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Are the K–2 Common Core State Standards for Mathematics Developmentally Appropriate?

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In this article, I (a) illustrate how the K–2 CCSSM reflect the major findings from research studies carried out over the last 30 years on early mathematical abilities that indicate these standards are developmentally appropriate for young children, and (b) offer insights into some types of instructional strategies (e.g., student-centered approaches, assessment methods) that teachers can use to implement the standards to foster young children's mathematical abilities and dispositions without detriment to children's natural development. I conclude that the K–2 CCSSM can be used as a tool to understand children's natural ways of thinking and encourage innovative learning and teaching in school settings. The K–2 CCSSM also provide a common referent for early mathematics educators to discuss how to improve early mathematics education.

Although the *Common Core State Standards for Mathematics* (CCSSM; National Governors Associate Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) have received significant support from mathematics educators, policy makers, and government agencies, they have also been criticized for certain sequencing and instructional decisions being made. Some parents, school administrators and early childhood educators in particular, have asked whether or not the K–2 standards are developmentally appropriate for children at these grade levels (e.g., Carlsson-Paige, as cited in Khrais, 2014; Moore, 2014; Strauss, 2013, 2014). In this article, I address this question by examining

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some potential benefits of the K–2 CCSSM for improving early childhood mathematics education. Specifically, I aim to convey the view that the CCSSM can be a useful tool for teachers rather than being a constraint. I work from the position that the K–2 CCSSM provide a common referent for early childhood educators to understand the mathematical knowledge and skills that young children have developed and are able to learn at specific ages.

The first section of this article synthesizes some of the major concerns that have been voiced about the lack of appropriateness of the K–2 CCSSM for young children. The second section illustrates how the K–2 CCSSM reflect some of the more important findings from studies carried out over the last several decades on early mathematical competencies and how these standards are developmentally appropriate for children between four and eight years of age. The third section offers insights into the types of instructional approaches and assessment methods that teachers could use, and are already implementing, to foster children's mathematical abilities. I argue that the CCSSM can serve as a useful tool to encourage and cultivate innovative teaching and learning in school settings and provide a common language for a country to discuss how to enhance early childhood education. I also point out how teachers' transition to the CCSSM implies challenges. Successful implementation of standards demands public support, and to achieve this support both widespread understanding of the CCSSM and coordinated participation of multiple stakeholders are necessary (Kilpatrick, 1997). This article contributes to this public discussion by providing insights into early mathematical development and effective instructional strategies for young learners. These insights can help parents, early childhood educators, and school administrators understand how the implementation of the K–2 CCSSM is beneficial for improving young children's education.

Recent Critiques of the K–2 CCSSM

Despite the wide acceptance of the CCSSM, concerns have been raised about the implementation of the K–2 mathematics standards at these grade levels. Overall, critics claim that the K–2 CCSSM are not developmentally appropriate for students from kindergarten through second grade. Two main critiques related to *appropriateness* have emerged since the standards were adopted in several states: (a) young children's lack of readiness for the CCSSM, and (b) irrelevance of instructional practices according to the CCSSM. In this section, I summarize these critiques to contextualize the relationship between the CCSSM and early childhood education discussed in the following sections.

Young Children's Lack of Readiness for the CCSSM

Opponents of the CCSSM argue that children between 4 and 8 years of age lack readiness to meet the K–2 mathematical standards at these grades, and that there is no evidence to support the fact that learning mathematics at early ages would lead to later success in school (Carlsson-Paige, as cited in Khrais, 2014; Strauss, 2013, 2014). For instance, Moore (2014) claims that the K–2 CCSSM were created from the top down. According to this criticism, the K–2 mathematics standards are not grounded on research studies about young children's capabilities and only consider the skills children should have at the end of the school process. Thus, young children are expected to learn skills they are not ready for.

These criticisms are mainly focused on Kindergarten CCSSM. The most debated standard is KCC1, the first of the Kindergarten *Counting and Cardinality* standards: "Count to 100 by ones and by tens" (NGA & CCSSO, 2010, p. 11). Critics contend that kindergarteners may not be ready to meet this standard, and claim it encourages memorizing words instead of understanding numbers (Carlsson-Paige, as cited in Khrais, 2014; Strauss, 2014). Interestingly, opponents do not mention other standards in this way.

