

Exploring the Relationship between Questioning, Enacted Mathematical Tasks, and Mathematical Discourse in Elementary School Mathematics

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This study examined the mathematical discourse of elementary school teachers and their students while participating in a year-long professional development project focused on implementing reform-based mathematics curriculum. The teacher participants included 12 teachers, two from each grade level from Kindergarten through Grade 5. Field notes were collected during observations and were analyzed as raw qualitative data. Inductive qualitative analyses of classroom observation data indicated that the level of enacted mathematical tasks and teachers' questioning strategies influenced the types of mathematical communication in classrooms. Quantitative analyses of data regarding teachers' level of questioning and implementation of mathematical tasks found an increase in high-level questions and tasks, but the increase was not statistically significant. Implications for the design of professional development and further research in this area are shared.

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Researchers and educational leaders have cited issues with mathematics achievement and students' mathematical learning in the United States (Hiebert & Stigler, 2000; OECD, 2012; U.S. Department of Education [USDE], 2008). While a number of reforms have been initiated and supported, there is still a lack of empirical evidence on how these reforms influence students' mathematical achievement (OECD, 2012). In the past decade, calls for reforms in mathematics education include supporting teachers to pose cognitively-demanding mathematical tasks (Stein, Grover, & Henningsen, 1996; Stein, Remillard, & Smith, 2007), ask high-level questions to support students' discourse and reasoning about mathematics (Hufferd-Ackles, Fuson, & Sherin, 2004), and use their knowledge of students' understanding to design future instruction (Joyner & Muri, 2011; William, 2010). These types of pedagogies are encompassed in the broader phrase, standards-based teaching, which refers to the NCTM *Principles and Standards for School Mathematics* (2000).

Several researchers have examined the influence of standards-based instruction on student achievement (Reys, Reys, Lapan, Holliday, & Wasman, 2003; Tarr et al., 2008). In a large scale study with thousands of students, eighth-graders who had been taught with standards-based curriculum resources in conjunction with standards-based pedagogies significantly outperformed their peers who had been taught with a standards-based curriculum with traditional pedagogies or taught with a traditional curriculum (Tarr et al., 2008). Further, Reys, Lapan, Holliday, and Wasman (2003) found that eighth graders who were taught with standards-based curricula for at least two years did equally as well or better than the comparison district. Further, Post et al. (2008) found that students in grades 5 to 8 who were taught using standard-based instruction showed greater achievement on open-ended test questions and problem-solving items compared to items assessing procedural knowledge. Clearly, standards-based instruction and the use of standards-based resources have the potential to support students' mathematics understanding.

With elementary school students, Smith and Smith (2006) found that third grade students who had participated in

standards-based instruction demonstrated a higher number of correct responses than fourth graders who had experienced traditional instruction on items focused on conceptual understanding, such as writing a word problem to match an equation and drawing a picture to match the problem. Further, recent studies found that elementary school students in Grades K-5 who were taught with standards-based curriculum scored higher when their teachers taught with standards-based pedagogies and had beliefs about mathematics teaching that aligned with standards-based ideas (Polly et al., 2015; Wang et al., 2013).

The studies on the use of standards-based pedagogies and curriculum indicate a positive influence on student learning on both large-scale assessments (Tarr et al., 2008) and curriculum-based assessments (Polly et al., 2015; Wang et al., 2013). ; However, there remains a need to more closely examine teachers' enacted pedagogies while using standards-based curricula, including the types of mathematical tasks and questions that teachers pose. This study considered teachers use of the standards-based mathematics curriculum *Investigations in Number, Data, and Space* (TERC, 2008) and examined the mathematical tasks and questions that teachers posed during classroom observations.

Teachers' Use of Mathematical Tasks

An integral component of standards-based instruction is the use of cognitively demanding mathematical tasks (National Council of Teachers of Mathematics [NCTM], 2014). Henningsen and Stein (1997) described mathematical tasks as problems and practice activities teachers provided students that require students work diligently for a certain work period with the goal of learning specific mathematical concepts. The seminal framework (Smith & Stein, 1998) separates tasks into those with low cognitive demand and high cognitive demand. Tasks with low demand involve recalling memorized information or carrying out a procedure without mathematical connections. Tasks with high cognitive demand include those in which students carry out a procedure but make mathematical

connections to representations, various strategies, and those that involve doing mathematics, such as solving a non-routine problem in which students must devise, carry out, and verify their strategies (NCTM, 2014; Smith & Stein, 1998). Tasks with a high-cognitive demand promote reasoning, students' exploration of mathematical concepts, and align with recommendations for standards-based pedagogies.

