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Underprepared Students' Performance on Algebra in a Double-Period High School Mathematics Program

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The primary goal of the Intensified Algebra I (IA) program is to enable mathematically underprepared students to successfully complete Algebra I in 9th grade and stay on track to meet increasingly rigorous high school mathematics graduation requirements. The program was designed to bring a range of both cognitive and non-cognitive supports to bear on underprepared students' learning of rigorous algebra content within an extended period of instruction, thus allowing them to catch up with their peers on a pathway toward more advanced mathematics courses. This study measured gains in IA students' overall mathematical performance based on a comprehensive multiple-choice assessment of algebra proficiency and a constructed response assessment. Results showed IA students' performance significantly increased over the academic year on both assessments. In addition, students' performance showed a consistently large improvement in three of six core content areas (Graphing Linear Equations, Functions and Graphs, and Solving Linear Equations) within the multiple-choice assessment. This study provides promising evidence of IA meeting its programmatic goal of supporting underprepared students learning of core algebra content in a function-based curriculum. Implications for curriculum implementation and ongoing development, and further research are discussed.

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The importance of students' success in Algebra I for college and career options has become a focal point of mathematics education research and practice over the last two decades, leading to the familiar characterization of introductory algebra as a gatekeeper course (Moses, 2001). The connection between success and failure in introductory algebra and other key academic outcomes is well documented. Students who are successful in introductory algebra are more likely to take advanced mathematics courses, graduate from high school, and achieve academic success beyond high school (Adelman, 2006; Ma & Wilkins, 2007). Conversely, students who fail algebra run a greater risk of dropping out of high school and losing future opportunities for academic and economic success, as well as full participation in democratic citizenry (Moses, 2001). It is in this context that the *Intensified Algebra I* (IA) program was developed with the aim of providing underprepared students the opportunity to learn in a meaningful way the core ideas of algebra, with adequate supports to be successful (Agile Mind, The Charles A. Dana Center, & University of Illinois at Chicago Learning Sciences Research Institute, 2013).

Researchers have documented extensively the difficulties students have encountered in learning algebra (e.g., Boulton-Lewis, Cooper, Atweh, Pillay, & Wills, 2001; Demana & Leitzel, 1988; MacGregor, 1996). In parallel, the structural approach to algebra used in most traditional programs has been identified as problematic (Cai, Nie, & Moyer, 2010). Such an approach introduces equation solving not only before variables and functions but it is also characterized by abstract work with symbols not grounded on conceptual foundations developed in problem situations. In response, several alternative approaches to the teaching of algebra have been developed in the last decades (e.g., Mason, 1996; Schwartz & Yerushalmy, 1992). One such approach is functions-based, which is one of the central aspects of the IA design. In this approach, functional relationships are introduced in situations and contexts before equation solving, and contextualized functions are used as a way to provide meaning to equations.

This paper focuses on IA students' performance to better understand the extent to which IA supports students' acquisition of key mathematics knowledge and skills in the domain of algebra. Specially, the research questions guiding the report are:

1. Do Intensified Algebra students show significant performance gains in mathematics content typically associated with introductory high school algebra courses?
2. Do Intensified Algebra students show improvement in their ability to solve non- routine algebraic problems?
3. How does IA students' performance vary across six different sub-strands of algebra content?

Algebra as a Gatekeeper Course

Recent changes in federal and state education policy call for a substantial increase in the breadth and depth of mathematical knowledge students must acquire in order to graduate from high school. A growing number of states now require all students to master—on an exit examination—the content of Algebra I and Geometry (Kober et al., 2006), and these requirements are increasing. Currently, over 40 states and the District of Columbia have adopted the Common Core State Standards for Mathematics (CCSSM), which call for high school graduates to complete three years of rigorous mathematics courses, including content typically taught in Algebra II, to ensure they are college- and career-ready (Council of Chief State School Officers, 2010).

One reason for these changes is that success in mathematics has important consequences for students' college and career options. In fact, research suggests courses, such as Algebra I, serve as gatekeepers to more advanced mathematics and can affect mathematics achievement in high school and beyond (Adelman, 2006; Ma & Wilkins, 2007). In addition, the National Educational Longitudinal Study (NELS) indicated students taking rigorous high school mathematics courses are much more likely to go to college than those who do not (Chapman, Laird, Ifill, & KewalRamani, 2011). The NELS data specifically showed 83 percent of students that took Algebra I and Geometry went to college within two years of graduating from high school. This percentage dropped to 36 percent for those who did not take Algebra I and Geometry. Identifying and testing the impact on student learning of programs intended to support underprepared students' success in algebra is a key to aiding students in attainment of subsequent college and career opportunities.

