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Preparing Secondary Mathematics Teachers: A Focus on Modeling in Algebra

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This study addressed the opportunities to learn (OTL) modeling in algebra provided to secondary mathematics pre-service teachers (PSTs). To investigate these OTL, we interviewed five instructors of required mathematics and mathematics education courses that had the potential to include opportunities for PSTs to learn algebra at three universities. We also interviewed a group of three to four PSTs at each of the universities. We coded the interview transcripts using an analytic framework developed based on related literature and policy documents. We report the similarities and differences in perspectives among instructors and PSTs related to modeling at each university, along with comparisons of OTL across universities.

Algebra has long been considered a foundation for advanced mathematics and a gatekeeper for high school students to enter a college or university for an advanced degree (e.g., Kilpatrick & Izsák, 2008). Usiskin (1987) proposed that every student should have the chance to learn about algebra, even before high school; and students' learning of algebra has been described as a "civil right" (e.g., Moses & Cobb, 2001). Policy recommendations aimed at improving K-12 mathematics education, including algebra, are regularly revised to reflect current research about mathematics learning and

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teaching. For example, the National Council of Teachers of Mathematics (NCTM) first released *Curriculum and Evaluation Standards* in 1989, and later, the *Principles and Standards for School Mathematics* in 2000, which contain standards for mathematics teaching including how to teach algebra in ways that are more engaging and relevant to students. For example, NCTM promotes opportunities for students at all levels to model real-world phenomena and to make connections between algebra and other mathematical subjects. Through modeling, students can learn algebra in real-world contexts and make connections between algebra and other subjects, such as geometry and statistics. Modeling, therefore, can be a tool for students to effectively learn algebra.

Recently, the *Common Core State Standards for Mathematics (CCSSM),* adopted by 43 states, has provided guidance for K-12 mathematical content, emphasizing modeling in the standards for mathematical practice (e.g., Model with mathematics) (National Governor's Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010). The process of modeling described in *CCSSM* includes formulating a model by selecting variables and representations, reflecting on the results to improve the model, evaluating the model, and reporting on the conclusions. In addition to these processes of modeling, several purposes of modeling have been described in other policy documents (e.g., NCTM, 2009; Conference Board of the Mathematical Sciences [CBMS], 2012). For example, students can solve physical and social problems, and gain new knowledge by engaging with mathematical modeling (NCTM, 2009). Students can also make mathematical connections as they solve real-life problems (CBMS, 2012) and use multiple representations when describing the behavior of a system (NGA & CCSSO, 2010).

The new expectations from *CCSSM* related to modeling raise questions about whether teachers are being prepared to teach modeling in middle and high school mathematics classrooms. At the same time, a recent review of reports on teacher preparation by the National Research Council (NRC) suggested that there is a lack of quantitative and qualitative data about mathematics teacher education programs, and recommended systematic research about the content of mathematics teacher education (NRC, 2010). Thus, it is important to describe the experiences and opportunities of secondary mathematics pre-service teachers (PSTs) in their teacher education programs. To this end, we report preliminary findings from a study focused on modeling, conducted as part of a larger project, *Preparing to Teach Algebra (PTA)*, which investigated PSTs' opportunities to learn (OTL) about *(1) algebra, (2) algebra teaching, (3) issues in achieving equity in algebra learning, and (4) the algebra, functions, and modeling standards and mathematical practices described in CCSSM*. In this paper, we report on the findings specifically related to opportunities that secondary mathematics teacher education programs provide to learn about modeling in algebra. Although not all participants reported opportunities that were algebra-specific or a comprehensive list of opportunities to learn about modeling, the results of this work can provide information about what is being done in specific programs and could highlight notable opportunities to learn algebra through modeling that could be useful to other teacher education programs. Specifically, our research question for this study is "What are the opportunities to learn modeling in algebra as described by instructors and pre-service secondary mathematics teachers at three universities?"

Review of Related Literature

In this section, we first describe literature related to OTL, followed by a summary of research related to the education of mathematics teachers. Finally, we provide evidence of the need for this study by describing existing studies that address the purposes and processes of modeling in mathematics education.

Opportunity to Learn (OTL)

OTL emerged from international comparative studies when researchers at the International Association for the Evaluation of Educational Achievement sought to ensure valid comparisons of mathematics achievement among countries (McDonnell, 1995). OTL was described as a measure of the opportunities that students had to learn about a particular concept (Burnstein, 1993; McDonnell, 1995); it was initially used as a methodological tool for reliability comparisons. Later, OTL was further conceptualized to inform policy related to equal opportunity because it was correlated with student achievement (Floden, 2002). Especially in relation to access to mathematics for all students, OTL was considered a factor of multiple elements including course taking, teacher quality, and high-stakes assessments (Anderson & Tate, 2008). With OTL, researchers and policy makers can contextualize and give more nuanced explanations of student achievement data (Tornroos, 2005).