Irrelevance of CCSSM-based Instructional Practices

The second critique is that the K–2 CCSSM have led to the implementation of instructional practices that are not relevant to the developmental needs of children in grades K–2. Critics claim that the standards counter research findings on the best approaches to teach early-grade children such as play-based activities, hands-on exploration, and problem-solving situations (e.g., Strauss, 2013, 2014). According to this criticism, these approaches have been forced out, giving way to direct, traditional instruction based on drill and practice methods. In turn, critics argue, drill and practice diminish students' capacity for creative thinking (e.g., Moore, 2014). Critics also affirm that the use of the K–2 CCSSM has crowded out children's precious time, leaving little room for social, emotional, and artistic development (e.g., Moore, 2014).

One significant concern raised is that implementing the K–2 CCSSM has led to the use of multiple-choice, standardized testing in early grades, and that these tests are harmful for young children's development (e.g., Moore, 2014; Strauss, 2013, 2014). Critics claim that testing does not fit into early childhood education because standardized test indicators do not measure how young children learn and not all children develop at the same time (e.g., Moore, 2014). These critics suggest that *observation* and *listening to* are suitable methods to assess young children's learning. Overall, critics do not cite specific pages where the CCSSM mandates instructional methods such as drill and practice and standardized tests for K–2 grade levels.

CCSSM as a Tool to Understand Young Children's Mathematical Abilities

Claims regarding young children's lack of readiness to meet the K–2 CCSSM are erroneous, groundless beliefs. Since the early 1980s developmental and educational research has shown that when school life begins, at about age 4, children have already developed a wide variety of mathematical skills through their daily encounters with mathematics. For example, babies are able to discriminate, add and subtract small quantities (Wynn, 1995), and toddlers can understand the principles for enumerating objects (Gelman & Meck, 1983). Similarly, preschoolers are able to discover portions of the verbal sequence of counting by themselves (Fuson, 1988; Ginsburg, 2014), spontaneously display a range of sophisticated strategies for addition and subtraction problems (Baroody & Dowker, 2003; Fuson, 1988, 1992; Nunes & Bryant, 1996), identify and describe shapes such as triangles, circles and squares (Clements & Sarama, 2000) and read simple maps using distances and angles (Huttenlocher, Newcombe & Vasilyeva, 1999; Spelke, Gilmore & McCarthy, 2011). Ginsburg (2014) states: "A living, everyday form of math exists in the lives and minds of children from the ages of 2 to 4 before the onset of formal schooling" (p. 53). The problem according to Ginsburg, is that when children enter school their experience with mathematics becomes poor or their teachers do not bring mathematics into the classroom.

The K–2 CCSSM were based on three important findings about young children´s mathematical learning that are synthesized in Clements and Sarama (2014). First, the mathematics that children learn in early years predicts mathematical achievement in later years; moreover, early mathematics skills also predict later success in literacy (Clements & Sarama, 2014; Sarama & Clements, 2009). For instance, Duncan et al. (2007) found that school-entry skills, such as early mathematics and reading, were statistically significant predictors of later mathematics and reading achievement. In addition, the results showed that "early math skills have the greatest predictive power" (Duncan et al., 2007, p. 1428). Sarama, Lange, Clements and Wolfe (2012) also demonstrated that participation in the pre-kindergarten mathematics curriculum program, Building Blocks, significantly impacted children's mathematics achievement and oral language competencies such as the ability to recall and use relevant words from a story, construct complex sentences when retelling a story, retell a story independently of prompts and, make inferences about aspects of a story. Moreover, Uttal et al. (2013) revealed that promoting early spatial thinking improved skills required to enter STEM careers (see also Newcombe, 2010). All these studies have shown that fostering early experiences with mathematics is crucial for later success in literacy, mathematics and other STEM disciplines.

The second finding is that young children have the potential to learn abstract, challenging and interesting mathematics by using their own ways of reasoning and strategizing if they are nurtured through developmentally appropriate instructional activities (Balfanz, Ginsburg & Greens, 2003; Clements & Sarama, 2014; Ginsburg, 2014; Sarama & Clements, 2009). Researchers have stressed that *early mathematics* does not mean a push-down curriculum from higher grades (Balfanz, Ginsburg & Greens, 2003; Sarama & Clements, 2009). Early mathematics means that young children are allowed to use their prior knowledge and informal methods to make sense of their mathematical world. It implies that children can access different mathematical concepts informally from an early age if they are encouraged to use their own forms of reasoning. These forms of reasoning may include observation, metaphors and analogies, discovery of patterns and relationships, visualization and imagery, and the creation of informal strategies to understand and solve mathematical problems. For instance, Fuson, Smith, and Lo Cicero (1997) found that first-grade children initially made groups of 16 objects to represent the numeral 16 or drew 24 sticks to represent the numeral 24; however, after engaging in activities of addition and subtraction of two-digit numbers by regrouping with objects (e.g., ten-sticks) and drawings, they were able to count by tens and ones to add and subtract these numbers. Through the use of manipulatives, the young learners developed an abstract informal idea of place value that was more powerful than counting only by ones.