Students' Difficulties in Learning Algebra

Over the past two decades, the learning and teaching of algebra has increasingly become a central component of the mathematics education research agenda (Gutiérrez & Boero, 2006; Stacey, Chick, & Kendal, 2004). As discussed earlier, algebra is often considered a gatekeeper to accessing, and ultimately understanding, more advanced mathematics (National Council of Teachers of Mathematics, 2009; National Mathematics Advisory Panel, 2008; U.S. National Research Council, 2001). Given the importance of algebra in school mathematics, researchers have documented extensively the difficulties students have encountered in learning algebra. For example, research has shown students do not seem to understand the equal sign indicates a relationship between the quantities on both sides of the sign, but rather believe it only represents a unidirectional operator that produces an output on the right side resulting from the input on the left (e.g., Booth, 1984; Kieran, 1981; Vergnaud, 1985).

Other research has demonstrated students often focus on finding particular answers (e.g., Booth, 1984), do not recognize the use of commutative and distributive properties in their work on algebra (e.g., Boulton-Lewis et al., 2001; Demana & Leitzel, 1988; MacGregor, 1996), and do not use mathematical symbols to express relationships among quantities (e.g., Vergnaud, 1985; Wagner, 1981). Still, other researchers have found students often do not comprehend the use of letters as generalized numbers or as variables (e.g., Booth, 1984; Kuchemann, 1981; Vergnaud, 1985), have difficulty operating on unknowns (e.g., N. Bednarz, 2001; Nadine Bednarz & Janvier, 1996; Filloy & Rojano, 1989; Steinberg, Sleeman, & Ktorza, 1990) , and often fail to understand that equivalent transformations on both sides of an equation do not alter its truth value (e.g., N. Bednarz, 2001; Nadine Bednarz & Janvier, 1996; Filloy & Rojano, 1989; Steinberg et al., 1990) . Together, this body of research illustrates that students have difficulty engaging with algebraic concepts typical of Algebra I courses.

IA Design Principles

In response to the high failure rates encountered in algebra nationally, many U.S. school districts adopted the strategy of increasing the amount of instructional time in algebra for underprepared students (i.e., making Algebra 1 a double-period course). Despite the extra time in algebra classes, however, few districts implementing the strategy realized anticipated improvement in 9th grade algebra performance. Moreover, Algebra 1 courses for underprepared students typically consist of watered-down algebra content (Ansalone, 2001; Schmidt, 2008) along with focus on ineffective strategies for promoting remedial mathematics skill development, e.g., reteaching prerequisites as new learning (Finnan & Swanson, 2000; Silver, 1998). The result is, even if students pass these less rigorous Algebra 1 courses, they are ill-prepared for success in subsequent mathematics courses (ACT, 2007; Adelman, 2006). To address these broad challenges, a research and development project was initiated in 2008 to increase the success rates of 9th grade students who are enrolled in Algebra 1 but were significantly underprepared for high school mathematics. The project used a design-based research approach (Bannan-Ritland & Baek, 2008; Barab & Squire, 2004) to develop and study a comprehensive algebra program for students 1-3 years behind based on middle school mathematics achievement measures and/or diagnostic assessments upon entering high school that could be successfully implemented at scale. The resulting program, Intensified Algebra I (IA), is a comprehensive program for students enrolled in double-period algebra classes, used by 38,000 students and 925 teachers in 420 schools in 15 states as of 2014-15. The program blends text-based and technology-based interfaces for students and teachers, and provides teachers with a comprehensive set of professional development supports.

Algebra Core and Functions-based Approach

To counter the remediation focused, pre-algebraic approaches typically enacted in algebra classrooms with underprepared learners, the scope and sequence of Intensified Algebra was developed to be commensurate with the content of a typical, introductory algebra course, and was based on a review of state and national standards and recommendations (e.g., Achieve, 2008; College Board, 2006; National Council of Teachers of Mathematics, 2000; National Mathematics Advisory Panel, 2008). Subsequent revisions to the program reflect the grade-level content of the Common Core State Standards for Mathematics (CCSS-M) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), and other college readiness standards.

In addition to inclusion of on-grade-level-content, the algebra core in Intensified Algebra also emphasizes explicit connections between algebraic procedures and their underlying concepts. The value of students' development of conceptual understanding has long been supported in mathematics education, with particular benefits noted for struggling students in mathematics (Boaler & Humphreys, 2005). Conceptual development is facilitated by drawing students' attention to connections among ideas, facts, and procedures (Hiebert & Grouws, 2007; Kilpatrick, Swafford, & Findell, 2001) as well as their fluency with multiple representations (Brenner et al., 1997; Koedinger & Nathan, 2004). Based on these principles, IA lessons are generally sequenced so students first develop foundational concepts by working with contextual representations early in a unit, and then progress to more abstract, symbolic procedures later in the unit when they can be built upon an established conceptual foundation. This progression is characteristic of a functions-based approach (Yerushalmy & Chazan, 2002), in which ideas about growth patterns, relationships, and qualitative behavior of functions are established prior to performing symbolic manipulations with those functions.