The OTL construct was expanded by other researchers, to include time engaged in academic tasks and activities (e.g., Tate 1995 described OTL as the amount of time spent on working toward a particular learning goal), as well as detailed accounts of content coverage. For example, the Beginning Teacher Evaluation Study utilized teacher logs and classroom observations to investigate aspects of students' opportunities to learn particular mathematical concepts (Fisher et al., 1980). In the Second International Mathematics Study, researchers utilized a questionnaire completed by teachers to measure students' OTL. However, the lack of validation about what was enacted in classrooms was proposed as a shortcoming of this approach. In addition, the nature of the teachers' responses could have also been a result of varying conceptions of OTL (Floden, 2002; Mayer, 1999). The Trends in International Mathematics and Science Study (TIMSS) improved on this approach by refining OTL surveys and conducting video-taped observations. However, the video-taping approach had limitations of feasibility and cost.

Tornroos (2005), building on the three-level curriculum framework used in the TIMSS studies, comprised of *intended* (consisting of written learning goals or standards set by stakeholders), *implemented* (application of intended curriculum), and *attained* curriculum (results achieved in assessment measures); also proposed a *potentially implemented* curriculum, which encompassed course materials as they are used in the classroom, as a potential measure of middle-grades students' OTL. Tornroos (2005) analyzed textbooks and survey data which captured content coverage reported by teachers as measures of OTL. He found that an item-based analysis of topics emphasized in class texts proved to be an effective measure of OTL. There was also a positive correlation between students' achievement on the TIMSS assessment and the emphasis of related topics in the texts. All of these studies have focused on investigating the correlation between students' OTL and their achievement on national or international assessments.

Schmidt, Cogan and Houang (2011) proposed that OTL is not only important to understand how and what students learn, but it also offers insight into the learning opportunities available to PSTs who will eventually teach. Other studies have sought to investigate correlations between PSTs' OTL and their achievement on comparative assessments. For example, the Mathematics Teaching in the 21st Century study (Schmidt et al., 2007), which investigated PSTs' opportunities across six countries, used institutional, faculty, and PST surveys, as well as document analyses of course offerings. Researchers found that PSTs' OTL gave insight into both their content knowledge and their knowledge of students' learning (Schmidt et al., 2008). In related work, findings from the Teacher Education and Development Study in Mathematics (TEDS-M) also highlighted the connection between content experienced in courses and PSTs' achievement on content knowledge and pedagogical content knowledge assessments. Researchers in this study described OTL using data collected from surveys completed by PST participants (Schmidt et al., 2011). This indicates that participants' reported perspectives can be appropriate measures of OTL.

For the purposes of our study, we used OTL to describe experiences and corresponding courses in teacher education programs in preparation to teach algebra. These experiences included but were not limited to class activities and tasks, field experiences, assignments, formative and summative assessments, and instructional practices. Tate (1995) in his critique of existing OTL frameworks noted the limitations of these frameworks as tools for examining the OTL of African American students in urban schooling contexts. He noted that these frameworks should include components beyond content toward the fiscal, cultural, and pedagogical contexts where students learn mathematics. From a sociocultural perspective, Gee (2008) also noted that OTL should not be limited to specific content coverage, but also encompass opportunities for students to participate in meaningful activities related to learning goals. Although our study does not focus on the cultural and fiscal contexts of the teacher education programs, we report here findings related to content, pedagogical activities, and experiences of potential OTL from the perspectives of PSTs and instructors of required courses.

To be specific, we investigated PSTs' opportunities to learn about modeling in algebra using instructor interviews and PSTs' focus group interviews. These perspectives will provide information about the OTL that students have in their teacher education programs. We do not intend to make claims about the quality of teacher education programs, what PSTs learned, their effectiveness in teaching algebra, or the impact on their students' achievement. Floden's (2002) characterization of OTL as "a basis for considering current practice and possible alternatives" (p. 49) aligns with our project goal. We specifically focus on documenting PSTs' opportunities to learn about modeling in algebra and learning to teach algebraic modeling. Such content knowledge and pedagogical knowledge have been highlighted as critical components for PSTs as described in the following section.

