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Reframing Research on Methods Courses to Inform Mathematics Teacher Educators′ Practice

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Calls have been made for the creation of a shared knowledge base in mathematics teacher education with the power to inform the design of scholarly inquiry and mathematics teacher educators' (MTEs) scholarly practices. Focusing on mathematics methods courses, we summarize and contribute to literature documenting activities MTEs use in mathematics methods courses. We suggest two strands of a research program that have the potential to structure and inform the development of MTEs' scholarly inquiry and practices. A summary of findings from research studies exploring one type of methods course activity is shared. The research is explored for its potential to contribute to two different strands of the research program and to MTEs' practices. Recommendations and implications for inquiry that supports MTEs' practices are provided.

Researchers studying mathematics methods courses have identified broad differences in emphases and instructional approaches (Harder & Talbot, 1997; Taylor & Ronau, 2006; Watanabe & Yarnevich, 1999). Particularly salient to mathematics teacher educators (MTEs) are questions regarding which course activities and experiences might affect prospective teachers' (PTs') future practices. To support

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MTEs' work, there is a need for research that explores connections between course activities and experiences of PTs and that explores contexts of such activities. Arbaugh and Taylor (2008) addressed this need with a call for a mathematics teacher education research program. Lee and Mewborn (2009) extended this call by suggesting the field undertake scholarly inquiry to support the design and development of MTEs' scholarly practices.

The development of a research program into methods courses and practices of MTEs, with well-defined areas of inquiry, addresses the calls of Arbaugh and Taylor (2008) and Lee and Mewborn (2009). The purpose of this paper is to introduce two interrelated strands (learning and landscape) contributing to a mathematics teacher education research program. The *learning strand* focuses on exploration of links between activities MTEs create and implement in methods courses and understanding PTs' experiences and development as a result of these activities. The *landscape strand* focuses on understanding MTEs' practices and the development of those practices. To support our discussion of these strands, we begin by summarizing existing research regarding activities employed in methods courses and present findings from a survey of activities employed by MTEs. We then discuss the landscape and learning strands. Possibilities of research within both of these strands are explored through a summary of empirical research reports addressing one type of methods course activity, dynamic interactions. We conclude by discussing how the field might build from these two strands to support the development of scholarly inquiry and practices in mathematics teacher education.

Scholarly Inquiry and Scholarly Practices

Bergsten and Grevholm (2008) describe MTE practices as derived from "previous experiences as teachers in school or in university, or 'anew' based on research and experimentations" (p. 233). These approaches are necessary because the research base on mathematics teacher preparation is emergent. Mewborn (2005), following Cooney (1994), called for the development of "frameworks in the areas of mathematics teaching and teacher education that would parallel" (p. 4) those used in studying student learning. Yet links between MTEs' practices and the "development of knowledge in teaching" (Jaworski, 2008b, p. 2) are difficult to discern. MTEs often rely on *practical knowledge,* developed through participation and reflection on daily practice (Arbaugh & Taylor, 2008), that is locally productive in work with PTs and school personnel. Stakeholders, however, tend to view such knowledge as unsanctioned or anecdotal (Grossman, 2008). To elevate the status of MTEs' practical knowledge, we align with Mewborn (2005) and suggest that mathematics teacher education needs to produce a knowledge base for and about MTEs' practices.

Lee and Mewborn (2009) emphasized the interplay between research and practice in mathematics teacher education in their discussion of the significance of MTEs' *scholarly inquiry* and *scholarly practices*. The researchers describe scholarly inquiry as the explorations of "issues and practices through systematic data collection and analysis that yields theoretically-grounded and empirically-based findings" and scholarly practices as "adapted from empirical studies of the teaching and learning of mathematics and the preparation of mathematics teachers" (p. 3). Lee and Mewborn's descriptions of scholarly inquiry and practices illustrate how the two areas of scholarship are interrelated, in that MTEs use empirical studies to build practices that are labeled scholarly. In addition, scholarly practices can inform directions for scholarly inquiry regarding PTs' mathematics teaching and learning.