In the field of early geometry, Sinclair and collaborators showed using digital interactive technologies that facilitate visual-dynamic transformation of geometrical objects allowed kindergarten and first-grade children to understand the concept of triangle (Sinclair & Moss, 2012) and intuitively discover complex mathematical relationships such as parallelism (Sinclair, de Freitas & Ferrara, 2013) and symmetry (Ng &

Sinclair, 2015). Children also used informal forms of probing (Sinclair, Moss & Jones, 2010). Moreover, second-grade children can understand informal ideas of congruence and similarity through both the visualization of multiple dynamic representations and the coordination of different communication modes such as talking, gesturing, and touching, while working with digital multimodal technologies (Otálora, 2016a, 2016b). Research on early algebra further supports these ideas about young children's abilities (Blanton & Kaput, 2011; Brizuela, Blanton, Sawrey, Newman-Owens & Gardiner, 2015; Schliemann, Carraher & Brizuela, 2007). For instance, Brizuela et al. (2015) found that 6-year-olds were able to use variable notations to represent algebraic ideas within problem-solving situations that were developmentally appropriate for young learners. Schliemann et al. (2007) note that early algebra activities play an important role "in expanding students' mathematical reasoning and in helping them develop and use algebra notations and tools to solve problems" (p. 145). Overall, these studies suggest that educators can foster early mathematical development by implementing challenging learning environments that enable young children to use their own ways of reasoning and existing knowledge meaningfully.

The third finding is that young children follow particular paths of learning for different mathematical domains (e.g., number, geometry, algebraic thinking, and measurement). These paths, or *mathematical learning trajectories* (MLTs; Clements & Sarama, 2004, 2014; Confrey, Maloney & Corley, 2014; Daro, Mosher & Corcoran, 2011) can be defined as:

> Descriptions of children's thinking and learning in a specific mathematical domain, and a related conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain. (Clements & Sarama, 2004, p. 83)

According to this definition, a learning trajectory is a natural developmental progression in a specific area of knowledge (e.g., geometry). This trajectory includes a description of children’s levels of thinking and understanding over long periods of time and across series of instructional activities previously created to promote the move from one level to another more sophisticated level (Clements & Sarama, 2004, 2014; Confrey et al., 2014). Descriptions of MLTs are based on prior research on young children’s mathematical competencies; in turn, 18 MLTs informed the construction of the K–8 CCSSM (see Confrey et al., 2014; Daro et al., 2011). Specifically, Clements and Sarama (2014) describe 10 MLTs related to early mathematics education.

Contrary to critics’ claims about lack of readiness, these three key findings indicate that the mathematics children learn in their early years will positively impact their future learning. Furthermore, children enter school ready to learn challenging, important and interesting mathematics because they are already on a developmental path (Gelman & Brenneman, 2004) and have the capacity to construct new knowledge related to their prior experiences. These findings also support the idea that using learning trajectories to understand how standards are organized may help educators guide instruction based on the K–2 CCSSM. Sarama and Clements (2009) explain that MLTs are a “useful tool” (p. 63) for teachers to understand and support the development of children's mathematical reasoning: “When teachers understand these paths and offer activities based on children's progress along them, they build math learning environments that are developmentally appropriate and particularly effective” (p. 63). The CCSSM is the operationalization of the MLTs approach into curriculum design. Teachers could use these MLTs, organized into strands of standards, as a tool to understand where their students are, what learning could follow, and implement instructional practices accordingly.

Appropriateness of Counting and Cardinality Standards

As mentioned in the first section, opponents of the CCSSM claim kindergarteners are not ready to meet the Counting and Cardinality standard KCC1 requiring them to count to 100 by ones and tens, and that this standard is focused solely on vocabulary learning. However, critics do not cite evidence from research to support this claim and do not refer to other specific K–2 CCSSM standards. Moreover, they do not emphasize the relationship between the KCC1 standard and the other standards in the Counting and Cardinality strand. Because the sets of standards were crafted as components of MLTs, it is important to refer to all the standards of the strand (e.g., Counting and Cardinality), both within and across grade levels. For instance, KCC1 through KCC3 are focused on the verbal and written sequence of numerals, whereas KCC4 through KCC7 are focused on understanding the abstract principles of counting and comparing quantities. Thus, the KCC4–KCC7 set of standards, that focus on numerical understanding, complements the KCC1–KCC3 set related to learning the verbal and written number system.

