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Public Conceptions of Algorithms and Representations in the Common Core State Standards for Mathematics

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Algorithms and representations have been an important aspect of the work of mathematics, especially for understanding concepts and communicating ideas about concepts and mathematical relationships. They have played a key role in various mathematics standards documents, including the Common Core State Standards for Mathematics. However, there have been some public misunderstandings about the standards and the role that algorithms and representations have in the teaching and learning of mathematics. In this article, I will first look at how algorithms and representations are discussed in the standards, and then examine and unpack some of the public conceptions around algorithms and representations.

"The ways in which mathematical ideas are represented are fundamental to how people can understand and use those ideas" (National Council of Teachers of Mathematics [NCTM], 2000, p. 67). Representations play an important role in supporting students' understandings of mathematical concepts, which can afford students opportunities to develop conceptual understandings of algorithms. Over the past few decades, a great deal of research has been conducted on the use of algorithms and representations in mathematics (Goldin, 2002; Lesh & Harel, 2003; Lesh, Post, & Behr, 1987; Orrill, Sexton, Lee, & Gerde, 2008). These studies illustrate the significance of being able to create and use different representations (pictures, drawings, tables, graphs, etc.) as a way of understanding mathematical relationships and supporting conceptual understanding of algorithms. As a result, both

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representations and algorithms have played an important role in various sets of mathematics standards (NCTM, 2014; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010; NCTM, 2000). In this article, I will first consider the role algorithms and representations such as the number line have in the Common Core State Standards for Mathematics (CCSSM; NGA & CCSSO, 2010) and then I will examine the public perceptions concerning the use of algorithms and number lines.

Representations, algorithms, and strategies are all related constructs and will be discussed throughout this piece, so it is important to note how I distinguish between them. Strategies are the approaches that one takes when attempting to solve a problem, but these strategies are not pre-determined, formulaic, or outlined. In contrast, according to Mauer (1998), an algorithm is a "precise, systematic method for solving a class of problems…takes input, follows a determinate set of rules, and in a finite number of steps gives output that provides a conclusive answer" (p. 21). For instance, the standard algorithm for subtraction in the United States involves lining up the numbers to be subtracted, starting with the ones place, seeing if the subtraction can take place, and if not, involves taking a ten and then decomposing it into ten ones, and then the subtraction continues in this fashion. In this piece, representing shall refer to "the act of capturing a mathematical concept or relationship in some form and to the form itself" (NCTM, 2000, p. 67) and representations shall align with Goldin and Shteingold's (2001) use of external representations (e.g., figures, drawings, diagrams, graphs, charts, number lines, etc.). Representations are often used as ways of communicating mathematical ideas and help support the conceptual understanding of algorithms or are used in conjunction with strategies; but a representation by itself in isolation has no meaning. Formulas, equations, and Cartesian graphs are all specific representations that can only be analyzed in their respective contexts as they are part of "a wider system within which meanings and conventions have been established" (Goldin & Shteingold, 2001, p. 1).

Algorithms and Representations in the CCSSM

Algorithms and representations played an important role in both sets of NCTM (1989, 2000) standards documents, and that tradition has been continued with the CCSSM. It is important to note that across all the standards, the first specific mention of a representation is in the second grade standards, which introduce the use of the number line as a way of representing whole numbers as lengths (NGA & CCSSO, 2010, p. 20). Beyond this specific use of a representation, there are numerous standards that require students to explore ideas through the use of multiple representations (e.g., drawings, equations, modeling with objects, graphs), especially when exploring and analyzing data, but the choice of representations is left for teachers and students to decide. Most often students and teachers will use number lines as a representation of choice because it supports the development of the concepts of number, place value, and even standard algorithms. However, there are many parents and other members of the public who believe that the CCSSM strictly require students to only use the number line (and no other representations) when operating with numbers. If students have good number sense, fluidity in constructing and deconstructing numbers, and the ability to recognize relationships with the aid of the number line, they will be able to understand the mathematics behind standard algorithms and will be able to use them in more powerful ways (McCallum, 2015). In the next section, using the standards from grades 2-4, I will demonstrate the relationship between the number line and standard algorithms and their role in the CCSSM.