The empirical research base MTEs need to build scholarly practices is growing slowly, particularly in comparison to knowledge about students' mathematics learning. For example, research on students' development of proportional reasoning (e.g., Lamon, 1993) has motivated the creation of classroom activities and curricula to support student development. Recognizing the critical role of teachers in supporting students' proportional reasoning, researchers have in turn, begun exploring teachers' knowledge of proportional reasoning for teaching (e.g., Lobato, Orrill, Bruken, & Jacobson, 2011; Orrill & Burke, 2012). Although tentative findings have been reached, generating curricula for MTEs has only just begun (Lobato & Ellis, 2010). Still more remote are findings that would reveal how MTEs might effectively use such resources to support the development of PTs' knowledge of proportional reasoning for teaching. This example illustrates the temporal challenge of moving from findings in student learning to activities to support PTs. A more direct research agenda is needed, aimed at mathematics teacher education and focused on MTEs' practices and PTs' development.

Practices in Methods

Several researchers have attempted to gain perspective on MTEs' practices (Harder & Talbot, 1997; Taylor & Ronau, 2006; Watanabe & Yarnevich, 1999). In 1997, Harder and Talbot identified school district stakeholders' concerns about teacher preparation as motivation for exploring "how mathematics methods courses are taught" (p. 3). To address this concern they examined course syllabi from members of the Association of Mathematics Teacher Educators (AMTE) for evidence of activities and PTs' experiences used in methods courses. Writing assignments such as the development of "papers on technology, a position on current issues, textbook analysis, curriculum" (p. 6) were most common. Additional course assignments found in the syllabi were PT presentations or micro-teaching and the development of lesson or unit plans. Harder and Talbot concluded that MTEs commonly asked PTs to share evidence of their thinking yet the focus of PTs' thinking may vary greatly. This finding suggests that what PTs learn may vary as well.

Watanabe and Yarnevich (1999) were motivated by differences in mathematics teaching advocated in the late 1990s as compared to 20 years earlier. To explore what "different groups of people believe should be taught in preservice mathematics methods courses" (p. 2), the authors developed a survey based on the *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics, 1991). In contrast to the work of Harder and Talbot (1997), Watanabe and Yarnevich (1999) focused on what should be taught in elementary methods rather than how mathematics methods should be taught. Insights were drawn from PTs, supervisors of PTs, in-service teachers, and AMTE members who taught elementary methods. Participants scored a list of topics on a Likert scale from 1–4, with 4 suggesting that the "topic must be included" (p. 2). Standard deviations on topics rated by methods instructors were higher than for other subgroups of participants, suggesting substantial variation in methods instructors' assessment of what topics should be included in methods courses. Watanabe and Yarnevich further identified topics with mean scores among methods instructors of less than 3.0 and among teachers and supervisors of more than 3.0. Two such topics were *unit* planning and *yearly* planning. This difference in the views of MTEs and supervisors prompted Watanabe and Yarnevich to suggest that methods instructors strive to meet the planning expectations of schools and build PTs' skills in "long-range planning" (p. 6). In contrast, but consistent with the findings of Harder and Talbot (1997), methods instructors, supervisors, and in-service teachers all identified *lesson* planning as important (mean greater than 3.5).

In 2006, Taylor and Ronau again examined AMTE members' syllabi to gain insights into "activities in a typical mathematics methods course," "common goals" in such courses, and "common characteristics" of these courses (p. 12). They examined 58 syllabi, which were categorized by grade level, and reported:

> There were 22 elementary syllabi, of which 15 addressed grades K-6 (or some subset of this) and seven addressed grades K-8. Five of the syllabi addressed middle school, 27 addressed secondary, which included two syllabi addressing middle-secondary (3-12), and 25 addressed 7-12 (or some subset). Finally, four syllabi outlined K-12 methods. (p. 12)

Although Taylor and Ronau did not differentiate their findings by grade band, they reported that the syllabi "revealed literally dozens of different activities," (p. 13) pointing to substantial variation in methods courses. These activities were developed into categories including case analyses, lessons and lesson planning, and reflections. Taylor and Ronau further inferred course goals from evidence in the syllabi and looked for links between these goals and activities. They found "[s]yllabi that labeled goals and assignments in ways that connected them were rare" (p. 14). Taylor and Ronau used the inferred goals and existing research in mathematics education to propose a framework for categories of goals or objectives in methods courses. They suggested further research exploring variation in methods courses to understand the benefits and risks of including some activities and not others.