Mathematics Teacher Education

Educational researchers, policy makers, and professional organizations have long recommended that beginning teachers need both content knowledge and pedagogical knowledge (Ball & Cohen, 1999; NRC, 2010; Shulman, 1986). Shulman (1986) described the close connection between these types of knowledge; when defining pedagogical content knowledge, he stated that "pedagogical knowledge, which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching" (p. 9). Unfortunately, recent surveys show that many teachers may be unprepared to teach mathematics because of their lack of content knowledge; for instance, among those who taught Algebra I in 2007, only 44% held a degree in mathematics (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). Teachers also need pedagogical content knowledge, such as the ability to recognize students' common errors and address their challenges (Knuth, 2000). In order to facilitate this learning, the National Council on Teacher Quality recommended changes for teacher education programs, including the facilitation of the connection between mathematics courses, methods courses, and fieldwork that PSTs are required to take in their programs (Greenberg & Walsh, 2008). In a study of pre-service elementary teachers, Mewborn (1990) found that beginning mathematics teachers do not necessarily apply knowledge gained from methods courses to their classroom teaching. In order to connect PSTs' course work with their teaching, they may need more practical knowledge, such as modifying instruction based on evidence of student learning (Hiebert, Morris, Berk, & Jansen, 2007).

In addition, more systematic studies of mathematics teacher education are needed, especially those that look across programs (Adler, Ball, Krainer, Lin, & Novotna, 2005), as well as those that examine how teacher education programs impact the practices of future and practicing teachers based on policy recommendations (Confrey & Krupa, 2010; Sztajn, Marrongelle, & Smith, 2011). Furthermore, research that investigates the influence of new policy standards, including *CCSSM*, on teacher education programs is needed. For example, Garfunkel et al. (2011) stated the need for studies related to the types of OTL aligned with these policy standards that are provided to future teachers in their programs. Because of the attention to modeling in NCTM (2000) and *CCSSM*, we were interested in investigating the OTL related to modeling in these programs.

In terms of research in algebra teaching, Kieran (2007) mentioned the need to go beyond studying about algebra teachers, to studying about teaching algebra and its connection to students' learning. Some studies have explored teachers' use of algebraic tasks and their beliefs about students' learning of algebra. For instance, Lobato, Ellis, and Muñoz (2003) described how the instructional activities used by a teacher affect the ways in which students generalize algebraic concepts. They illustrated that a teacher's use of function tables might lead to students' misconceptions of slope when the teacher simply increases 'x' values by 1 in each pair to show linear relationships. In another study examining teachers' beliefs about student learning, Nathan and Koedinger (2000) reported that teachers overestimated students' ability to solve equations without context, whereas they underestimated students' ability to solve contextualized problems. The authors highlighted the discrepancy between teachers' content knowledge of algebra and their knowledge of assessing students' competencies in solving problems in context. Our study examined opportunities to learn both content and pedagogical knowledge related to algebraic modeling in teacher education programs. In the next section, we describe problem solving in context, including the importance of mathematical modeling for helping students learn mathematics.

Purposes of Modeling

Mathematical modeling can be described as interpreting a real-world context mathematically, solving the problem using mathematical representations, predicting the real-world, and verifying the conclusion (e.g., Gravemeijer, 2004; Lesh & Doerr, 2003; NGA & CCSSO, 2010). Several benefits can be gained from teaching with a modeling perspective. First, connecting mathematics with real-world situations helps students approach physical and social problems mathematically (NCTM, 2009). For example, students can apply proportional reasoning to compare the value of several products with different prices per unit of weight to buy a product when they have limited funds. Modeling also helps students use appropriate mathematics to understand contexts better and to improve their decision making in everyday life (NGA & CCSSO, 2010). In addition, mathematical modeling allows students to make connections between different mathematical topics since realistic problems are often complex and require an integration of mathematical skills (CBMS, 2012; NGA & CCSSO, 2010). Furthermore, when students use mathematical concepts in novel ways to solve problems in context, they can develop the ability to effectively use knowledge in new contexts (NCTM, 2009). The ability to integrate knowledge and apply it to new situations has also been highlighted as an increasing need in the workforce (Partnership for 21st Century Skills, 2007). Finally, students can understand and describe the behavior of a system through modeling. Models can shed light on a natural system or event when students use multiple representations. For example, students can investigate the behavior of rapid bacterial growth by modeling it with an exponential function (NGA & CCSSO, 2010). With these purposes of modeling in mind, it is crucial for teachers to understand what processes students experience when they engage with mathematical modeling. We will discuss the processes of modeling described by researchers and a policy document (i.e., *CCSSM*) in the following section.

Processes of Modeling

Lesh and Doerr (2003) described four processes of modeling (see Figure 1). In the first process, *description,* students connect real world with the model world. The second process, *manipulation,* requires students to identify the ways in which they generate actions to address the problem, such as using representations (e.g., table, diagrams) to manipulate the model. In the third step, *prediction,* students are expected to connect appropriate results to the real world by making predictions in the problem's context based on the mathematical model context. The final step, *verification*, requires students to reflect on the usefulness of manipulation and predictions. Lesh and Doerr (2003) emphasized the need for multiple cycles to refine the model as students reflect and find alternative ways of reasoning.