Algorithms and Representations in the Content Standards

As mentioned earlier, the first time the CCSSM names the use of a specific representation is in the second grade in the Measurement and Data strand, but it is important to note that representations do appear as early as the kindergarten standards. According to Standard 2.MD.B.6, students are to relate addition and subtraction to length. The standard reads, "Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2…, and represent whole-number sums and differences within 100 on a number line diagram" (NGA & CCSSO, 2010, p. 20). This standard attempts to develop the concept of a number as a length; each point on a number line represents a different magnitude and these magnitudes accumulate to create larger or smaller magnitudes. The points on the number line then are not numbers themselves, but rather labels of the lengths from 0. This standard also relates the operations of addition and subtraction with the concept of length and distance on the number line. Furthermore, students in second grade are expected to understand place value. The use of the number line to represent operations with numbers, such as addition and subtraction, necessitates a strong understanding of place value. Thus it is appropriate that the number line would be introduced in this grade before introducing the standard algorithms for addition and subtraction (Otten & De Araujo, 2015).

Although the number line is the only representation explicitly named in the second grade standards, the standards indicate that it should not be the only representation students see. For example, in the Number and Operations in Base Ten strand for second grade 2.NBT.B.7, students are to "add and subtract within 1000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction" (NGA & CCSSO, 2010, p. 19). Drawings, concrete models, and strategies could include a wide array of approaches aside from the standard algorithm or the number line, and phrases such as this one appear in standards for later grades as well.

In third grade, the Number and Operations strand expands on the standards from second grade by focusing on multi-digit arithmetic. For example, standard 3.NBT.A.2 of this strand states, "[F]luently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction" (NGA & CCSSO, 2010, p. 24). This standard builds on the standards from second grade with additional emphasis on fluency. Specifically, instead of focusing on concrete models or drawings, it refers to the use of strategies and algorithms. The standards encourage teachers to push students to have a more-developed number sense to determine which strategies and algorithms are more efficient.

Additionally, number lines are expanded in third grade to include fractions. Students are expected to develop an understanding of fractions in relation to the number line:

- 3.NF.A.2.A: Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

- 3.NF.A.2.B: Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line (NGA & CCSSO, 2010, p. 24).

Figures 1 and 2 illustrate what both of the standards above describe. In figure 1, to place 1/2 on the number line, the interval from 0 to 1 is partitioned into two (or generally *b*) equal parts, each part of size 1/2. The endpoint of the part based at 0 locates 1/2 (depicted as 1/*b*) on the number line. Similarly, to place 3/4 on the number line (see Figure 2), three (or generally *a*) lengths of 1/4 are marked off from 0, and the resulting interval is now of length 3/4, with the endpoint of the interval being where 3/4 is located (depicted as *a*/*b*). In second grade, students are expected to use the number line to represent whole numbers as lengths and the operations of addition and subtraction as finding distances between numbers. In third grade, students expand the concept of number to include fractions, and as such, need to be able to use the number line representation with fractions.

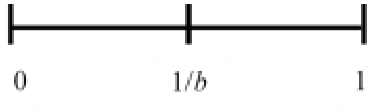


Figure 1. Number line partitioned into two equal parts representing the location of 1/2 on the number line.

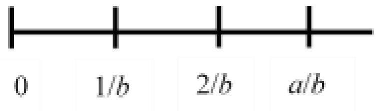


Figure 2. Number line marked by fourths representing 3/4 on the number line.

Standard algorithms first appear in the fourth grade standards. In the Numbers and Operations in Base Ten strand, under using place value understanding and properties of operations to perform multi-digit arithmetic, standard 4.NBT.B.4 states, “[F]luently add and subtract multi-digit numbers using the standard algorithm” (NGA & CCSSO, 2010, p. 29). Figure 3 offers an example of the standard algorithms for addition and subtraction to which this standard refers. In the addition algorithm (shown on the left in Figure 3), the two numbers 293 and 172 are aligned so that corresponding place values are in the same columns. Then, starting with the ones place on the far right and continuing to the left, the digits in each place value are added together and the total is recorded (e.g., 3 + 2 = 5). If the total in a column is greater than 10, the digit in the ones place is recorded and a 10 is regrouped with the place value to the left, as is the case in the tens column where 9 + 7 = 16. The 6 is recorded and 10 is regrouped in the hundred’s column (notated by the 1 above the 2 in the hundreds place). The subtraction algorithm (shown on the right in Figure 3) functions in a similar manner except one subtracts instead of adds the digits in each column. If the top digit is smaller than the bottom digit, as is the case in the ones column (i.e., 5 is smaller than 8), a 10 is decomposed and added to the ones column so that the subtraction can be carried out (e.g., 5 becomes 15 with the addition of 10 ones and 7 groups of 10 is reduced to 6 groups of 10).

