler, C. (2013). What Do Teachers Say? Student Difficulties Solving Mathematical Word Problems in Grades 2-5. *International Electronic Journal of Mathematics Education, 8*(1), 3-19. Retrieved from http://www.iejme.com/

Polya, G. (1945, 2009). *How to solve it.* Princeton, NJ: Princeton University Press.

Seeley, C. (2006). *President's Corner: Teaching to the Test*. National Council of Teachers of Mathematics. Retrieved from http://www.nctm.org/about/content.aspx?id=842

Sherman, H. J., Richardson, L. I., & Yard, G. J. (2009). *Teaching Learners Who Struggle with Mathematics*, (2nd ed.). Upper Saddle River, NJ: Pearson Education, Inc.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*, 521–539.

Texas Education Agency. (2006). *Texas Essential Knowledge and Skills for Mathematics.* Retrieved from http://ritter.tea.state.tx.us/rules/tac/chapter111/ch111a.html.

Van de Walle, J. A. (2010). *Elementary and middle school mathematics*, (7th ed.). Boston: Allyn and Bacon.

Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., …& Ogbuehi, P. (2012). *Improving mathematical problem solving in grades 4 through 8: A practice guide.* Retrieved from http://ies.ed.gov/ncee/wwc/pdf/practice_guides/mps_pg_052212.pdf