Researchers have noted that even when teachers pose cognitively demanding mathematical tasks, students interacted with these tasks in low cognitively demanding ways (Hsu, 2013; Polly & Hannafin, 2011). Teachers may decrease the cognitive demand of the task by over directing or doing too much of the task themselves, instead of allowing students to explore the task (Hsu, 2013; Polly & Hannafin, 2011).

Teachers' Use of Questioning

Teachers effective use of questions has the potential to generate students' responses about their mathematical thinking, problem solving, and strategies (Hufferd-Ackles et al., 2004). The process that students go through while describing and explaining their thinking allows students to recognize misunderstandings, internalize problem solving strategies, and develop new understandings (Chi, 2000; Saxe, Gearhart, Note & Paduano, 1993). Schuster and Anderson (2005) suggested that "good questions" serve as an invitation for students to engage in thoughtful communication and move into a more active role in the classroom. Bennett (2010) proposed that although mathematical communication strengthens student understanding, creating this environment is a challenge for new teachers and students. The types of questions used in the classroom to elicit students' thinking have been an area of increased research.

Further, Franke et al. (2009) found that uncovering a student's strategy often required multiple specific questions that build on an element of the student's explanation. These types of questions helped initial incomplete or ambiguous explanations become more focused and complete. This research suggested that teachers asking leading questions and

assuming much of the mathematics work failed to provide students with the opportunities to build on their own understanding. Questioning is used to facilitate classroom discourse that offers room for students to build their understanding.

Mathematical Discourse

The Standards for Mathematical Practice in the Common Core Mathematics Standards (CCSSI, 2011) recommended that teachers provide opportunities for students to construct viable arguments, critique the reasoning of others, attend to precision while communicating, and reason quantitatively while solving and discussing mathematical tasks. These echo ideas shared previously in the *Principles and Standards* (NCTM, 2000) and other seminal works, such as the National Research Council's *Adding it Up* (NRC, 2001). One case study highlighted the importance of questioning in creating mathematical discourse, and discovered several factors that help teachers facilitate their change to reform-oriented practices including: (a) research-based mathematics curriculum *Children's Math Worlds* (Fuson et al., 1997), (b) the reform-focused school culture, and (c) weekly feedback from the researchers (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008).

Jingzi, Normandia, and Greer (2005) conducted research through classroom observations to analyze teachers' and students' mathematical discourse. The analysis focused on discerning the level of theoretical understanding demonstrated by students. The discourse analysis revealed students were able to express mathematical concepts at the level of describing actions and a sequence of steps. Students encountered difficulty as they tried to move from this type of discourse, which is less cognitively demanding, to expressing conceptual knowledge, reasons behind actions and defending their choices. Students' discourse remained on the procedural level and when pushed by teachers to explain further the concepts, principles, or methods employed in their solution, students hesitated. Jingzi et al. (2005) noted students' reluctance and failure to articulate the next level of understanding led teachers to intervene and

finish the task for the students. The focus on the use of questions, mathematical tasks, and the level of mathematical discourse in the classroom was examined to further students' mathematical understanding.

Mathematical tasks, questions, and opportunities for discourse are important elements of standards-based mathematics instruction. While research indicates potential for these elements to positively influence students' mathematical understanding, there is a further need to closely look at the interplay between mathematical tasks, questions, and discourse in mathematics classrooms. To this end, this study examined the following research questions in elementary school classrooms:

- How do teachers use questions to engage in mathematical discourse with students?
- How do teachers pose cognitive demanding mathematical tasks in their classroom?