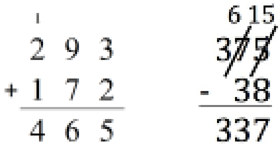


Figure 3. Standard algorithms for addition and subtraction in the United States.

Fourth grade students are also expected to expand their understanding of number to include decimal representations of fractions and to understand how these decimals can represent lengths as well as points on the number line (see standard 4.NF.6; Figure 4). This use of the number line is also applied in the Measurement and Data strand of the fourth grade standards. Specifically, the second standard in the Measurement and Data strand requires solving a variety of word problems and representing quantities using diagrams such as a number line featuring a measurement scale (NGA & CCSSO, 2010, p. 31). For the operations of multiplication and division, the standards reference equations, rectangular arrays, and area models as possible representations and strategies (see Figure 5).

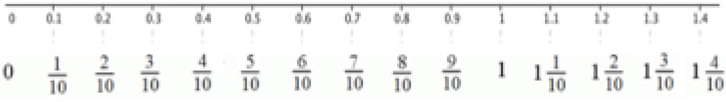


Figure 4. Fractions and their decimal equivalents as represented on a number line.

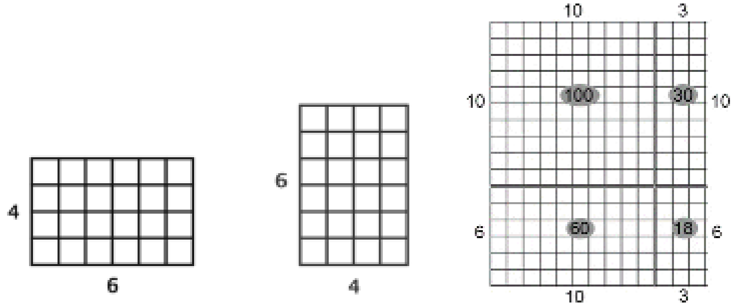


Figure 5. Rectangular arrays (left and middle) and an area model for multiplication (right).

The multiplication standard algorithm, however, is not explicitly introduced until fifth grade (NGA & CCSSO, 2010, p. 35) and the division standard algorithm is not explicitly introduced until sixth grade (NGA & CCSSO, 2010, p. 42). Figure 6 shows examples of the multiplication (i.e., 13 x 12) and division (i.e., 257 ÷ 6) standard algorithms in the United States. In the multiplication algorithm, (on the left in Figure 6) the two numbers are aligned in the same manner as in the addition and subtraction standard algorithms. Then, starting from the ones place in 12, 2 is multiplied by 3 above it, which equals 6, and then the 2 is multiplied by one 10, which results in two tens, combined resulting in 26, which is recorded. Next, the one ten in 12, multiplied by 3, is 30, and then the one 10 multiplied by another one 10 results in 100, the combined result being 130. Lastly, the two values 26 and 130 are added together, giving a total of 156. In the division algorithm (on the right in Figure 6), start with the place value column farthest left in 257 and divide the 2 by 6, which does not result in a whole number. So, building on the 2 to include the next place value, makes the new value 25. The division of 25 by 6 results in 4 with one left over (the 4 is recorded in the tens place above 257). Then, multiple 4 times 6 to obtain 24. A zero is placed in the ones column to subtract 240 from 257, which results in 17. The process is repeated with 6 and 17. Afterwards, there are no numbers left to divide (5 divided by 6 does not result in a whole number), meaning that 257 divided by 6 equals 42 with a remainder of 5.

$$
\begin{array}{r}
13 \\
\underline{\times 12} \\
26 \\
\underline{+130} \\
156
\end{array}
\qquad\qquad
\begin{array}{rr}
 & 42 \\
\cline{2-2}
6 & \big) 257 \\
 & \underline{-240} \\
 & 17 \\
 & \underline{-\ \ 12} \\
 & 5
\end{array}
$$

Figure 6. Standard algorithms for multiplication and division in the United States.