CCSSM outlines six processes involved in modeling in school mathematics. In the first process, students identify and select variables that represent important features in a real world problem. Second, students formulate a model by creating representations (e.g., graph, table) that explain relationships between the variables. Next, students make computations to draw conclusions using the relationships. Then, they interpret and validate the conclusions based on the original context, and improve the model if necessary. Last, students should report on the results and the justifications behind them. Additionally, *CCSSM* suggests that many of these steps are iterative, as indicated in Figure 2.

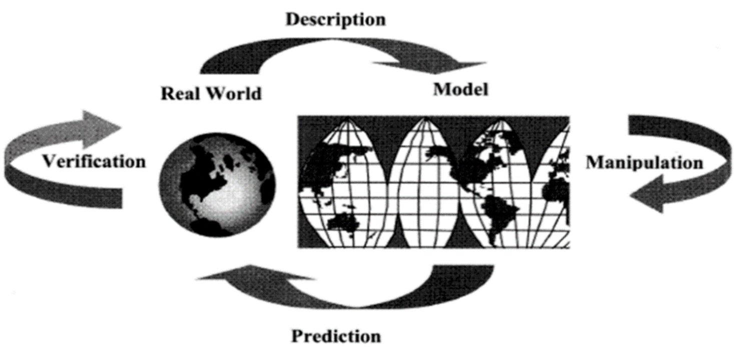

Figure 1. Processes of Modeling (Lesh & Doerr, 2003, p.17).

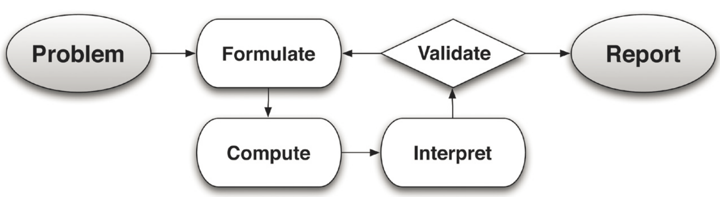

Figure 2. Processes of modeling (NGA & CCSSO, 2010, p. 72).

The processes of modeling described by Lesh and Doerr (2003) are related to the processes of modeling described in *CCSSM*. Similarities between the two sets of processes include the need for interpreting the problem, using representations to describe and solve the problem, and reflecting on the processes. Both sources addressed the need for multiple cycles of the processes even though the former requires them while the latter recommends them if they are applicable. We utilized the processes of modeling described in *CCSSM* because one of the purposes for the larger study was to consider to what extent teacher education programs prepare PSTs to work in the context of the current standards.

Methods

Setting and Participants

This study was conducted as part of the *PTA* project which investigated *opportunities that secondary mathematics PSTs have to learn about algebra; algebra teaching; issues in achieving equity in algebra learning; and the algebra, functions, and modeling standards and mathematical practices described in CCSSM*. Pilot data was collected at three universities which we will reference as Universities A, B, and C. University A is a Master's degree-granting institution in which mathematics educators and mathematicians are on the faculty in the Mathematics Department; it offers a four-year secondary teacher education program. University B is a Ph.D.-granting institution with a five-year undergraduate program in which PSTs complete a full-year student teaching internship during the final year; mathematicians and mathematics educators are housed in separate departments in different colleges. University C is also a Ph.D.-granting institution with mathematics instructors and mathematics education instructors in separate departments in different colleges; unlike University B, however, University C has a four-year secondary teacher education program. Universities A and C certify PSTs to teach in grades 5-12 whereas University B certifies PSTs to teach students in grades 7-12. Table 1 shows the number of required courses and credits for mathematics and mathematics education courses that PSTs take to be eligible for initial certification in each program.

Table 1
Number of Required Courses and Credits for Mathematics and Mathematics Education Courses

	University A	University B	University C
Mathematics Courses (Credits)	13 (45)	12 (36)	13 (46)
Mathematics Education Courses (Credits)	5 (15)	4 (17)	3 (17)

At each site, project researchers interviewed five or six instructors of a combination of required mathematics and mathematics education courses and a group of PSTs who were in the final year of their secondary teacher education program. Because we only interviewed a subset of the faculty teaching required courses and a small number of PSTs, we do not intend to make conclusive claims about the teacher education programs at these universities; rather, we explore PSTs' unique experiences related to modeling in several mathematics and methods courses at each university. Table 2 includes the names of the courses taught by the instructors who participated in the study.