Methodology

Context of the Study

This study focused on examining the instructional practices of elementary school teachers from two high-need school districts in the southeastern United States. Each district educated a large percentage of students who qualified for the federal free and/or reduced lunch program. During the year of this study, each teacher participated in a yearlong mathematics professional development project titled, Content Development for Investigations (CoDE-I), funded by the state's Mathematics Science Partnership (MSP) grant program. The grant involved teacher-participants from two districts. District One was a large district with 88 elementary schools that include urban and suburban areas. District Two was a neighboring suburban district with five elementary schools (grades K through 4), and one district-wide intermediate school (grades 5 and 6).

The focus of the CoDE-I project was to support elementary school teachers' use of the standards-based curriculum, specifically *Investigations in Number, Data, and Space*

curricula (TERC, 2008). Additional goals of the MSP grant program were (a) to increase teachers' mathematics content knowledge, (b) improve student learning outcomes, (c) create a cohort of teacher-leaders who will lead school based professional development on standards based math curriculum, and (d) to create a successful model of professional development.

Each year of the CoDE-I project, teachers applied to be selected as participants through procedures defined by their school districts. District One teachers were selected based on the number of applicants at their school, with attention paid to selecting large numbers of teachers at each school. District Two teachers were selected in an effort to have nearly every teacher eventually participate in the grant by the end of the third year. On average District One selected 200 teachers per year and District Two selected between 30 and 40 teachers per year.

The summer institute was facilitated by district and teacher-leaders from the school districts and a mathematics education professor at the partnering university with additional insight from a university mathematician. The professional development focused on building teachers' knowledge of mathematics content and pedagogies through solving mathematical tasks and analyzing materials in *Investigations* (TERC, 2008). Teachers from the two school districts participated in the professional development separately and on different days, but the overall content and focus of the professional development remained consistent. Data from both cohorts II (2010-2011) and III (2011-2012) were used in this study.

Selected teachers participated in approximately 70 hours of professional development, which included a 48-hour summer institute followed by approximately 30 hours of school-based professional development. During the summer institute, teachers explored cognitively demanding mathematical tasks (in the context of number sense, fractions, and algebra concepts). Teachers also spent considerable amount of time unpacking the State Standards and analyzing the *Investigations* curriculum to better understand the mathematical ideas and

how those ideas developed across lessons. Teachers also explored ways that they could differentiate the curriculum to meet the needs of both high-achieving and struggling students. During the school year, professional development activities included face-to-face workshops, which were similar to the summer sessions, as well as classroom-based experiences. These classroom experiences included analyzing student work samples and assessment data, video tapes of mathematical discussions, and facilitating a planning session for their colleagues that included a synergy of content, mathematics standards, standards-based pedagogies, and resources from the *Investigations* curriculum.

Mathematical discourse frequently emerged during workshop sessions. Professional development facilitators and participants discussed and considered the extent to which the tasks and activities in *Investigations* provided opportunities for elementary school students to discuss and reason about mathematical strategies. The participants also made connections between mathematical concepts. Within these activities the sixth Mathematical Practice (CCSSI, 2011), “attend to precision” was frequently used as a lens for examining the curriculum.

Teacher Participants

This study included 48 randomly selected participants, 24 participants from Cohort II and 24 participants from Cohort III. The 24 teachers in each cohort consisted of two teachers per district from Kindergarten through Grade Five. The 24 Cohort II teachers were chosen from a pool of 153 teachers, and the 24 Cohort III teachers were chosen from 182 teachers. All teachers held a teachers license in either Early Childhood Education (Birth through Kindergarten) or Elementary Education (Kindergarten through Grade Six). The 48 teachers varied in age from 23 to 54 and had a diverse range of teaching experience from one to 32 years.

Data Collection

The research team observed each teacher twice during the year, once in the fall between October and December, and the second observation in early spring between February and March. Each of the observed lessons lasted between 60 and 90 minutes. Every observation was conducted by one of three researchers. The researcher(s) completed an observation profile, in which they recorded field notes of all observations, noting specific interactions between the teacher and student, types of questions asked by the teacher, and an overall description of the classroom environment. The observation protocol had been developed, piloted and refined with Cohort I teachers.

All researchers who completed the observations engaged in conversations with other project staff prior to this study about the aspects of mathematics instruction included in the observation protocol. Furthermore, all researchers were involved in the piloting and refinement of the observation protocol prior to the study. During the study, inter-rated reliability checks were conducted at two points. These two checks involved all three researchers observing the same mathematics lesson, using the protocol, and taking notes. The two classrooms observed were first grade and fifth grade. The protocol and notes were examined and the inter-rater reliability was 91% for both observations.