In summary, from the second through fourth grade standards, one gets a sense of the role that algorithms and representations play throughout the standards. One of the common perceptions is that the number line is the only method required by the standards for solving problems involving operations with numbers and that no other representations nor algorithms are included. Although the number line is the only representation the standards explicitly require, that representation is carefully chosen for its connection to developing the concept of number as students are introduced to different types of numbers. Across the grade levels, representations help students explore and analyze relationships, and students have the freedom to choose the representations they wish to use. This setup allows the flexibility for them to develop their reasoning and thinking skills; for example, consider the problem 23 x 10. A student may at first need to draw out 23 objects iterated 10 times and count the total when they reason through the multiplication problem, but as they think about the relationships (particularly multiplying by 10), they can consider the relationships and ideas about grouping they have learned so that they can think of different, more efficient ways to solve the problem that do not rely on drawing out all of the objects. Children who are exposed to different representations, strategies, and algorithms will have a better time understanding certain mathematical concepts (e.g., multiplication and division of fractions) in later grades.

Public Distortions Concerning Number Lines and Algorithms in the Common Core

Recently there have been numerous public debates about various aspects of the CCSSM, one of which involves the role of number lines and algorithms in teaching and learning mathematics. There is some parental concern that the pacing of curricula is either too slow or that the curricula become very tedious as students spend more time focusing on understanding the arithmetic operations rather than learning to use standard algorithms for calculations (Rich, 2014). In other words, students and parents are being asked to move beyond simply knowing *how*, to understanding the *why* behind the mathematics. Researchers (e.g., Lesh & Harel, 2003; Mauer,

1998) have found that students need to be proficient at thinking and reasoning, not only at calculating, especially in the current digital era where computation is a trivial skill that can be easily performed with the aid of technology. However, there are people who disagree with this approach to mathematics; some are unsure how these new approaches are helpful for developing meaningful understandings. For example, the Internet hosts numerous examples of parents struggling to make sense of the types of problems that their children are bringing home from school (Nelson, 2014). These parents often believe that the way they learned mathematics is perhaps the best way for their children to learn mathematics, and in many cases, this perspective implies a greater focus on arithmetic, calculations and the use of standard algorithms, rather than on understanding the meaning behind these procedures (Otten & De Araujo, 2015). However, parents' struggles can also be attributed to parents' misunderstanding the values these new approaches offer. They may only able to compare their children's current experiences with their own past experiences in mathematics and may not understand the reasons and intent for how they were taught or how their children are currently being taught. This concern may be related to broader issues of the CCSSM involving their implementation: lack of communication with the public and the misalignment of curricula to these standards. As a result, moving forward requires textbook publishers to focus more on the alignment of curricula to the CCSSM and mathematics educators and researchers on communicating to parents and teachers the benefits of using algorithms and representations to develop a deeper understanding of mathematics. In the next section, I will explore and attempt to address the issue of lack of communication.

The Case of Jack and the Number Line

One large-scale example of the debate surrounding the CCSSM occurred when a frustrated parent posted a picture of one of his child's homework problem (Figure 7; Patriot Post, 2014) on a social media website; it sparked immediate responses from the public and even garnered news coverage across the nation (Heitin, 2014). The problem reads, "Jack used the number line below to solve 427 - 316. Find his error. Then write a letter to Jack telling him what he did right, and what he should do to fix his mistake."

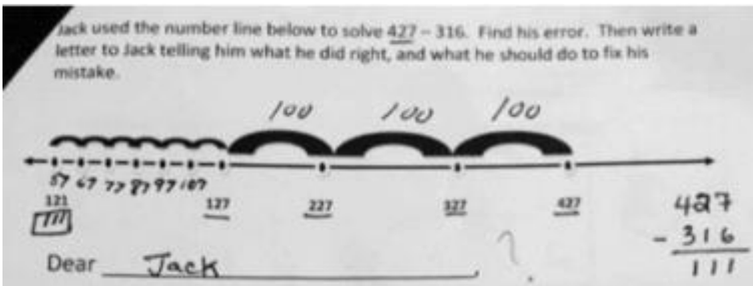


Figure 7. Common Core mathematics problem posted online by Jeff Severt (Patriot Post, 2014).