Table 2
Mathematics and Mathematics Education Courses

University	Mathematics Courses	Mathematics Education Courses
A	*Structure of Algebra*	*Secondary Math Methods*
	Linear Algebra	*Middle School Math Methods*
	Secondary Math from an Advanced Perspective	
B	*Real Analysis*	*Secondary Math Methods I*
	Mathematics Capstone for Secondary Teachers	*Secondary Math Methods III*
		Secondary Math Methods IV
	Linear Algebra	
C	*Abstract Algebra*	*Secondary Math Methods*
	Differential Equation	*Middle School Math Methods*
		Student Teaching Seminar

Note. Course titles have been standardized to protect institutional anonymity.

Data Sources and Procedures

Semi-structured interviews were conducted with 16 course instructors (five or six per university) to gain their perspectives on the opportunities that PSTs had to learn about modeling and to learn about how to teach modeling in their course. One focus group interview with three or four PSTs was also conducted at each university (three PSTs from University B, four PSTs from Universities A and C). The focus group interviews were conducted in order to capture the perceptions of PSTs related to their opportunities to learn about and teaching modeling during their secondary mathematics education program. A focus group methodology was utilized due to its potential to encourage group interactions and, therefore, inspire participants' thoughts and gain diverse perspectives among the participants in an informal setting (Creswell, 2009). Whenever possible, the focus group questions were designed to be similar to and have the same sequence as the questions asked in the instructor interviews.

During the interviews, the course instructors and PSTs were given a brief description of the modeling strand in *CCSSM* in order to help them become familiar with new terms and *CCSSM* in case they were not familiar with the new standards related to modeling. While we reminded participants to focus on algebra, some opportunities discussed were not algebra-specific and it is unlikely that participants provided a comprehensive list of opportunities to learn about modeling. We have chosen to use our participants' words in reporting whenever possible to reduce the impact of an additional layer of interpretation.

Course materials were also collected to provide additional evidence of PSTs' opportunities to engage with modeling in their programs. The course materials provided to the research team varied greatly; examples included: course syllabi, assignments, tests, exams, quizzes, homework assignments, daily lesson plans, textbook references, and/or reading lists. These documents were reviewed prior to the interviews to enable the researchers to ask related follow-up questions, but were not used in data analysis. For example, if an instructor included a modeling activity in his/her course, we asked for more details about the activity, including whether or not it was algebra-related. The next section describes how these data sources were analyzed by the research team.

Data Analysis

The *PTA* research group, (i.e., three principle investigators, eight graduate research assistants, and three undergraduate assistants), developed analytic frameworks based on literature and professional recommendations related to specific areas of importance in algebra teaching and learning (e.g., contexts and modeling, reasoning and proof). In our initial efforts, we created a framework that outlined significant themes from literature for learning about algebra, teaching algebra, and recommendations from *CCSSM*. The section of this framework related to modeling (i.e., purposes and processes of modeling) was used as the analytic framework for this paper.

During preliminary data analysis, we refined and narrowed the initial framework related to the purposes of modeling to the following three purposes: (1) *use mathematical modeling in real world contexts*; (2) *use problem-solving approaches to investigate and understand mathematics*; and (3) *understand and describe the behavior of a system or event*. These three purposes were selected as the final categories for coding, because they emerged during the instructor interviews and focus group interviews. The processes of modeling came directly from the modeling strand in *CCSSM* (NGA & CCSSO, 2010). The processes of modeling are presented in Figure 2.

After deciding the categories for coding (e.g., three purposes and six processes of modeling), two graduate research assistants individually coded the 15 instructor and 3 focus group interview transcripts and documented direct quotes related to each category. We considered opportunities to learn about modeling as a learning activity or an assignment related to learning modeling or learning how to teach modeling. Based on this approach, we coded any excerpt from the transcripts which included a participant's description of a problem or task related to modeling.

We then organized the quotes from the transcripts into analytic documents identified for instructors, PSTs, and universities. Then, we compared individual codes and came to consensus for each OTL in order to establish reliability. After each of the two graduate researchers wrote a brief summary about what they noticed from their coding, they discussed individual quotes and summaries to settle discrepancies in categorizing relevant excerpts from the transcripts. Finally, we integrated the summaries to create the narrative of results and discussion below. This process of developing an analytic framework, coding direct quotes based on our research questions, and building consensus established reliability and validity of the case study (Yin, 2003).

Findings

In this section we describe each of the purposes and processes of modeling using participants' voices. We focus on the perspectives of PSTs and instructors' opportunities to learn about contexts and modeling at each university. We also compare OTL across universities.

OTL from the Perspectives of Instructors and PSTs

There are similarities and differences between the perspectives of the instructors and PSTs in terms of opportunities to learn. Table 3 shows the extent to which each group of participants provided examples of opportunities that PSTs have to engage with the purposes and processes of modeling. The numbers in the table represent participant responses (e.g., relevant activities, assignments, tasks, and discussions) related to each aspect of OTL about modeling. We found that 14 of the 16 instructors interviewed (approximately 93%) provided some responses about algebraic modeling in their course.