Data Analysis

Data from the observation protocol were examined quantitatively to look for relationships between teachers' use of high level questions and the tasks they employed during instruction. The observation protocol included items that pertained to questioning and items related to teacher's use of mathematical activities (see Table 1 and Table 2). Items were rated on a Likert-type scale from 1 to 5 with 1 = "Minimal" and 5 = "Advanced." Minimal was defined as, "The teacher does not demonstrate the behavior of interest and any similarity is incidental" and advanced was defined as, "The teacher

frequently displays the behavior of interest and it is a well-developed and intentional part of practice.” A mean score for use of questions and activities was computed for each teacher for both observations. T-tests were used to examine the difference in the means from Observation 1 to Observation 2 for both questioning and activities.

Field notes from the observations were analyzed qualitatively using thematic analysis (Coffey & Atkinson, 1996). The researchers organized the data into categories by themes (Ezzy, 2003). Initial data analysis using Atlas.ti software (2015) led to eight different codes. The data was then reduced to only include the five codes related to interactions between the teacher and their students: (a) teacher questions, (b) teachers actions, (c) teacher directions, (d) teacher discussion, and (e) student responses. The revised data using the reduced data was then analyzed several times in order to find more specific themes and to verify findings. The multiple sources of data listed above were used to triangulate the results.

Findings

In this section we highlight two major themes that emerged from our analysis. They include the ways that teacher questioning influenced discourse and the enactment of mathematical tasks.

Teacher Questioning Influenced Levels of Mathematical Discourse

The questioning items from the observation protocol were analyzed and the mean score was computed for both observations for each teacher (see Table 1). The protocol examined questioning that provides opportunity for students to engage in open discussion, explain, and create new questions. Questioning in this manner extends the mathematical discourse in the classroom from a lower level of stating answers to a higher level of discussion. The mean score for all observed teachers for Observation 1 was 3.86 and the mean score for

Observation 2 was 4.08. This indicated some growth in teacher questioning from Observation 1 to Observation 2, but a paired sample t-test showed the mean difference was not statistically significant ($p = .210$). Although the improvement in observed teacher questioning was not statistically significant, most teachers demonstrated better questioning abilities by the second observation according to the protocol.

Table 1
Observation Protocol Items Means for High-Level Questioning

Item	Observation 1		Observation 2	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Engages students in an open-ended discussion about their use of different strategies for solving mathematics problems.	3.77	1.07	3.91	1.23
Through modeling or discussion, encourages the use of multiple strategies for solving mathematics problems.	4.05	1.01	4.21	.86
Creates a classroom environment where student-led discussions are welcome.	4.09	.98	4.19	.98
Asks high-level cognitive questions to check for student understanding.	3.72	1.1	4.05	1.07
Asks high-level cognitive questions to extend student learning.	3.50	1.24	3.93	1.25
Gave opportunities for students to explain their responses or solution strategies.	3.91	1.38	4.23	1.13
Total	3.86		4.08	

Note. Items were rated on a Likert-scale from 1 to 5 with 1 = “Minimal” and 5 = “Advanced.” Minimal was defined as, “The teacher does not demonstrate the behavior of interest and any similarity is incidental” and advanced was defined as, “The teacher frequently displays the behavior of interest and it is a well-developed and intentional part of practice.

Through the qualitative analysis of observation notes, additional evidence of the teachers' use of questioning for promoting discourse was found. For example, in a second grade classroom, the teacher provided a visual of a combination of shapes and angles and followed with high level

questioning. This example shows how follow-up questions can be used to encourage justification. The teacher allowed time for students to observe the picture and focused their attention on the angles within the visual aide.

Teacher: Looking at this picture there are many types of angles, what types do you notice? Come to the board please and explain.

Student: A right angle.

Teacher: Why is it a right angle?

Student: It's 90 degrees, I see a line segment, I see a straight angle, it's straight, the "a" is right here and the "b" is right here.

Teacher: How many degrees are in a straight angle?

Student: 180.