The purpose of this problem is not only to calculate 427 - 316 but also to analyze a hypothetical student's work in solving the problem and to communicate appropriately the validity of said student's approach. An accepted response to this problem could be that a student explains that Jack used the appropriate strategy of counting down on a number line from 427 by 3 hundreds, but then he counted by 6 tens incorrectly; so after counting the 3 hundreds, he would need to count 1 ten, and then 6 ones. The parent who posted the problem (hereon referred to as the Jack problem) used his educational background in engineering as the focal point of his argument. He claimed that with all of the higher-level mathematics courses he had taken in his undergraduate degree, he could not make sense of this approach. Instead, he argued that Jack should have used the standard algorithm for subtraction, which after rapid calculation would have resulted in an answer. Stephen Colbert, best known for his satirical news program, *The Colbert Report*, supported this position claiming, "Folks, that word problem couldn't be easier to solve. All you have to do is check the semicircles on the same-side arrow, put the two numbers up in it, and bing-bang math!" (Stewart, Purcell, & Colbert, 2014). However, what the parent, the comedian, and thousands of others overlooked was the purpose of the problem. It was not only about calculating the difference between two numbers and using the number line to do so; it was also about analyzing another person's mathematical reasoning, which attends to the ability to critique the reasoning of others as well as construct viable arguments, both of which together constitute one of the eight Standards for Mathematical Practice in the CCSSM (NGA & CCSSO, 2010).

The Standards for Mathematical Practice in the CCSSM encompass the activities and behaviors that mathematicians and others who engage in mathematics employ. For example, professor Andrew Wiles was able to solve and prove Fermat's Last Theorem, a problem that had been lacking a proof since 1637. Wiles had to make sense of the theorem and persevere in solving it, trying many different ideas and following numerous strategies in order to come up with the proof, and make sure that his arguments in his proof were valid and viable. All of these practices are important because they help to foster more than just mathematical ability, but critical thinking and communication of mathematical ideas as well (Koestler, Felton, Bieda, & Otten, 2013). The Jack problem discussed in the previous paragraph is an example of a task that focuses on not only developing mathematical ability, but also analyzing another's work and argument. Analyzing another person's mathematics can be a difficult task because it requires one to not only determine if a mathematical approach is valid, but then also to determine the person's thinking and reasoning behind their mathematics. Such a skill is powerful because it is not limited to mathematics; it is useful in almost every single discipline and even in daily life. For instance, computer programmers work together in teams, and people can be assigned different parts of the same program. If something does not function correctly in one person's code, someone would need to analyze the code that came before theirs to see if there are different ways of approaching the task this program needs to execute or if they made any errors in the code. They can then suggest the modifications that need to be made to another member of the team in order to make the code function properly.

Even though much of the importance of this task is tied to its incorporation of the Standards for Mathematical Practice, it is worth considering the mathematical affordances of the number line model as well. According to Nelson (2014), “The idea behind using a number line for subtraction is that students get a visual representation of what subtraction is: figuring out the ‘distance’ between two numbers” (Can you teach number sense, para. 5). With this particular problem, she expressed that a clearer representation of the problem was needed (Figure 8 as described by Nelson, 2014), which to some degree addresses the parent’s concerns with the particular representation in Figure 7.

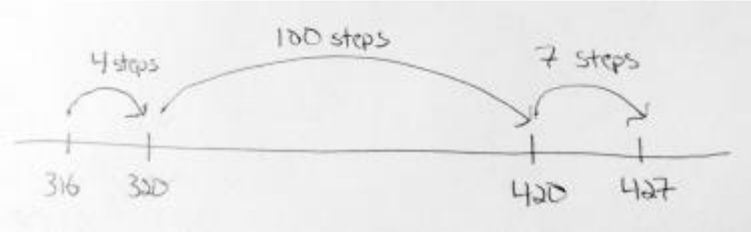


Figure 8. Alternative use of the number line to model the problem 427 - 316.

In this representation, the numbers in question, 316 and 427, are at the ends of the number line. What Nelson (2014) proposes is that in this representation, one would start at 316 and realize that the closest 10 is 320, which is 4 steps away. Then, it is 100 steps away from 320 to 420, and finally it is 7 steps from 420 to 427. So, the distance between 316 and 427 is 4 + 100 + 7, which results in 111. Her proposal for this representation (Figure 8) attempts to resolve the issues in how Jack’s number line was used not only to represent the distance between two numbers, but also to emphasize place value. Her representation encourages the use of making tens and hundreds when adding, which are numbers that young children are generally familiar with early on and help to solidify the foundation of place value.