Table 3
Frequency of Modeling OTL Mentioned by Participants

Purpose of modeling	**PST**	**MI**	**MEI**
Use mathematical modeling in real-world contexts	5	6	7
Use problem-solving approaches to investigate and understand math	2	0	3
Understand and describe the behavior of a system or event	0	0	1
Total	7	6	11
Process of modeling (from *CCSSM*)			
Identifying and selecting variables	0	3	4
Formulating a model by creating and selecting appropriate representations	2	3	5
Analyzing and performing operations to draw conclusions	0	2	1
Interpreting the results of the mathematics	0	1	3
Validating the conclusions, possibly improving the model	0	1	2
Reporting on the conclusions and the reasoning behind them	0	0	1
Total	2	10	15

Note. MI refers to Mathematics Instructors and MEI refers to Mathematics Education Instructors.

The results indicated that the set of mathematics education instructors addressed all areas of the framework at least once, while PSTs addressed the fewest purposes and processes. In addition, the most frequently addressed category among all participants was the purpose of modeling, *using mathematical modeling in real-world contexts*. This suggests that participants view using mathematical modeling in real-world context as an important purpose of modeling.

The first two purposes of modeling were most frequently addressed by both instructors and PSTs. For instance, PSTs and an instructor from University C highlighted opportunities to learn about modeling in a Differential Equations course. The instructor stated, “In Differential Equations they get to see these methods working together to solve problems that they can see that are related to real world problems.” A PST who took this course at University C remembered these experiences, “My mind comes into Diff EQ, like my professor… tons of modeling kind of, we did like tank, two tanks in lottery.” Other focus group participants also remembered this OTL.

Modeling was often described by instructors and PSTs as solving problems in realistic contexts. Comparing the concentration of orange juice mixtures using proportional reasoning and rolling dice to solve probability problems were mentioned by PSTs as examples of opportunities to learn about modeling. Several participants also emphasized that an important aspect of formulating a model is creating and selecting appropriate representations. For example, several instructors and PSTs mentioned drawing a picture, using manipulatives, or making a chart or table.

In contrast, there were opportunities that PSTs mentioned but instructors did not address; this difference may be attributed to PSTs’ opportunity to look across courses. For example, using technology for modeling was mentioned by two PSTs, but was not included by any of the interviewed instructors. One PST from University A said, “I'm trying to think of math software. We did stuff with fish modeling [and] population [modeling]. When we did modeling like when you have a loan paying off your loan and things like that. Doing that in Excel.”

The connections between algebraic concepts and geometric modeling were mentioned by PSTs, but not instructors. For instance, when PSTs were asked to share examples of their experiences regarding learning about modeling, a PST from University B remarked,

> I don't think you can extricate geometry from algebra because when we're doing geometry we have things like distance formula. Algebra –you need it for things – any kind of distance in the shapes. You can stick on a coordinate plan. I mean we're always doing algebra.

Likewise, there were ideas that instructors mentioned that were not explicitly addressed by PSTs. Table 1 indicates that instructors' responses focused more on the processes of modeling than on the purposes of modeling, while PSTs' responses were more related to the purposes of modeling. Even though both groups of participants had a chance to look at the processes of modeling described in *CCSSM* during the interview, PSTs did not report their opportunities to learn about the processes of modeling except for *Formulating a model by creating and selecting appropriate representations*. Instructors, on the other hand, described specific examples of the other modeling processes. For example, several instructors discussed identifying and selecting variables, one of the modeling processes described in *CCSSM*. A Seminar instructor from University C stated that "word problem….once students write you know x+y =7 you know to try and ask them what the 'x' stands you know what does the 'x' mean? What does the 'y' mean? What does 'x+y' mean?" Here the instructor emphasizes the importance of students appropriately selecting and understanding the meaning of particular variables in the context of word problems. An Abstract Algebra instructor from University C mentioned that his students had opportunities to analyze and perform operations to draw conclusions: "They get a lot of chances, to carry out certain operations of the model execute plans, apply symmetries to this model and then draw conclusions from that." A Linear Algebra instructor from University A also emphasized interpreting the results of the mathematics, the fourth process mentioned in *CCSSM*: "Once you solve your system of linear equations, [you] have to really interpret, what this solution really means in terms of the real world problem."

Both instructors and PSTs' perspectives on OTL were more about learning about modeling than learning how to teach modeling. For instance one PST from University A talked about his/her opportunities to use modeling in calculus, "Well in Calculus maybe when we did like the trough problems and like the volume problems;" and another PST from University C mentioned how modeling came up in the discrete mathematics course s/he took; "what about the discrete that we had to do modeling? We had to do diagrams and counting."