The teacher's initial question elicited a response about what angles the students were seeing in the picture and requested students to come to the board to explain. The student correctly identified the angle to be a right angle, but the teacher sought justification in his reasoning by asking, "Why is it a right angle?" The student indicated that a straight line intersects the straight line. Therefore equal angles exist on each side. By asking the student how many degrees are in a straight line, the teacher encouraged the student to identify prior knowledge about line segments, as well as required him demonstrate the mental division in which he engaged to get 90 degrees for each angle. The combination of higher-level questions with fact-based questions served to help the students reason through their thinking. The lesson continued and the students worked collaboratively to define acute and obtuse angles.

Another example of how the teacher's questioning influenced the classroom discourse occurred in a fifth grade classroom. The teacher posed a multi-step problem and allowed students to work in groups to explore the task while supporting their work with high-level questioning. The task was:

One hundred students have lockers 1 to 100. Student 1 opens all of the lockers. Student 2 closes all of the lockers with numbers that are multiples of 2. Student 3 changes the

status of the lockers that have numbers that are multiples of 3. How many lockers were open, switched the most times, and only opened twice?

The mathematical discourse between this teacher and her students proceeded as such during this lesson:

Teacher: How are you going to keep track? So what strategy do we have?

Student: [Explains that she crossed out all of the multiples of 2 on her chart.]

Teacher: Ok, so how are you going to keep track? That's a good strategy to start but you have to keep track.

Student: I could cross the box out a different way if the status changes.

Teacher: That's a good strategy. Explain to your group what you're doing.

Student: I can count each line to see which ones are changed.

Teacher: ok.... [reiterates strategy] All right so how are you going to keep track [moves tables]. I think the ones that aren't two are the ones that are closed. [Teacher explains again.]

Student: So the ones aren't two, they're open?

Teacher: So you have a strategy that you started but, is it going to be the best strategy so that you know which ones are closed and open the most?

Student: [One student explains that she'll write a circle or a square around the numbers on her chart either open or closed.]

Teacher: You guys started out with a pretty good strategy but you have to make sure it fits all of the questions in the problem. How can you use your chart? Can I share a couple strategies that were shared over here? One group will use O's and C's. Another group is going to mark it out and count the lines. Talk between the four of you and decide which to use. [Teacher moves to other end of the table, suggests that this group also skip count by 3 to be able to answer questions.]

The questioning throughout the lesson encouraged students to critically think through whether their strategy would provide the answers, and provided the opportunity for students to evaluate whether their method was the best strategy to work through the problem. Students explained their process and each new and different strategy was discussed and considered. As students began to engage in creating a system and collaborating with one another the teacher asked, “Is it going to be the best strategy so that you know which ones are closed and open the most?” This question encouraged the use of multiple strategies for solving problems and moved students to the next level of evaluation of the strategies that had been generated. The students will need to consider their system, knowledge of multiples, and start testing their assumptions.

Finally, compare and contrasts questions were seen as helpful in influencing productive discourse. During an observation of another first grade classroom, students offered characteristics of geometric shapes while the teacher recorded their ideas on a poster board.

Teacher: What else do you know?

Students: All the corners are facing each other.

[The teacher solicits information about a triangle using short comparison questions such as “more or less” “different or the same”]

Students: It's flat. They are both flat, looks like a slide, if you put another one with it, it makes a square, and it is a closed figure.

Teacher: Why is it a closed figure?

Students: It has no open sides.

Teacher: How is this - rectangle- different than a square?

Students: The sides are different.

This first grade geometry unit was designed to help students establish characteristics of certain shapes by using a compare and contrast strategy. The teacher utilizes short comparison questions that provide an opportunity for students to move from one-word responses to a response that contains several observations pertaining to geometry. Instead of paraphrasing

student answers with more details about shapes, the teacher used questioning to encourage students to access their prior knowledge and refine their definitions and characteristics of triangles, rectangles, and squares. Thus, the students appeared to build understanding of geometric shapes through discussion, as opposed to direct instruction, where the teacher's questioning was a key aspect to providing students with the opportunity engages in a higher level of discourse.