Finally, the types of activities presented characterize the algebra core of the program. Research on mathematical learning has established conceptual understanding develops through high cognitive demand tasks that involve important mathematics, enacted in a way that maintains their level of cognitive demand (Bottge, Heinrichs, Chan, & Serlin, 2001; Henningsen & Stein, 1997; Stein, Grover, & Henningsen, 1996). IA provides instructional tasks and professional support emphasizing the enactment of high cognitive demand tasks in the classroom, and leveraging the extended time for partner-based exploration subsequent connection to formal mathematical ideas.

Architecture of Supports

Research on mathematics learning among underprepared students has highlighted promising areas for providing support for their learning of rigorous mathematics. Substantial evidence indicates struggling learners characteristically experience difficulties with aspects of executive functioning, including memory, attention, and self-regulation (Montague & Applegate, 1993; Swanson & Jerman, 2006; Swanson & Sáez, 2003). Further, these students typically demonstrate a limited repertoire of metacognitive strategies and low motivation (Montague, 2007). They also demonstrate difficulty organizing and interpreting information and complex processes (Swanson & Deshler, 2003; Swanson & Hoskyn, 2001). Metacognitive and cognitive strategies support students becoming self-directed learners promote students' problem solving capabilities and mathematics achievement (Bransford, Brown, & Cocking, 2000; Fuson, Kalchman, & Bransford, 2005).

In addition, students' beliefs about their intelligence—as a fixed trait (fixed mindset) or one that can grow over time (malleable mindset)—influence their motivation to engage in academic tasks, and consequently, their success, especially in challenging subjects (Dweck, 2007; Good & Dweck, 2006). Malleable-mindset interventions, which explicitly teach students about the brain, its functions, and intellectual development is the result of effort and learning, have increased student achievement in middle school mathematics (Aronson, Fried, & Good, 2002; Blackwell, Trzesniewski, & Dweck, 2007; Good, Aronson, & Inzlicht, 2003).

Based on these identified areas of need, the IA design integrates several foundational support structures. To help students organize and self-monitor their learning, the program integrates well-defined, daily learning routines, as well as a set of graphic organizers that make students' mathematical thinking visible (Swanson & Deshler, 2003; Swanson & Hoskyn, 2001). To address students' misconceptions and provide just-in-time opportunities for review and practice (Caple, 1996; Rittle-Johnson, Siegler, & Alibali, 2001; Rohrer & Taylor, 2007), daily, timely, spaced practice is provided specifically addressing skills needed for new learning. Teacher supports and worked examples also provide ways to address common errors and misconceptions as they arise, rather than through the wholesale reteaching of previously learned content. To increase students' abilities to make conceptual connections and engage with difficult content, visual animations and representations are integrated into daily instruction. These are aimed specifically at connecting various representations for the foundation of the functions-based approach.

Finally, a set of social-motivational tools have been integrated into the program, based on research about students' beliefs about themselves, and its role in academic learning and motivation (Aronson, Fried, & Good, 2002; Blackwell, Trzesniewski, & Dweck, 2007; Dweck, 2007; Fuson, Kalchman, & Bransford, 2005; Good & Dweck, 2006. Building from this research, the Dana Center and Agile Mind developed the Academic Youth Development (AYD) program, for entering ninth graders to support students' aspirations for high achievement by teaching them theories of malleable intelligence, strategies for goal setting and effective effort, how to learn, and effective communication strategies and applying them to mathematics. Collectively, these AYD materials have been adapted and strategically incorporated into IA's scope and sequence to directly address issues of motivation and mindset associated with learning disabilities students and underprepared learners.

Methods

Participants

A total of 153 students of diverse background from 28 urban schools across the United States took part in the IA course during the 2011-2012 academic year. Majority of the schools were located in Illinois and Texas and most of the students were in the ninth-grade. Parents' consent and students' assent rates were low in participating schools, which, in turn, resulted in a relatively small sample. Low rates were due mostly because of non-returned parental consent forms despite multiple attempts by the research team. For analysis purposes, the sample size is reduced to 78 students who were present during the multiple administrations of each of the two assessments.

Instruments

Multiple-choice assessment. The multiple-choice assessment was designed to measure students' proficiency with concepts and skills of the Algebra I course. Specifically, it was modeled after the Acuity™ Algebra Proficiency Test[1] because Acuity™ is a widely used comprehensive instrument for Algebra I concepts and skills. The Acuity™ Algebra Proficiency Test was developed and published by CTB/McGraw-Hill in 2007 and consists of 32-items intended for students who have completed Algebra I.