OTL Comparisons across Universities

Looking across universities, we found similarities and differences (See Table 4). We report these comparisons acknowledging, as stated previously, that all required experiences were not included in our study. All of the professors, except one Real Analysis instructor at University B, mentioned at least one OTL related to modeling in their course. Not surprisingly, the most common purpose of modeling mentioned across the universities was "Use mathematical modeling in real-world context." In terms of the modeling processes, opportunities to learn related to creating and selecting appropriate representations was most common. According to our framework, University C provided their PSTs with the greatest number of OTL related to modeling in the courses included in the study.

Table 4
Themes Addressed by Participants at Universities A, B, and C

Purposes of modeling	**A**	**B**	**C**
Use mathematical modeling in real-world contexts	5	5	8
Use problem-solving approaches to investigate and understand math	2	2	1
Understand and describe the behavior of a system or event	0	1	0
Total	7	8	9
Processes of modeling (from *CCSSM*)			
Identifying and selecting variables	1	2	4
Formulating a model by creating and selecting appropriate representations	3	2	5
Analyzing and performing operations to draw conclusions	0	1	2
Interpreting the results of the mathematics	1	1	2
Validating the conclusions, possibly improving the model	0	0	3
Reporting on the conclusions and the reasoning behind them	0	0	1
Total	5	6	17

Most of the opportunities for PSTs to engage in modeling or learn about modeling involved completing tasks or math problems related to the content of the specific course (e.g., modeling for secondary school algebra in methods courses, differential equations, and linear algebra in college mathematics courses). This notion was reinforced by statements from both instructors and PSTs at all three universities.

For example, a Secondary Math Methods instructor from University A stated:

> I don't assess students' ability to do [modeling]. We look when we analyze textbooks, we look at textbooks that have a very strong modeling approach, and we talk about the advantages and disadvantages probably of that, perhaps an introduction to a particular topic and then we talk about using real world contexts and perhaps real world data as an introduction to a lesson about a topic or as a conclusion application.

A PST from University B also talked about using modeling in his/her Statistics course, "There was Stats, which was all about modeling because that's what statistics is." Also, an Abstract Algebra instructor from University C discussed a modeling problem from his course:

> If you want to make a necklace with two different colors of beads and eight beads on the necklace how many different necklaces are there? How do we do it? Well we think of the beads as lying on the vertices of a regular octagon in that case and the symmetries of the octagon is acting on the necklace and, of course, you can't tell them apart; that is, if you rotate the beads around it's the same necklace

There were also some unique responses across universities. For example, one of the purposes of modeling, *understand and describe the behavior of a system or event*, was only mentioned by a Secondary Methods instructor at University B:

> So understanding that the work of proportional reasoning isn't just, let's teach kids how to set up two fractions and equal sign and solve an equation like that. That it extends to, that it defines a class of contexts of models of

> phenomena that have a consistent set of behaviors. I think [this] gets at some of these ideas of modeling.

Another aspect of modeling, *validating the conclusions, possibly improving the model,* was only mentioned by a Differential Equations instructor at University C:

> Maybe we talked about that a little bit about what's being left out of the model. When you model harmonic motion, you are assuming the resistant force is linear and I talk about why we assume that and what the alternatives might be.

Overall, there were variations between instructors and PSTs, across universities, and between mathematics and methods instructors. Many of the examples provided involved PSTs engaging with modeling for their own learning and actively participating in the process. The examples that PSTs provided addressed some aspects of the purposes and processes in the modeling framework, which include using mathematical modeling in real-life contexts, using problem-solving approaches to investigate mathematics, and formulating appropriate representational descriptions. On the other hand, mathematics education instructors addressed all the themes, often with specific examples, while mathematics instructors gave examples regarding 6 of the 9 categories addressing the OTL about modeling.

Discussion and Implications

To conclude, we will summarize and highlight some of our results and discuss limitations and implications of the work described in this paper. We will also discuss the differences we found between different groups, first PSTs and instructors, mathematics and methods instructors, then universities. Our results indicated that the teacher education programs at University A, B, and C offered diverse opportunities to learn about modeling. Based on the perspectives of the participants of our study, some of these opportunities include performing contextual tasks, problem solving, using a variety of tools, and planning lessons that incorporate modeling.