Prior research has shown that young children are ready to meet the Counting and Cardinality standards, and this learning requires more than memorization of vocabulary. Regarding the KCC1 and the KCC2 standards, developmental psychologists have found that babies between 5 and 7 months are able to recognize and distinguish among small quantities (e.g., Starkey, Spelke & Gelman, 1983; Wynn, 1995), and between 7 and 9 months they can recognize addition and subtraction operations with both small quantities (e.g., Wynn, 1995) and large quantities (e.g., McCrink & Wynn, 2004). These findings show how babies develop quantitative reasoning that can support further development of counting processes.

Furthermore, during their first years of age children have numerous daily experiences with quantities and number words that allow them to develop counting skills without instruction. For example, in feeding situations, a mother gives spoons of compote to her baby while counting, “one, two, three…”; in party situations, 2 to 4-year-old children share with friends an equal number of cookies or count on the same number of cupcakes and refreshments with help from their parents. Young children have similar experiences relating quantities to written numbers; for instance, they interpret numbers on calendar sheets and enjoy following numbers of the floors in an elevator (Ginsburg, 2014). Ginsburg argues that "spoken and written number words permeate children's everyday world" (p. 56). Early childhood researchers have shown that around 2 years of age children can recognize and name small quantities (e.g., saying "two" when they see two dogs) and subsequently develop counting strategies, although their sequence can be short and may not include all the numerals (Nunes & Bryant, 1996). Ginsburg (2014) claims that between 3 and 5 years of age children begin inferring and understanding the generative characteristic of the verbal system of counting. They naturally discover that after saying, "twelve," the sequence has a pattern which repeats part of the first verbal sequence learned (*one* to *nine*) but adds a new word either at the beginning or the end of the number. For instance, from 13 to 19 they should add the suffix *teen* to each number word in this way: thir*teen*, four*teen*, fif*teen*, six*teen*, etc. If children discover this rule, they can learn new number words in the range 13–19 by themselves. According to Ginsburg, the range 13–19 might be challenging for English-speaking children (e.g., *thirteen* does not contain *three* while *sixteen* contains *six*). However, with help from their parents or teachers, children are able to discover this pattern. When children reach the number *twenty*, they discover a new repetition of the first sequence by adding the word *twenty* at the beginning, so they reproduce by themselves the correct sequence, *twenty*-one, *twenty*-two, *twenty*-three, etc. After *twenty*, the sequence involves a generative rule that is characteristic of the base-ten system, which is the place-value principle. Children begin understanding this concept during counting and number writing activities (Fuson, 1992).

These findings show that children do not learn to count by a simple process of memorizing words, as they can infer the generative rule from the first part of the sequence (e.g., 1 to 19, 20 to 29) and use this knowledge to find out the rest of the sequence. Therefore, kindergarteners can learn to count to 100 by ones and tens as required in KCC1, if they are encouraged to discover the generative nature of the verbal system of counting (Ginsburg, 2014). This also means that teachers do not have to teach each number from 1 to 100, but they could implement play-based activities that include counting within different ranges (1 to 10, 1 to 20, 20 to 50, 50 to 100, etc.) and help their children by saying the new words (e.g., *twenty*, *thirty*, *forty*, etc.). For instance, when children reach *nineteen* in their counting procedure, the teacher can provide the words, “twenty, twenty-one, twenty-two…” and then stay silent so that children can try to continue by themselves, and when children reach “twenty-nine,” the teacher can provide the words “thirty, thirty-one, thirty…” and so on. Later, teachers should encourage children to count by tens using this same principle, helping them discover the generative rule: twen*ty*, thir*ty*, for*ty*, etc. Researchers have found that with experience 5-years-old children can count on from numbers other than one; for instance, using their fingers and hands they can count on from 10 (Fuson, 1992). This also helps students understand the structure of the number system. Standard KCC2 in Counting and Cardinality explicitly addresses this ability: “Count forward beginning from a given number within the known sequence (instead of having to begin at 1)” (NGA & CCSSO, 2010, p. 11). Counting is more than memorization as it depends on children’s ability to discover the patterns of verbal and written number systems they use every day.