Teacher Enactment of Mathematical Tasks

The observation protocol items related to teachers' use of cognitively demanding mathematical tasks were analyzed for both observations of each teacher. The protocol for mathematical tasks examined: (a) opportunities to solve complex problems, (b) use appropriate mathematical representations with manipulatives or other materials, (c) make conjectures about mathematics, and (d) develop conceptual understanding. The mean score of all teachers on Observation 1 was 4.26 and the mean score for all teachers on Observation 2 was 4.36 (See Table 2). While this indicates some growth, a paired sample t-test showed difference was not statistically significant ($p=0.36$).

Although the growth from Observation 1 to Observation 2 was not significant, teachers using the *Investigations* curriculum included activities that allowed students to explore cognitively demanding mathematical tasks. These tasks offered students the opportunity to use more efficient and effective ways of problem solving. The tasks included mathematical concepts that could be applied to other problems. Students explored these tasks in whole group, small group, and individual settings.

This observation from a fifth grade classroom begins with a task that required groups of students to determine the amount of squares that are on each chart. In this example the teacher used tasks to generate multiple strategies. The teacher began, "How can we figure out how many squares are on here? The task for today is to figure out how many squares are on the paper." Teacher then puts students in groups and gives each

group a chart. Students are instructed to determine the number of squares and then explain how they came to that conclusion. The teacher then asked students to share their group's answers. As the groups shared, the teacher wrote their strategies on the board and gave directions for students to write the number one in the first square and 10,000 in the last square. She then tells students, "We need to choose some landmark numbers." Students talked about easy to work with landmark numbers and one student responded, "Mostly I would say factors of 10,000 so numbers like 10." The teacher prompted, "anything else... maybe 5's?" Students agreed and one student suggested 100's and 2's. Another student said "maybe 1,000's." The task allowed the students to apply the concept of landmark numbers. One student suggested during the task that landmark numbers are directly related to factors. The knowledge of landmark numbers and factors form a strategy that was applied to this particular task and other similar problems.

Table 2.

Observation Protocol Item Means for Cognitively Demanding Activities

Item	Observation 1		Observation 2	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Provides opportunities for solving complex problems and/or tasks.	4.23	.94	4.42	.82
Provides opportunities for students to develop appropriate mathematical representations using manipulatives or other materials.	4.52	.85	4.60	.54
Provided opportunities for students to make conjectures about mathematics ideas.	4.23	.94	4.33	.72
Fostered the development of conceptual understanding.	4.14	1.07	4.30	.83
Total	4.26		4.36	

The next example is also from a fifth grade classroom. The example shows how sometimes teachers implemented tasks in

the form of mathematical games to facilitate learning. Games were often used in groups or as a whole class in order to work with mathematical concepts. The teacher started with a whole class game called “Guess my Fraction.” The teacher began the task by naming a fraction that represented an observation of the classroom. The students observed the classroom, articulated their observation in fraction form, and hoped their observation matched the teacher’s fraction.

Teacher: First you have to guess the fraction I'm thinking about....

Student: $1/6$...

Teacher: That's not my rule, but what made you think of that.

Student: $3/6$

Student: $4/6$,

Teacher: $4/6$ is my fraction. Would you like to guess my full rule?

Student: $4/6$ are wearing jackets.

Teacher: not quite.

Student: $4/6$ are wearing light blue

[Guessing continues until the rule of $4/6$ of the students are wearing hoods is identified]

Teacher: Writes $4/6$ on the board. How do I make it an equivalent fraction?

Student: $2/3$ You could do four two times and it would be eight and you could divide four into six and then you could divide three into six and get two....

Teacher: are you dividing by three,

Student: no, two....

Teacher: good

Student: I have another one, $8/12$if you do four times two....

Teacher: [Calls six more people to the front.] What fractions do you see, discuss....What fraction were you thinking of....

Student $3/6$...

Teacher: If I wanted to reduce $3/6$, what is it?

Student: $1/2$

Teacher: What fraction am I thinking about and what's my rule?

[Teacher launches into another Guess my Fraction task, which leads to the next lesson on percent]

The task allowed students to explore fractions and express their strategy for making equivalent fractions. Presenting this task offered students the opportunity to observe their classroom and evaluate the parts of whole relationships around the room. In this example, students observed several representations of a fraction and the activity was then extended to equivalent fractions. The teacher facilitated the task by asking prompting questions that allowed the students to be part of the discussion and explain their thinking, this is important to progress students through the task. The task is used to foster engagement as students move on to transforming fractions into percent. These concepts can then be applied to other similar problems during mathematics instruction.