To construct two parallel forms of the IA multiple-choice assessment, we first surveyed all released National Assessment of Educational Progress (NAEP) items matching the math

[1]AgileAssessment is a bank of formative assessment items developed by our IA technology partner, Agile Mind, that are related to their other middle and high school mathematics courses.

concept and/or skill represented in a particular Acuity™ item. If no comparable item was found in this item bank we selected items from the AgileMind Assessments item bank. Two researchers constructed the tests. For each Acuity™ item, one researcher proposed an initial selection after which the other researcher accepted or rejected the selection. If the item was rejected an alternative was suggested. This iterative process continued until both researchers agreed on each item for both forms/versions of the test. Two sample problems are shown in Figure 1.

Sample Problem 1

A plumber charges customers $48 for each hour worked, plus an additional $9 for travel. If h represents the number of hours worked, which of the expressions could be used to calculate the plumber's total charge in dollars? *(Students select the correct answer from five different choices)*

Sample Problem 2

x	y
0	-1
1	2
2	5
3	8
10	29

Which of the following represents the relationship between x and y shown in the table above? *(Students select the correct answer from five different choices)*

Figure 1. Sample Items in the IA Multiple Choice Assessment.

In addition to measuring change in students' overall performance according to the multiple choice assessment, we also wanted to investigate the extent to which there were specific areas within the content of Algebra I that showed greater change from pre- to post- assessment. To do so we grouped items according to the following content areas: Variables, Expressions, Formulas (VEF), Solving Linear Equations (SLE), Graphing Linear Equations (GLE), Functions and Graphs (FG), Quadratic Equations and Functions (QEF), and Geometry (G).

Constructed response assessment. The constructed response assessment was designed to provide a measure of students' understanding of a variety of concepts and skills central to Algebra I. To design the constructed response IA assessment, we first surveyed tasks from the MARS project (http://map.mathshell.org/materials/index.php) through the Silicon Valley Mathematics Initiative (http://www.svmimac.org/). Two researchers constructed the two parallel forms of the assessment. For each form, one researcher proposed an initial selection after which the other researcher accepted or rejected the selection. If a task was rejected an alternative was suggested. This iterative process continued until both researchers agreed on each task for both forms of the assessment.

Table 1.
Selected Tasks and Their Corresponding Skills

Form	Task Name	Skills and Concepts
C	Toy Trains	Find and use a number pattern Find an algebraic expression for a number pattern
	Lawn Mowing	Solve a practical problem involving ratios Use proportional reasoning
	Going to Town	Interpret and complete a distance/time graph for a described situation
	Vacations	Analyze relationships using graphs and algebra
	Picking Apples	Work out costs from given rules
D	Patchwork Quilt	Recognize and extend a number pattern Express a rule using algebra
	Photographs	Use proportion in a real life geometric context
	Bike Ride	Interpret a distance/time graph
	Shelves	Solve problems in a spatial context Identify and distinguish the four point graphs related to this situation
	Buying Chips and Candy	Form and solve a pair of linear equations in a practical situation

Selected tasks and skills were intended to be measured are included in Table 1. Specifically, two tasks (i.e., Toy Trains and Patchwork Quilt), one in each form of the assessment, were intended to measure students' skills and knowledge about linear functions. Each of the constructed response problems has several questions or prompts organized in sub-parts.

Data Collection

The two versions of the multiple-choice assessment and of the constructed response assessment (i.e., Form C and Form D) were administered at the beginning (pretest) and end (posttest) of the 2011-2012 school year. There were 33 multiple-choice items on each version of the multiple-choice test.[2] There were five constructed response problems on each version of the constructed response assessment, each consisting of multiple parts. Students were randomly given a version of the pretest, and then given the alternate version for the posttest. For purposes of this study, we analyzed data from students who took Form C for the pretest and Form D for the posttest and vice versa excluding students who took the same form as both pre and posttests. This reduced the total number of students for analysis purposes to 153 on the multiple-choice assessment of which 87 took Form C for the pretest and Form D for the posttest and 66 took Form D for the pretest and Form C for the posttest. On the constructed response assessment, the same procedure reduced the total sample to 128, 72 who went from Form C to Form D, pre to post, and 56 students who went from Form D to Form C, pre to posttest.

The pretest administration occurred at the beginning of the fall semester, before much instruction on the algebra content had occurred. The posttest was administered at the end of the school year when most of the instruction was complete.

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[2] The original test had 32 items, after reviewing pilot data, one item was given two question prompts.

Reliability of Instruments

Reliability of the multiple choice and constructed response assessments was determined using Cronbach's alpha to measure internal consistency. Table 2 shows the Cronbach's alpha for the assessments varied from .48 to .80 with lower reliability at pretest than posttest. This is expected given the small sample as well unfamiliarity with some of the specific content prior to actual instruction. However, the reliabilities at posttest for both forms are acceptable and in the range reported for the Acuity instrument following instruction in Algebra I.