The Jack problem is an issue that William McCallum and Jason Zimba, two lead writers of the CCSSM, discussed in an article of *The Hechinger Report* (Garland, 2014). According to Garland (2014), both McCallum and Zimba claim that the Jack problem is not what the CCSSM requires and not something that they would include in any curricular materials they would write. The standards that this Jack problem attempts to address are from second grade (NGA & CCSSO, 2010):

- 2NBT.A.1-A.4: Understand place value (e.g., 100 can be thought of as ten tens).
- 2NBT.B.5-B.9: Use place value understanding and properties of operations to add and subtract (e.g., explain why addition and subtraction strategies work).
- 2MD.B.5-MD.B.7: Relate addition and subtraction to length (e.g., represent whole numbers as lengths on the number line, and represent whole-number sums and differences on the number line).

In the interview reported in Garland (2014), McCallum claims that in the problem a student is supposed to recognize that the issue was around confusion of place value, and Zimba claims that attempting to combine standards dealing with place value and the number line simultaneously can be confusing. Both mathematicians assert that this particular task does not reflect any issues with the CCSSM themselves. Instead, they assert that this task is the byproduct of a lack of quality control for educational materials created in the name of the CCSSM. Without a mechanism for quality control, some materials will inevitably be more aligned to the standards and of a high quality, and many other materials will be loosely or not aligned at all to the standards and of low quality. This sentiment was echoed by Schoenfeld (2014) who argued, “the Common Core is not a curriculum…the vast majority of materials currently labeled ‘Common Core’ don’t come close to that standard” (What do Common Core curricula look like, para. 1). The Jack problem provides an example of this; just because a problem was taken from curricular materials that claimed to be aligned to the CCSSM does not necessarily mean that it actually adheres to the standards and is a well-developed problem.

Thus, there needs to be more attention focused towards the connection between the CCSSM and the problems in curricular materials that are aligned to said standards, specifically the role that representations play in curricular materials and whether or not these representations are used in mathematically meaningful ways that help to develop students' understanding of the concepts underlying them.

Algorithms vs. Number Line: The Case of Number Sense

Since the Jack problem surfaced, many other parents and teachers have taken to the Internet as an outlet for sharing mathematics problems that claim to be CCSSM problems with which they have serious concerns. What seems to be underlying the debates surrounding these problems is the issue of including representations, such as the number line, before exposing students to the standard algorithms in the CCSSM. When parents see their children bringing mathematics problems home requiring them to use methods other than the standard algorithms with which they are not familiar, they are frustrated because they may not understand these other methods or the merit of using such methods. For instance, one of the common arguments is that standard algorithms are much easier and faster to use and that some of the other methods are more cumbersome and time-consuming. However, this argument fails to consider that standard algorithms are more complicated than they appear. According to Samuel Otten (2014), adults who claim standard algorithms are easier have the benefit of having already learned the algorithms, but this is not the case for students who are seeing the algorithms for the first time. Additionally, these adults may have learned the algorithms through memorization or rote drill, and while some are able to use the algorithms, may not have a conceptual understanding of the algorithms (e.g., how the standard algorithm for subtraction tends to hide or obscure the regrouping of numbers that occurs during subtraction; Otten, 2014).

Recently, NBC News (2014) aired a segment on the Common Core, that specifically focused on the example of 34 -

9. They discuss solving the problem in two ways, the first using the standard algorithm for subtraction and the second using the number line. The side-by-side comparison of the two methods is illustrated in Figure 9.

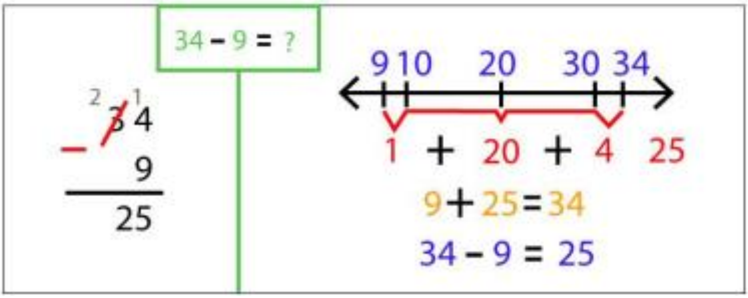


Figure 9. Side-by-side comparison of two methods of solving the subtraction problem 34 - 9. Recreated from "Nightly News with Brian Williams," NBC News (2014).