Discussion and Implications

The Common Core State Standards (CCSS) in Mathematics (CCSSI, 2011) includes standards for mathematical practice that emphasizes the National Council for Teachers of Mathematics (NCTM, 2000) process standards of problem solving, reasoning, proof, communication, representation and making connections. The NCTM Standards also calls for teachers to provide students with opportunities to communicate about mathematical concepts in a clear and coherent manner. The CCSSI (2011) stated, "Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments." This initiative further defined proficient students as being able to justify their conclusions, communicate them to others, and respond to the arguments of others. The CCSS and NCTM standards stress higher-level reasoning and understanding, which is a weakness for students in the United States. Among the 34 OECD countries, the United States

performed below average in 2012 in mathematics and is ranked 27th. OECD (2012) analysis of the United States performance indicated our students are weak in performing mathematics tasks with higher cognitive demands. Additionally, it is noted that our students have problems with mathematical literacy tasks where the students have to use mathematics they have learned in a well-founded manner.

This study examined how teachers used high-level questions and cognitively demanding tasks to engage students in mathematical discourse and the exploration of their ideas. Our study supports Schuster and Anderson's (2005) work that suggested creating an environment for mathematical discourse is important for student learning and remains an area that requires further development for teachers. The observation field notes of teacher questioning and student response provided evidence of students' mathematical discourse focusing more heavily on justifying their strategies and making connections between topics rather than summarizing steps or providing answers. This type of discourse connected with questioning and tasks addresses the weaknesses noted in OCED (2012). The OCED (2012) report also mentioned that U.S. students' strengths lie in cognitively less-demanding mathematical skills. In order to increase student achievement on cognitively higher-level problems, teachers must present students with opportunities to engage in challenging tasks and discourse that encourages reasoning and justification.

In this study the tasks provided a space for students to interact and increase their mathematical discourse. Student discourse included sharing strategies, procedures, ideas and questions. The quantitative results showed growth in this area; however, the lack of significance indicates further research needs to be done to examine enacted tasks and discourse. The observations used in this study were of teachers primarily using *Investigations*, a standards-based curriculum. Our qualitative findings illustrate the types of cognitively demanding tasks and higher-level questions found in standards-based curriculum. This study also offers some insight into the connection between teachers' use of high-level questioning, their enactment of

cognitively demanding mathematical tasks, and classroom discourse.

Limitations

Exposure time to standards-based instruction and use of a standards-based assessment were not accounted for in this study and may be factors to examine in future research. Timing of the data collection of this study varied based on coordinating schedules, which made collection more liberal. The teachers within this study were visited in the fall and spring of the school year. However, the day of the observation was selected from mutual convenience of both parties. Whether the teachers were just beginning a unit or toward the end was not considered in planning the observation. The other factor that may have influenced the t-test findings is the timing of the second observation. The second observation occurred in the spring of the school year. In the state where the study took place end of grade testing occurs in May with teachers preparing for those tests two or more months ahead of time.

Another limitation that we acknowledge in this study is the claims about the intentions and goals of the activities that teachers enacted. This study was not designed to examine teachers' intended and espoused practices, and as a result there are limitations in terms of the generalizations about how these mathematical tasks and discussions influenced students' mathematical understanding. Lastly, while this study established data collection from many teachers, only two observations were conducted. This means that while researchers observed teachers in their classroom, the small number of observations may or may not be representative of typical classroom practice.

Future Research

In future research, more teachers should be observed for their use of questioning with relation to their enactment of mathematical tasks. In our study, tasks and questions came together in a collaborative effort between the teacher and the

whole class. The part of the unit being studied and the time frame in the school year connected to high stakes testing may be two factors that could impact the mathematical discourse that is observed. The mathematical achievement of the students could also be impacted by the previously mentioned factors. Similarly, the timing of the students' assessment and the observation should be planned to occur during a much shorter window to allow less time for greater internal validity. Finally, teacher pedagogical practices should be explored with other student assessments to understand the scope and generalizability of the results.

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