Nearly every participant mentioned opportunities for PSTs to learn about and use modeling. However, the PSTs did not provide the same level of detail as instructors. This was perhaps because they were expected to reflect on all program courses, while instructors were focused on one course that they may have taught multiple times. When both sets of participants (i.e., instructors and PSTs) looked at the purposes and processes of modeling described in *CCSSM* during the interview, PSTs did not describe the last purpose of modeling, *understand and describe the behavior of a system or event*. Since this purpose of modeling is also recommended by *CCSSM* (NGA & CCSSO, 2010), instructors may need to be more explicit about modeling as describing the behavior of a system or event. In terms of the process of modeling, PSTs described one aspect of the process of modeling, *formulating models by selecting and creating different representations*, but did not mention the other processes of modeling. Some of the OTLs described by instructors might be possible examples of how PSTs can be more prepared to teach the modeling processes to their students. For example, a Seminar instructor at University C described that she emphasized the importance of PSTs identifying the meaning of particular variables by asking questions, such as "What does the 'x' and 'y' mean?" in the context of modeling problems.

Both PSTs and instructors reported opportunities that were more related to opportunities to learn about modeling for PSTs' own mathematical understanding or modeling related to non-algebraic concepts, rather than how to teach modeling. Future teachers need to have meaningful experiences during their teacher education programs that allow them to integrate their mathematical content and pedagogical knowledge to support their students (Silverman & Thompson, 2008). Thus, in preparation to teach algebraic modeling, PSTs would likely benefit from purposefully chosen experiences that would facilitate their integration of knowledge related to modeling and pedagogy needed to support their students' learning of modeling in their own classrooms.

Several of the mathematics instructors mentioned that they had very little experience with *CCSSM*. This does not mean that they do not address aspects of modeling in their courses, but they may have not made changes based on the standards. We found that methods instructors provided examples and anecdotes about both the purposes and processes of modeling described in *CCSSM*, while mathematics instructors' responses focused more on the process of modeling than on the purposes of modeling. Not surprisingly, methods instructors also discussed the pedagogical aspects of modeling, while mathematics instructors addressed this category to a lesser extent. A future study could investigate the cultural and fiscal contexts of the programs and how the curriculum and the background of the instructor may influence the focus on and conceptualization of modeling in a course. In addition, there is the need for more conversations and collaborations across programs and departments. As the *Mathematical Education of Teachers* recommended, partnerships between mathematics faculty and mathematics education faculty are necessary, emphasizing that there are opportunities for growth in both communities. Mathematics education faculty should consider mathematics departments' missions, while mathematics faculty could become more involved in mathematics education by designing courses for PSTs that are supervised by faculty with expertise in teacher education (CBMS, 2012).

In this study, we noted some unique opportunities that were provided for future teachers that could be useful for programs with goals of emphasizing purposes and processes of modeling. A Secondary Math Methods instructor described a textbook analysis activity where students analyzed and discussed the implications of using textbooks with a strong modeling focus. Textbooks may be used by teachers as guides, resources to draw on, or materials to interpret; in other words, they could impact the enacted curriculum in one way or another (Remillard, 2005). Thus, the emphases of a particular course are influenced by the written curriculum and finding curricula that align with particular goals of new teachers might support the enactment of lessons that could meet those objectives. A Differential Equations instructor from University C highlighted one of the opportunities to learn about validating conclusions and modeling- one of the modeling processes described in the

CCSSM. Students in the course were given the opportunity to discuss what could be missing from a particular model and shared some of the simplifying assumptions which could help unpack or explain the solution. This opportunity highlights that brief discussions of the appropriateness of solutions and suggestions for improvement of existing models may help future teachers experience the modeling process as learners in their content courses; perhaps even in courses that do not explicitly emphasize algebraic modeling.

This study was limited to some extent by the fact that the courses included across universities were not uniform; rather, the courses were a subset of those required. However, efforts were made to include a balance of mathematics and mathematics education courses. There were varying perceptions of modeling which could have also influenced what was reported by instructors and PSTs as opportunities to learn about algebraic modeling. For example, an Abstract Algebra instructor at University C considered constructing proof as a model, whereas a Capstone Mathematics instructor from University B mentioned, “I've never modeled anything I guess. I've always done pure math. I've always thought of math as just, I've never applied math. Ever.” Thus, some of our results could be a result of varying perceptions, rather than a lack or an abundance of opportunities to report. In future work, we will investigate OTL related to other areas important for algebra teaching, including the nature and structure of algebra, reasoning and proof, use of tools and technology, and equity issues related to algebra learning.

In this paper, we set out to describe the OTL related to modeling from the perspectives of instructors and PSTs at three universities. These reported opportunities varied across several dimensions, including the ways in which participants conceived of modeling, the type of courses in which the opportunities were provided, and who reported the OTL (i.e., PSTs, mathematics instructors, or mathematics education instructors). These findings represent a first step toward a more thorough understanding of the preparation of algebra teachers.

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