The broadcaster discussing the problem claimed that the standard algorithm requires fewer steps to complete the subtraction than the use of a number line, and as a result is a faster and easier method. If one carefully considers what is required to use the standard algorithm in this problem, there are really six steps involved. First, 9 cannot be subtracted from 4, so the 3 is crossed out, leaving a 2. Then the 4 becomes a 14. Fourteen minus 9 is 5. Then the 2 is brought down, resulting in 25. As Otten (2014) discusses in his response video to the NBC News (2014) segment, there is a lot happening behind the standard algorithm that cannot be seen, because the algorithm itself is meant to be more efficient. A 10 is not "borrowed" from the 30, rather regrouping is taking place; the 34 is regrouped into 20 + 14. This deconstruction of numbers is developed earlier on in the CCSSM so that this type of thinking can be applied when students are introduced to the standard algorithm. Additionally, deconstruction of numbers does not reduce the algorithm to a simple series of steps that students memorize and follow. Instead, students are building their number sense and deeper understanding of operations with numbers. For instance, if students were asked to subtract 199 from 4004, the use of the standard algorithm would be cumbersome and time consuming (each place value in 4004 would have to be regrouped, which is a lot of crossing out). Alternatively, the student could use their number sense to realize that 199 is 1 away from 200, and that 1 should also be added to 4004 to become 4005, resulting in the problem 4005 - 200, which is likely much simpler to solve (see Figure 10).

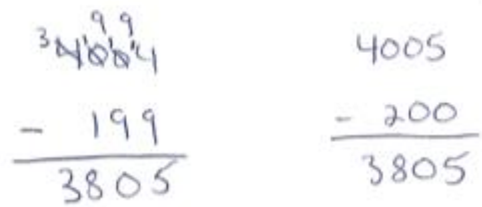
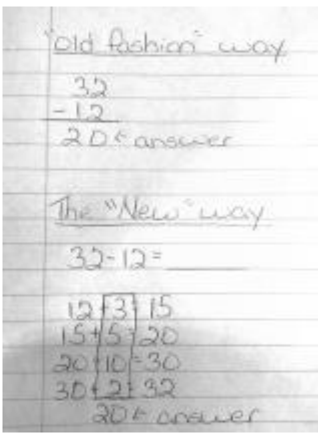


Figure 10. Using the standard algorithm to solve 4004 - 199, and using number sense to solve an easier problem, 4005 - 200.

In contrast, using the number line helps to situate the numbers visually in relation to one another and can aid in construction and deconstruction of numbers alongside the standard algorithm. In Figure 9, a student would place 9 and 34 on a number line. Second, the student would start at 9 and notice that it is 1 away from 10. Next, to get from 10 to 30 is 20 (these numbers are benchmark numbers). Then, to get from 30 to 34 is another 4. The student then sums together 1, 20, and 4 to get 25. Using the number line for this problem required five steps, as opposed to the six steps that the standard algorithm used, and also builds number sense (e.g., grouping with tens and relationships between numbers). The subtraction problem is unpacked in terms of addition from one number to another by exploiting groupings of tens, whereas the number line helps students to understand the concept of subtraction as distances and the connection of addition and subtraction as inverse operations. It is important to mention that the last two lines below the number line, 9 + 25 = 34 and 34 - 9 = 25, are included in order to emphasize the additive strategy used to determine the answer to the subtraction problem. These lines were not just included as extraneous information but were meant to emphasize the underlying number sense of the strategy. The broadcaster, though, does not make this point clear.

In the NBC segment, the conclusion seemed to be that the standard algorithm was the superior method for solving the problem because it required fewer steps and used less space. A similar argument emerged after another image that widely spread among social media and websites across the Internet involved the subtraction problem 32 - 12 (see Figure 11; Common Core Math Memes, 2014).



"Old Fashion" Way

$$\begin{array}{r} 32 \\ -\ \underline{12} \\ 20 \end{array} \leftarrow \text{answer}$$

The "New" Way

32 – 12 = _____

12 + 3 = 15
15 + 5 = 20
20 + 10 = 30
30 + 2 = 32
20 ← answer

Figure 11. Standard algorithms vs. alternative approaches (Common Core Math Memes, 2014). On the left is the original image circulated on social media and the image on the right is a text recreation of the original for clarity.