> Syllabi that are clearly different from the de-facto consensus with respect to what they chose to include, or perhaps more strikingly, what they do not include, may offer quite different experiences for their students. We do not know if their students benefit from these differences or if they miss something crucial. (p. 14–15)

Research exploring methods courses provides insight into commonalities over time. Existing research has reported that lesson planning is an enduring activity that should be included in methods courses. Also identified in each research report is the variation of activities MTEs in methods courses.

Survey of Faculty: Exploring Important Activities

To explore whether the commonalities identified in studies of mathematics methods still stand, we asked MTEs to describe theoretical frameworks they used to structure their methods courses and 2 or 3 important activities they used in methods courses. In this paper, we report on the activities identified by MTEs. Seventy-nine MTEs responded to a call for participants distributed electronically via two mathematics education listserves between October 28 and December 5 of 2011. Many of the respondents reported teaching methods for more than one grade band: approximately 20% taught early childhood methods, 50% taught middle grades methods, 55% taught elementary grades methods, and 61% taught secondary methods.

Data Analysis

Atlas ti (Atlas.ti, 2011) was used to analyze the data. Electronic cards containing quotes from survey respondents were created and sorted into activity categories (see Table 1). The activity categories were developed using what Merriam (2009) described as an intuitive process "informed by the study's purpose, the investigator's orientation and knowledge, and the meanings made explicit by the participants themselves" (Merriam, 2009, p. 183-184). To illustrate our approach, we share representative quotes, from different participants, that were grouped to form one activity category: Interviews/interventions with K–12 students about their mathematical thinking.

- "Cognitive interviews with children"
- "A series of three interviews with the same Focus Student; the objective is to understand what a student understands, and consequently, develop another problem situation which will lead to advanced understanding, and then a third iteration; teacher candidates are to be consistent with the mathematical concept, which helps them develop an understanding of the components of a mathematical concept"
- "Child Interaction Project - PST selects to perform a child interaction either with one child 6 times or 2 children 3 times each to engage in a 30-minute session around a non-routine task. The role of the PST is to listen to the child's thinking, and the PST refrains from telling the child what to do. A report is generated following the 6 interactions to describe what the PST learned in the process of creating the plans of action, implementing the tasks, determining what comes next, and connecting to field experience, class activities and course readings."

- "Interviewing a student and analyze his/her understanding of whole number and place value."

For activity categories with a large number of quotes we explored potential subcategories.

Findings

We identified 16 activity categories from the survey responses (see Table 1). Three of these categories (Planning, Experiencing Mathematics, and Student Thinking) contained over 20 data points and differences in quotes that provoked the creation of activity sub-categories. N reflects the number of respondents identifying an activity as important. As respondents were asked to describe 2–3 important activities, the total of all responses does not sum to 79.

Table 1
Categories of Methods Activities Identified by Survey Respondents

Activities	N
1. Planning	
a. Lesson planning, general	23
b. Microteaching	13
c. Facilitating with undetermined audience	8
d. Unit planning	7
e. Microteaching and video	6
f. Existing lessons and activities	6
g. Lesson revision	4
h. Lesson study	4
i. Teaching K–12	2
2. Experiencing Mathematics	
a. Making sense of PTs' own mathematics	12
b. Problem solving	8
c. Real-world applications	5
3. Student Thinking	
a. Interviews/Interventions with K–12 students about their mathematical thinking	12
b. Analyzing student work/error analysis	8
c. Analyzing video of student thinking	2
4. Manipulatives/Technology	17
5. Assessment	12
6. Discourse-Focused	10
7. State and National Standards	9
8. Reading Reflections	8

Table 1

9.	Classroom Management/Environment	6
10.	Studying and Modeling Best Practices	6
11.	Task Analysis	6
12.	Video and Case Studies	5
13.	Addressing Individual Student Needs	4
14.	Diversity-Focused	3
15.	Philosophy	3
16.	Field Visits	2