The Counting and Cardinality standard KCC4 refers to the three principles of counting and states that when enumerating objects children must say the number names in the conventional order pointing out one and only one object for each name (one-to-one correspondence), understanding that the last number said represents the quantity of the entire group (cardinality) and that each successive number name represents a quantity that is one larger (ordinality; NGA & CCSSO, 2010, p. 11). It has been shown that 3 to 4-year-olds are able to detect errors in other people’s long verbal sequence while enumerating objects, even before they are able to generate a long sequence for themselves (Gelman & Meck, 1983). For example, when one child notices that his/her classmate enumerates twelve toys but skips one of the toys while saying, "eleven," this child recognizes that there is a counting error, even if he or she cannot recite the sequence through twelve. This indicates that children understand the principles that make the counting process correct even before they get to count objects correctly. Between 4 and 6 years of age, children increasingly develop their understanding of these principles of counting (Ginsburg, 2014; Nunes & Bryant, 1996). They also learn to answer the question, "How many?" regarding a group of objects (even more than 20) which is required in the standard KCC5. They can establish the cardinality of the group of objects and can compare groups of objects specifying which is bigger than, smaller than, or equal to the other (Nunes & Bryant, 1996), which is required in the standard KCC6. Young children are developmentally ready to learn these standards and teachers can support them using play-based activities about counting everyday objects. Children learn by ranges of numbers, so they first understand these principles in the range 1–10, and with additional experiences, in the range 11–20. Therefore, it is important that counting activities in kindergarten include various numerical ranges.

Not only can children recite numbers through 20 and count up to 20 objects utilizing the principles of counting, they can also write them as required by the Counting and Cardinality standard KCC3. Similarly, the ability to compare written numbers required by the standard KCC7 (NGA & CCSSO, 2010, p. 11), emerge at an early age as children see numbers in their everyday life and identify how these symbols are used to communicate quantities. Tolchinsky-Landsmann and Karmiloff-Smith (1992) asked 4 to 6-year-old children to count different groups of objects with different quantities, and after counting each group they had to write *how many* objects there were and *what* the objects were. At 4 years of age, children used letters to convey the name of the objects and used number notations to convey the number of objects, even when the notations did not correspond to the specific quantities. This outcome indicated that children knew various written numbers and what the function of the number notations was. Five and 6-year-olds increasingly used conventional number notations to represent quantities. These findings demonstrate that between 4 and 6 years of age children learn how to use number notations as communicative tools to convey information about a certain number of objects. Therefore, kindergarteners are ready to learn written numbers between 1 and 20 to represent and communicate quantities. All the abilities discussed so far, developed before children enter school and indicate their readiness to meet the kindergarten Counting and Cardinality standards.

Appropriateness of the Geometry Strand for K–2

Prior research on developmental psychology and early childhood education also supports the claim that young children are ready to meet the *Geometry* standards. The Geometry standards KG1, KG2, KG3, KG4 and KG6 in Kindergarten require that children be able to identify, describe, analyze, compare, create and compose shapes (NGA & CCSSO, 2010, p. 12). Researchers have found that, from the first year of life, children are sensitive to the structure of the space (Newcombe & Huttenlocher, 2000). For instance, babies as young as 5 months are able to identify distances between two objects, and during the first 2 years of life, they learn how to use distance information for locating objects (Newcombe & Huttenlocher, 2000). Between 3 and 6 years old, children learn how to use distance and angle information to solve tasks involving simple maps, with a variety of configurations of objects, both with or without perceptual cues (e.g., objects or color; Huttenlocher et al., 1999; Otálora & Taborda, 2015; Shusterman, Lee & Spelke, 2008; Spelke et al., 2011; Vasilyeva & Huttenlocher, 2004). Researchers also have shown that during preschool years, children begin identifying shapes. For instance, approximately 96% of 4 to 6-year-olds can identify circles, 87% can identify squares, 60% can identify triangles, and 54% can identify rectangles (Clements & Sarama, 2000). Moreover, at 4 years of age children begin identifying and describing shapes based on their defining properties (e.g., number of sides) rather than on non-defining properties (e.g., orientation; Clements & Sarama, 2014; Satlow

& Newcombe, 1998). These results indicate that kindergarteners are ready to meet the Geometry standards.