Table 2.
Reliability Analysis of Multiple Choice Assessment

Assessment	No. of Items	Mean	N	Std. Deviation	Cronbach's α
Pretest Form C	33	0.3176	87	0.1139	0.48
Pretest Form D	33	0.3512	66	0.1345	0.62
Posttest Form C	33	0.4600	66	0.1764	0.8
Posttest Form D	33	0.4482	87	0.1579	0.72

Table 3 shows the Cronbach's alpha for the constructed response assessments varied from .83 to .90, suggesting students were consistent in their performance on Form C and Form D on both the pretest and posttest.

Table 3.
Reliability Analysis of Constructed Response Assessment

Assessment	No. of Items	Mean	N	Std. Deviation	Cronbach's α
Pretest Form C	44	0.3730	72	0.1732	0.87
Pretest Form D	37	0.3797	56	0.1777	0.86
Posttest Form C	44	0.4944	56	0.2005	0.90
Posttest Form D	37	0.5351	72	0.1849	0.83

Results

Overall Student Performance

As mentioned before, due to the difference in sample sizes between the group of students who took the multiple choice assessment at both pretest and posttest (i.e., 153) and the group who took the constructed response assessment at both testing times (i.e., 128), as well as variation in the overlap between these groups, all further analyses are performed using the intersection of the two samples (all students who took both the multiple choice and the constructed response assessment at both the pretest and the posttest). This reduces the total sample size to 78 for purposes of analysis.

Multiple Choice Assessment

A repeated measures ANOVA was used to assess student performance from pretest to posttest and to assess differences between assessment versions and content areas. Results from the ANOVA were used to compute effect size, both classic eta squared ($\eta 2$) and partial eta squared ($\eta p2$). To briefly summarize, classic eta squared is a proportion of the total variation explained by the factor, and partial eta squared is a proportion of the explained variance, excluding variances produced by other factors outside of the analysis. Following the recommendations of Pierce, Block, and Aguinis (2004) and Levine and Hullett (2002), both are reported as each follows a different index when determining the strength of association. Forty-seven students took version C for the pretest and version D for the posttest, while 31 students took version D for the pretest and version C for the posttest. There was not a statistically significant main effect for version ($F[1,76] = 1.591$, $p > .05$). In addition, there were no statistically significant interactions found between time and version ($F[1,76] = .252$, $p > .05$), between content area and version ($F[5,72] = 2.210$, $p > .05$), and between time, content area, and version ($F[5,72] = 1.984$, $p > .05$). This outcome suggests no difference in student performance exists between versions for the pretest or posttest, overall or by content area. As such, version has been removed from the subsequent analyses.

Overall student performance at pretest was found to be significantly different from posttest ($F[1,77] = 31.780$, $p < .001$, $\eta 2 = 0.0553$, $\eta p2 = 0.292$), with students performing significantly better on the posttest. As seen in Table 4, students averaged 31.15% correct on the pretest with a standard deviation of 11.08%, and 44.56% correct on the posttest with a standard deviation of 16.82%, showing an average gain of 13.41% over the intervention period. Effect sizes of pretest to posttest were very large, accounting for 5.53% of the total variation in student performance and 29.2% of the variation excluding factors not accounted for in the analysis. Of the 77 students in the sample, 70.13% improved from pretest to posttest on the multiple-choice assessment.

Table 4
Relative Improvement Results

Test	Mean	Standard Deviation
Pretest	0.3115	0.1108
Posttest	0.4456	0.1682

Figures 2 and 3 show the distribution of the students' relative scores on the pretest and posttest.