At the top of the image, the problem is solved using the standard algorithm for subtraction, which seems to be quite simple. However, a quick glance at the bottom reveals that this simple problem that was solved quickly is now solved through the use of four other problems. Anyone seeing this image for the first time would have a hard time understanding why one would solve the problem with the bottom approach when the top approach seems less involved and quicker (Mehta, 2014). A closer look, though, will reveal more about the mathematics underlying these approaches. The approach taken utilizes benchmarks, recognizing that 12 is 3 away from 15, that 15 is 5 away from 20, that 20 is 10 away from 30, and that 30 is 2 away from 32. The important implications from problems presented in Figure 11 and others like it is that the CCSSM does not dictate methods for problem solving other than some attention to the standard algorithms and some attention to the number line. It is inaccurate to claim that the CCSSM require students to solve problems using the method shown at the bottom of Figure 11. For instance, an alternative solution using the number line is illustrated below in Figure 12, and this strategy would be an acceptable strategy that is aligned to the CCSSM. Using number sense, a student can first move from 32 to the closest ten, which is 30, by moving two spaces left on a number line. Then, 12 - 2 = 10, so the student only has to move 10 more spaces to the left, and going 10 left from 30 results in 20.

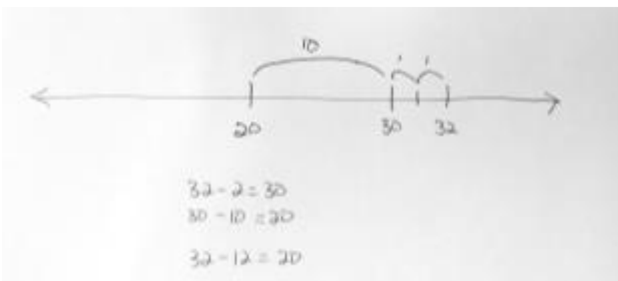


Figure 12. Alternative strategy for the problem 32 - 12 presented in Figure 11.

Oftentimes students can carry out procedures and calculate answers, but when asked about the mathematics behind their actions are unable to respond (Mehta, 2014). The CCSSM aim for students to develop number sense and alternative methods before learning the standard algorithms. The use of representations such as the number line is an important part of that development. Thus, Figure 11 has been widely misinterpreted as being representative of what strategies the CCSSM require students to use to solve problems. It is important to note that, the CCSSM do not dictate a specific way for teaching students to operate with numbers, and not only include exposure to alternative methods but include the use of standard algorithms as well (e.g., NGA & CCSSO, 2010, p. 29).

This section has explored some of the common public perceptions regarding the teaching of standard and alternative algorithms in conjunction with the use of representations. Underlying these perceptions is a larger issue regarding the implementation of the CCSSM with regards to communicating to parents the importance of algorithms and representations. Additionally, there are issues to be considered of curricula that appear to be aligned to the CCSSM when, in reality, are not (Schoenfeld, 2014). Some of the public perceptions arise because of misunderstandings about what the CCSSM are and what they are not. This article attempts to shed some light on these issues and also to provide a more holistic view regarding algorithms and representations such as the number line and their role in mathematics under the CCSSM.

Conclusion

Algorithms and representations are an important part of mathematics as their use affords students the opportunity for connection building among concepts and offers ways to communicate and organize mathematical ideas (Lesh & Harel, 2003; Lesh et al., 1987; NCTM, 2000). As such, they have been an explicit part of past standards documents and the current CCSSM. However, the widespread (mis)use of algorithms and representations in curricular materials claiming to be aligned to the CCSSM have contributed to a public misconception that the CCSSM demand particular approaches to teaching and learning mathematics. Instead of being based on knowledge of the CCSSM, this assumption is based on parents' perceptions from seeing their children's homework problems that involve the number lines or alternative strategies and assuming that these representations are the *only* ways the CCSSM dictate mathematics be taught. The standards do include the use of the number line, which is explicitly discussed, but it is not *the only* representation to support the development of number sense. The CCSSM introduce students to both informal and formal ways of reasoning about mathematics. Ideally, the learning is situated so that algorithms naturally follow the development of understanding (McCallum, 2015). In each of the grades, there are standards that discuss exposing students to a variety of representations and strategies for operating with numbers and for exploring and analyzing data. Thus, algorithms and representations are a necessary part of the standards, and it is important to realize that the standards are goals and expectations for each grade level, but do not specify how to teach particular topics and what materials are to be used. Schoenfeld (2014) claims the major issue is not necessarily with the standards themselves but with the curricular materials that are supposed to be aligned to the standards. Ultimately, we need to ensure that parents understand that the CCSSM are built upon years of research and to help them see the benefits and importance of going beyond procedural computation and fostering a deeper understanding of the mathematics (Otten & De Araujo, 2015).

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