Learning to identify and describe shapes based on defining properties can be challenging for young children when they have to work with atypical shapes (Clements, Battista & Sarama, 2001; Satlow & Newcombe, 1998). For instance, some kindergarten and first-grade students believe that rotating a shape (i.e., changing orientation) or dilating a shape (i.e., changing size) would change its name. This means that children tend to accurately identify triangles and explain that they have three sides and three angles if the triangles have a horizontal base (typical triangle), but do not recognize them as triangles when they are rotated or flipped (so that the base is no longer horizontal) or when the triangles are too skinny and too long (atypical triangles; Clements et al., 2001). Clements and Sarama (2014) affirm that this confusion “can last until age 8 if not well addressed educationally” (p. 145) and suggest that children’s accuracy to identify, describe and name shapes can improve between the 4 and 6 years of age with appropriate instructional support. Therefore, early education is important for helping students build on their informal mathematical understandings and overcome their intellectual challenges. For that reason, several Geometry standards in kindergarten and first grade such as KG2, which requires students to “correctly name shapes regardless of their orientations or overall size,” address these learning goals.

If teachers understand children’s progression and potential challenges in their natural development, they could create appropriate environments for helping students improve their abilities to identify, analyze, and describe shapes. For instance, teachers could use manipulatives or dynamic geometry software within play-based activities so that children can rotate, translate, or dilate shapes, becoming aware that a shape’s orientation or size could change but not the shape’s defining attributes (e.g., having three sides and three angles). Sinclair and Moss (2012) implemented geometry tasks in a kindergarten and first-grade classroom using the dynamic geometry software *The Geometer’s Sketchpad*®. The teacher continuously transformed a triangle making it skinny, long, fat, or flat and changing its orientation and position so that the students discovered the invariant properties of the shape, helping them change their perspective on what they could name *triangle*.

Some Geometry standards in kindergarten (i.e., KG5 and KG6), first grade (i.e., 1G2 and 1G3) and second grade (i.e., 2G2 and 2G3; NGA & CCSSO, 2010) refer to the composition and decomposition of geometric shapes, which are skills children develop at relatively early ages. For instance, between 3 and 4 years of age children can build shapes from other shapes (e.g., a flower built from combining one circle and four triangles). Children between 4 and 5 years of age combine simple shapes to compose parts of larger shapes (e.g., the arm of a person built from triangles and the body built from squares and one circle). Between 5 and 8 years old, they learn to compose two or three-dimensional shapes to create complex structures such as arches, towers and bridges (Clements & Sarama, 2014). These fundamental skills may prepare children for future STEM careers (Newcombe, 2010) and teachers can support them by implementing developmentally appropriate activities about building shapes from other shapes.

In this section, I showed that children begin schooling ready to meet the K-2 CCSSM as they have already developed mathematical abilities from the first year of life and that they can learn challenging mathematics during their first years of school with an adequate education. This means that the K–2 CCSSM are developmentally appropriate for young children. Far from being detrimental, implementing the CCSSM supports children’s creativity by encouraging them to use their own methods to discover patterns and develop their mathematical thinking. Therefore, educators could use the CCSSM as a tool to understand young children’s mathematical abilities.

CCSSM as a Tool to Innovate Teaching, Learning and Assessment

Claims regarding the implementation of direct instruction, drill and practice and standardized tests into K–2 classrooms as mandated by the CCSSM are also misconceptions. Like any standards, the CCSSM are the goals of school mathematics, outlining the knowledge and skills that children should achieve at the end of each grade, based on their own capabilities (Confrey et al., 2014); however, the CCSSM do not establish how to teach or which instructional practices teachers should use to help children meet each standard. The CCSSM state, "standards define what students should understand and be able to do" (NGA & CCSSO, 2010, p. 4), then clarify, "these Standards do not dictate curriculum or teaching methods." (NGA & CCSSO, 2010, p. 5). Decisions about how to teach, including guidance for curriculum implementation such as textbooks and materials, lesson organization, teaching strategies, and assessment methods, are not dictated by the CCSSM. Instead, teachers select instructional approaches for implementing the standards. For instance, some teachers may use drill and practice to implement the Geometry standards. However, it is likely that those teachers will have students who only master low-level skills, spend more time memorizing rules, become bored easily, and lose opportunities to develop creativity (Ginsburg, 2014). Other teachers may use guided play, hands-on activities, manipulatives, music, art, problem solving, collaborative exploration or digital interactive technologies to teach exactly the same standards. It is likely that the students of these innovative teachers will achieve at higher levels, acquire long-lasting skills in less time, and enjoy learning simultaneously (Ginsburg, 2014).