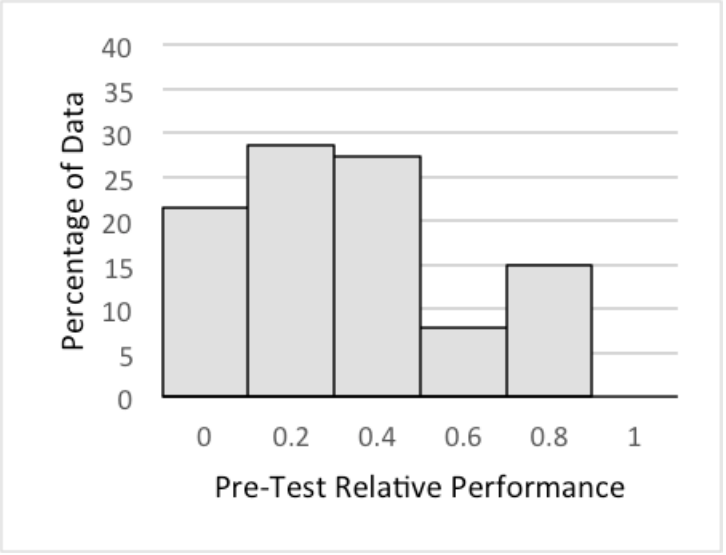

Figure 2. Distribution of Pretest Scores

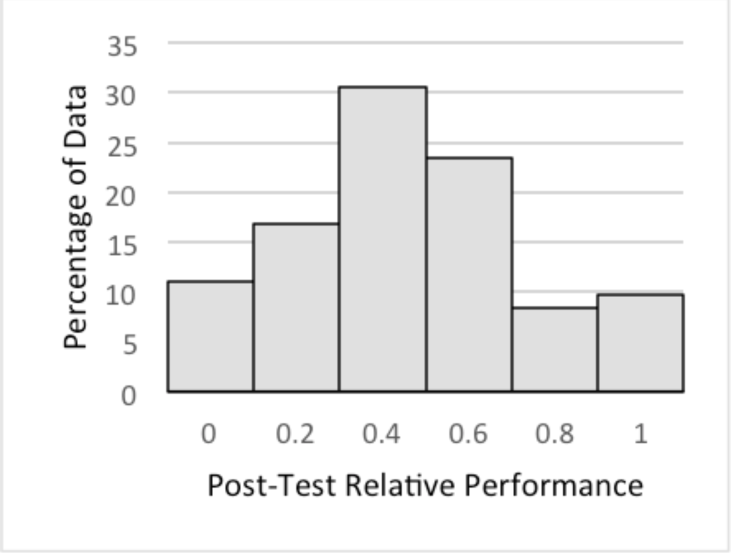

Figure 3. Distribution of Posttest Scores

As mentioned previously, the multiple choice items were categorized by content area, resulting in six domains: four items as Variables, Expressions, and Functions (VEF); eleven items as Solving Linear Equations (SLE); four items as Graphing Linear Equations (GLE); eight items as Functions and Graphs (FG); four items as Quadratic Equations and Functions (QEF); and two items as Geometry (G). Differences in student performance among these categories were found to be statistically significant ($F[3.841,295.758] = 13.696$, $p < .001$, $\eta 2 = 0.0669$, $\eta p2 = 0.151$). For both this main effect and the interaction between category and test, Maulchy's Test of Sphericity was violated ($X2 = 57.878$, $df = 14$, $p < .001$). As such, results from the Greenhouse-Geisser correction were used. Students' difference in performance between content areas had a medium effect size ($\eta 2 = 0.0669$), and the results suggest content area accounts for 15.1% of the variance in students' performance excluding other factors. Table 5 shows mean performance by content area for the pretest and posttest. Students performed highest on Solving Linear Equations, Graphing Linear Equations, and Functions & Graphs, averaging 50.3%, 46.2%, and 51.9% respectively.

Table 5
Descriptive statistics by content area and testing period

	Pre		Post				
	Mean	S.D.	Mean	S.D.	N	F[1,77]	Sig.
VEF	0.324	0.239	0.343	0.288	78	.231	.632
SLE	0.373	0.163	0.503	0.215	78	18.179	.0001
GLE	0.269	0.227	0.462	0.261	78	24.706	.0001
FG	0.298	0.188	0.519	0.258	78	47.978	.0001
QEF	0.250	0.221	0.304	0.244	78	2.109	.151
G	0.250	0.288	0.301	0.283	78	1.398	.241

The interaction between content area and performance at pretest and posttest was significant ($F[4.335,333.820] = 6.725$, $p < .001$, $\eta 2 = 0.0264$, $\eta p2 = 0.069$—results from Greenhouse-Geisser correction reported) indicating that student

performance changes from pretest to posttest differed depending on the content area. The interaction between content area and change from pre- to post- had a small effect size ($\eta p2 = 0.0264$), and the results suggest content areas account for 6.9% of the variance in students' performance excluding other factors. Figure 2 shows students' performance changes from pretest to posttest by content area. Three categories show a significant improvement from pre to post:
GLE ($F[1,77] = 18.179$, $p < .001$, $\eta 2 = 0.049$, $\eta p2 = 0.243$), FG ($F[1,77] = 24.706$, $p < .001$, $\eta 2 = 0.065$, $\eta p2 = 0.384$), and SLE ($F[1,77] = 47.978$, $p < .001$, $\eta 2 = 0.023$, $\eta p2 = 0.191$).