Although the CCSSM do not mandate how to teach, they emphasize eight Standards for Mathematical Practice (SMPs) that are useful to guide instruction: (a) make sense of problems and persevere in solving them, (b) reason abstractly and quantitatively, (c) construct viable arguments and critique the reasoning of others, (d) model with mathematics, (e) use appropriate tools strategically, (f) attend to precision, (g) look for and make use of structure, and (h) look for and express regularity in repeated reasoning (NGA & CCSSO, 2010). The mathematical practices are the same for all the grade levels and describe processes that all children are expected to experience and develop along with the fulfillment of the content standards.

Therefore, the description of the mathematical practices does not suggest the use of methods such as drill and practice. Instead, the mathematical practices encourage the use of active, student-centered instructional approaches, which presents an opportunity for teachers to enhance their pedagogical practices.

Research on CCSSM implementation is flourishing, but few studies examine the design of instructional strategies that implement the content standards and mathematical practices. However, some recent studies have shown important outcomes regarding both student learning and teacher transition to the CCSSM.

Learning Gains through Student-Centered Approaches

In recent years, early mathematics education researchers have examined how to foster the development of mathematical thinking and skills, through active, student-centered instructional approaches. Instead of memorizing algorithms and facts, these approaches encourage students to explore problems, engage in different forms of reasoning, create and try out their own strategies, explain their methods to others, use multiple representations, work with manipulatives, and discover mathematical patterns. For example, research on geometric thinking mediated by digital technologies shows how teachers can move away from traditional teaching and design innovative instructional strategies to implement the CCSSM (see Sinclair & Moss, 2012). Digital interactive technologies provide a range of options for young students to access and understand important mathematical concepts as those presented in the CCSSM within learning environments based on guided play, active exploration and problem solving. Implementing activities with these types of technologies allow young children to use their own ways of reasoning to understand and enjoy learning challenging mathematics (Otálora, 2016a, 2016b; Sinclair & Moss, 2012).

Sinclair and collaborators have investigated the use of dynamic geometry software such as *The Geometer's Sketchpad*® (hereinafter referred to as the Sketchpad; Jackiw, 2009) in K–2 to help children develop their discourse about shapes and infer complex mathematical relationships. For instance, Sinclair and Moss (2012) utilized Sketchpad in a kindergarten and first-grade mathematics classroom to teach children to identify, describe, name and analyze shapes as required in the kindergarten (e.g., KG1, KG2 and KG4) and first grade (e.g., 1G1) Geometry standards (NGA & CCSSO, 2010). The reported lesson illustrates how 4 and 5-year-olds talked and gestured about triangles by recalling their everyday experiences and discovered triangle's properties through the visualization of multiple dynamic representations of the geometrical shapes provided by the software and the interpretation of the visual feedback. Notably, the Sketchpad allowed young learners to interact with a variety of atypical shapes (e.g., rotated triangles, very skinny and long triangles, etc.), which is aligned with the KG2 and the 1G5 Geometry standards that require students to distinguish between defining attributes and non-defining attributes of shapes. Using Sketchpad facilitated the development of children's ability to identify and name a wide variety of triangles, within a learning environment that was developmentally appropriate.

Sinclair et al. (2013) also showed how using Sketchpad helped first-grade children access complex ideas such as parallel lines and intersecting lines. Observing and exploring dynamic representations of the lines helped children recall prior daily life experiences with parallel lines and make sense of the mathematical relationships. Children developed their mathematical discourse which includes speech, actions and gestures—genuine creative acts. This study revealed how young children are able to use their own ways of thinking to make sense of challenging mathematics. Moreover, this study illustrated how teachers can teach important mathematics and foster creative thinking while children enjoy learning.

Although Sinclair and Moss (2012) and Sinclair et al. (2013) are not explicit about the CCSSM, the goals of their implementation were aligned with the MLTs underlying the Geometry standards. Ng and Sinclair's (2015) study about how first-grade students understand symmetry within a dynamic geometry environment is explicit about the ways in which their research addresses these standards. The authors suggest that even when the K–2 standards do not emphasize symmetry, this concept is helpful for meeting the other geometry standards. Ng and Sinclair state, “symmetry can function powerfully as a tool to describe, recognize, classify and create both two- and three-dimensional figures” (p. 243). In this study the young children developed intuitive ways of reasoning about mirror symmetry while exploring, talking, gesturing and representing their findings through drawings.

Overall, Sinclair and collaborators’ studies have shown how specific goals related to the students’ learning trajectories underlying the content standards can be implemented along with some of the SMPs such as reason abstractly, construct viable arguments, and use tools appropriately (NGA & CCSSO, 2010). Using digital interactive technologies is an innovative way to implement the K–2 standards as this approach allows young children to reason intuitively about complex mathematical ideas through the exploration of visual-dynamic representations. These studies also have revealed that young children enjoy working with the technology, are less likely to become bored and have multiple opportunities to interact with their peers and teachers. Thus, these investigations illustrate how young learners can be taught interesting and important mathematics through student-centered approaches that foster their abilities to explore tasks, solve problems and discuss and explain their ideas through their own means. In this case, using learning trajectories to guide instruction has led to important innovations rather than drill and practice. Therefore, the CCSSM can be used as a tool for innovation in teaching and learning. In turn, these innovative learning practices can promote children’s creative thinking without depriving them of emotional engagement and socialization.

The Meaning and Means of Assessment at Early Grades

Criticisms stating that the CCSSM lead to the use of multiple-choice standardized tests at early grades are unfounded misconceptions. The CCSSM do not mandate timing or format of assessments. Due to the variability in young children's abilities and their different rates of learning (Siegler, 2007), traditional paper-based testing may not be an appropriate way to determine their progress and needs. However, with or without the CCSSM, it is important to use developmentally appropriate methods for assessing students' understandings, and progressions in their ways of thinking. Assessment can provide teachers with clues about children's learning gains and needs and the standards can be used as a reference to meet this goal.

Observing and listening to children closely for long periods of time and recording changes in their ways of thinking as they participate in the activities are more appropriate forms of assessment than multiple-choice tests (Belfanz et al., 2003; Ginsburg, 2014). Teachers can interview children while exploring tasks, and analyze how they talk about mathematics or follow shifts in their use of methods to solve the problems. These forms of assessment could be implemented by using the same activities that teachers utilize to teach children, for example, guided play, problem solving and exploration, hands-on activities, manipulatives, digital interactive technologies and other forms of active, student-centered learning. The difference is that teachers should use the MLTs in different domains (e.g., number and operations, geometry) as the reference for locating the current level of each child to determine individuals' needs.

For teachers to be able to locate their children's thinking levels they should carefully observe what children do, listening to their answers and analyzing their strategies, and then they should check the MLT specified in a strand of the K-2 standards to determine the child's level and identify new learning goals based on this developmental path (Clements & Sarama, 2014; Confrey et al., 2014; Ginsburg, 2014). Once teachers have identified all their students' abilities, they can design creative activities with various levels of complexity in order to meet the different needs of their groups and promote further development. Some children may need special attention, requiring teachers to implement specific interventions. Overall, these assessment strategies allow teachers to offer children the adequate support, and design compelling learning activities that are appropriate for different ages and levels of understanding. In this sense, the sets of MLTs detailed in the CCSSM can be seen as a helpful tool for assessment.

Conclusions

In this article, I have illustrated how the K–2 Math CCSSM are based on prior research on child development and early childhood mathematics education. Thus, I have argued that the CCSSM are developmentally appropriate for young children between 4 and 8 years of age and illustrated how these standards can be addressed at these grades without reducing time for play, exploration, active learning, and social interaction. Furthermore, I have described how these types of activities can be integrated with classroom practices fostering children's mathematical abilities at the same time as nurturing creativity, self-confidence, and social relationships.

It is possible to affirm that critics' issues do not reside in the CCSSM as they do not mandate ways of teaching and assessing. Alternatively, one persistent obstacle could be the limited understanding that some parents, school administrators, and early childhood educators have of how children learn mathematics and how to help them develop their mathematical skills at early grades. For a successful implementation of the CCSSM, these key stakeholders need support to understand the standards and their relationships with the children's MLTs. This support can be achieved through different means such as reading the research on which the standards are grounded, participating in public discussion of the standards and their implementation, and participating in professional development programs. As challenging as it may be to support stakeholders' understanding of the standards, it is promising that flourishing research on teacher transition to the CCSSM has shown that teachers hold positive perspectives on the implementation of the standards regarding student learning and their own teaching practices (Swars & Chestnutt, 2016). Overall, they believe that the CCSSM can lead to a positive change in mathematics education and consequently, are shifting their instructional practices from teacher-centered approaches towards more active, student-centered approaches. However, teachers also claim that they need professional development and curriculum materials to support decision-making processes during this transition (Swars & Chestnutt, 2016; Wilson & Downs, 2014). Professional development programs should help teachers improve their mathematical knowledge for teaching to young children, gain awareness of the ways in which young children develop mathematical competencies at different ages, and design and implement learning environments appropriate for the young learners to meet these standards. With this support, early childhood teachers and educators can use the K–2 CCSSM as a tool to guide the design of innovative, challenging school practices.

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