Student performance on Graphing Linear Equations had a medium effect size ($\eta 2 = 0.049$), and the domain accounts for 24.3% of the variance in students' performance excluding other factors. Student performance on Functions and Graphs had a mediun effect size ($\eta 2 = 0.065$), and the domain accounts for 38.4% of the variance in students' performance excluding other factors. Last, student performance on Solving Linear Equations had a small effect size ($\eta 2 = 0.023$), and the domain accounts for 19.1% of the variance in students' performance excluding other factors. Two categories show a small improvement from pre- to post- that was not significant: QEF and G. And finally, one category shows almost no change: VEF. Thus, the IA program had the most impact on the Graphing Linear Equations, Functions and Graphs, and Solving Linear Equations content areas. These are central to any curriculum in Algebra I, in general, and IA in particular. We expected to find a significant change in performance for the Variables, Equations and Formulas content area but failed to do observe such an outcome.

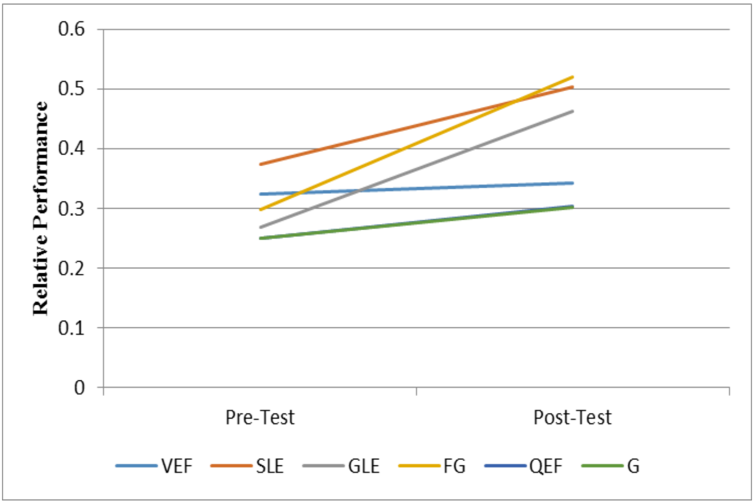

Figure 4. Performance by Category and Testing Period

Constructed Response Assessment

A repeated measures ANOVA was used to assess student performance at pretest and posttest and to assess differences between instrument versions on the constructed response assessment. No significant main effect was found for version ($F[1,76] = .190$, $p > .05$). Further, no significant interaction was found between time and version ($F[1,76] = .102$, $p > .05$). This outcome suggests no difference exists between student performance on the pretest or posttest between versions. As such, version has been removed from the subsequent analyses.

Overall the change in student performance from pretest to posttest was found to be significant ($F[1,77] = 43.346$, $p < .001$, $\eta p2 = 0.360$) with students performing significantly better on the posttest. The change over time from pre- to post- had a very large effect size ($\eta p2 = 0.360$). Of note, only partial eta squared is reported, as the analysis included one within-subject effect, making eta squared and partial eta squared identical. As seen in Table 6, students averaged 34.48% correct on the pretest with a standard deviation of 17.50% and 50.85% on the posttest with a standard deviation of 18.62%, showing a 16.37% improvement over the intervention period. Of the 77 students in the sample, 84.42% improved from pretest to posttest on the constructed-response assessment.

Table 6.
Constructed Response Performance

Test	Mean	Standard Deviation
Pretest	0.3448	0.1750
Posttest	0.5085	0.1862

Due to the nature of the constructed response assessment most of the tasks were too complex conceptually to categorize by a single content area. Furthermore, only one task on each version assessed Solving Linear Equations (SLE): The Toy Trains task on version C and the Patchwork Quilt task on version D. A repeated measures ANOVA was performed on this subset of tasks to examine differences between the two versions as well as growth over the intervention. Results indicated that while the students' improvement from pretest to posttest on the constructed response task on Solving Linear Equations was significant ($F[1,76] = 11.061$, $p = .001$), the interaction between performance pre to posttest and the version was also significant ($F[1, 76] = 26.449$, $p = .000$). One version was easier for the students than the other; in particular, students performed significantly better on the Toy Trains task on version C than on the Patchwork Quilt task on version D, whether on the pretest or posttest. As such, performance results at the task level for assessing content area change are not meaningful.

Implications

This study provides supporting evidence regarding the potential positive impact of adopting a comprehensive curriculum program for double-period algebra in its early stages of implementation. The program is specifically designed to support underprepared students' learning of rigorous algebra. Students who are enrolled in double-period algebra typically have experienced a persistent lack of success in school mathematics in their years leading up to 9th grade, and as a result may often see mathematics—particularly introductory algebra—as an endeavor neither worthwhile nor holding the possibility of success. With the implementation of double-period algebra policies, teachers face the daunting challenge of helping students regain ground in mathematics, while managing longer class periods with students who may no longer be invested in the enterprise.

While results from this study are from a relatively early iteration of IA's development/implementation cycle and are generated from a relatively small pilot program, the results demonstrate that students who have experienced persistent difficulty in learning mathematics are able to make significant gains with on-grade-level content in IA classrooms. These gains are evident in their performance both on multiple-choice items emphasizing discreet algebraic skills and procedures, and on constructed-response items emphasizing solving of open-ended, non-routine algebraic problems. As an ongoing design-based research project, IA continues through iterative cycles of implementation, revision, and testing. The results of this study therefore provide a baseline to compare against measures of student learning from future IA implementations that incorporate ongoing adjustments to the program, varied contexts of implementation, and potentially larger implementation samples.

Beyond the overall assessment results, the differences in student performance across specific sub-strands of content suggest students in IA classrooms may be learning more content in some domains compared with others. In particular, the post-hoc analyses of content-area performance on the multiple-choice instruments show significant pre to posttest gains in mean scores for the domains of Solving Linear Equations, Graphing Linear Equations, and Functions and Graphs in the multiple-choice assessment. Notably, extensive opportunities to learn in these three domains are provided throughout the scope and sequence of the IA program, indicating students' performance in these areas may benefit from this programmatic emphasis. Conversely and not unexpectedly, statistically significant gains in mean scores were not achieved in the Geometry domain, where far fewer opportunities to learn are provided, based on the algebraic focus of the program.

Further consideration, however, needs to be given to the algebra-related domains of Variables, Equations, and Functions (VEF) and Quadratic Equations and Functions (QEF). In both of these domains, students did not demonstrate significant changes from pre to posttest in the post-hoc analyses as expected. One possible explanation to be explored involves the positioning and treatment of these sub-strands within the IA scope and sequence. The VEF domain, for example, is emphasized early in the program, but it is unclear the degree to which opportunities to re-visit content in this domain are offered to students throughout the curriculum. In a similar vein, the QEF domain is addressed in the latter units of the program, where pacing issues often preclude implementation before the end of the academic year, particularly in the early years of program adoption. A targeted exploration of how instruction is implemented across particular content areas in IA, which focuses on both descriptive and quantitative accounts of opportunities to learn for students, may provide further understanding of differences of learning across algebraic content domains.

This direction of research could potentially be extended to contribute to the field more generally as well. By systematically tracing students' learning throughout the IA course to empirically validate or modify underlying learning trajectories for a functions approach to Algebra I, ongoing research of IA implementations could contribute to the development of more effective resources and tools for both instruction and assessment, particularly in the algebraic domains. A report from the Center for Policy and Research in Education (Daro, Mosher & Corcoran, 2011) highlighted the lack of learning trajectories for high school mathematical concepts, and has called for studies constructing and empirically validating such learning trajectories. Such research could contribute to filling this empirical gap.

In the broader research context, the learning outcomes demonstrated in this study add to a growing understanding in the field about the algebraic learning of underprepared students in double-period contexts. As implementations of double-period policies continue to be adopted by school districts, IA and other programs designed for students participating in extended models of instruction (e.g., Dubinsky & Wilson, 2013) will continue to provide a test-bed for studying the teaching and learning of rigorous algebra for underprepared learners in these contexts. Furthermore, while this study provides evidence underprepared students benefit from participating in a single year of double-period Algebra I instruction with IA, it does not provide a comparison to any other double-period algebra interventions currently being implemented in schools. In particular, further research is needed to address how performance of underprepared students in IA compares to similar students in classrooms where other programs/models are being implemented.[3] Beyond comparisons of standardized performance measures, any such exploration should also include a more nuanced investigation into what particular mathematics students learn in the context of each intervention, how students and teachers engage with the opportunities to learn provided, what affordances and benefits to students and teachers each approach provides, and under what contexts and conditions.

Finally, research on how to support underprepared ninth graders' success in Algebra I has increased importance as states and districts implement the Common Core State Standards for Mathematics (CCSS-M) (CSSO, 2010). The CCSS-M make major advances in K-12 mathematics curricula in a number of ways, one of which is including the study of linear functions and equations as core content of Grade 8 mathematics rather than as content for Algebra I. The study of linear functions has

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[3] E.g., See Transition to Advanced Mathematics (Neild, Byrnes & Sweet, 2011), a program developed at Johns Hopkins University, organized with a first semester of pre-algebra followed by a second semester of algebra; or the Talent Development Program developed at Johns Hopkins University; or the Transition to Algebra program, modeled as an additional support course to accompany a standard Algebra I course (Education Development Center, 2012).

historically been addressed as part of the introductory, 9th-grade algebra curriculum, with typically only a subset of students encountering the topic in an 8th-grade algebra course. The CCSS-M has shifted much of the introductory algebra content, including linear functions, into the standards for Grade 8. This shift is likely to result in more students being underprepared to learn linear functions (i.e., students who were underprepared to learn algebra in 9th grade will be more so in 8th grade, and likely in increasing numbers). Thus, learning how to better support underprepared students to understand linear functions and equations is a critical issue for CCSS-M implementation, as well as for mathematics education in general.

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