Similarities and a Difference

Two findings from the survey are consistent with findings from prior research (Harder & Talbot, 1997; Taylor & Ronau, 2006; Watanabe & Yarnevich, 1999). First, planning continues to be identified as an important activity in methods courses. Responses to our survey suggest that MTEs use a variety of forms of this activity to achieve what we hypothesize may be different goals. For example, some MTEs asked PTs to use existing lessons and activities in planning; other MTEs asked PTs to engage in lesson study as a planning activity. Second, MTEs identified a wide variety of important activities they use in methods. Beyond just identifying variation as a characteristic across methods courses, the survey results allowed us to begin to identify potential differences in PTs' experiences stemming from one activity category. For example, in the interviews/interventions activity subcategory, PTs in one methods class conduct "a series of three interviews with the same focus student," while those in another conduct interviews designed to gather evidence of "understanding of whole number and place value." This finding suggests that further exploration into MTEs' activities is needed to understand different instantiations of an activity and PTs' experiences with the activity.

Survey results also provided new insights into activities used in practice. In particular, our findings suggest that MTEs value engaging PTs in exploring K–12 student thinking, whether in static forms such as reviewing student work or in dynamic forms such as interviewing K–12 students. This category of activities, we call dynamic interactions, involves PTs in interacting with K-12 students and making sense of student work/thinking to respond. The inclusion of dynamic interactions as an activity used by MTEs in the survey results suggests MTEs are responding to PTs' needs by providing more authentic opportunities to learn to teach. It may also be true that MTEs are drawing from visions of teaching and learning mathematics described in research and reform and policy documents. Jaworski (2008a) notes that it is part of "the culture of MTE activity" (p. 355) that MTEs act in ways that promote their own development and that of others including reading and discussing research and reform documents. It is not clear from the survey responses whether the activities reported are scholarly practices drawn from explorations and understandings of research in mathematics education or practical knowledge built through systematic reflection on the needs of PTs. What is clear is that scholarly inquiry in mathematics teacher education should be expanded and summarized to support MTEs as they seek to develop their practices.

Building Scholarly Inquiry into a Productive Research Program

Drawing on the call for the development of a shared knowledge base (Arbaugh & Taylor, 2008), the need for further scholarly inquiry on which scholarly practices can be based (Lee & Mewborn, 2009), and findings from explorations of methods, we posit two strands for a research program that could serve to support the development and understanding of MTEs' practices. The first strand addresses the need for studies of MTE's scholarly practices and practical knowledge and their effect on PTs' development. Such studies would be part of what we call the *learning strand*, in that researchers are working to connect MTEs' teaching and PTs' learning. Studies contributing to the learning strand are commonly published in journals such as *Journal for Research in Mathematics Education* (JRME) and *Journal for Mathematics Teacher Education* (JMTE). Studies we classify as belonging to the learning strand often focus on the development or application of frames useful for examining outcomes of methods course activities and/or learning goals. Participants in these studies tend to be PTs rather than MTEs. Studies in the learning strand would include MTEs' goals for instruction and describe activities used to address such goals. In addition, studies summarizing or synthesizing existing research on PTs' experiences with an activity would belong to the learning strand. Studies in the learning strand provide MTEs with examples and insight about activities they might use as the basis for scholarly practice.

The second strand, *landscape strand,* differs from the learning strand in that it is comprised of reports that describe the landscape of MTEs' practice by either zooming in on specific details of MTEs' practice, or zooming out to look across descriptions of MTEs' practice. Our initial view is that three types of research would be included in the landscape strand. First, the landscape strand would include research summarizing commonalities and differences across a mathematics methods activity and MTEs' practice. Research reports of this type would highlight design features of an activity described in empirical studies and identify associated PT experiences. MTEs could use these reports to consider benefits of including or excluding an activity design feature to meet contextual needs and goals. A second type of report, exploring the development of MTEs' practices and contexts for those practices currently exists primarily in the domain of self-study research in teacher education (Chauvot, 2009) or in book chapters (Roth McDuffie, Drake, & Herbel-Eisenmann, 2008; Sanchez & Garcia, 2008). Local contexts such as university and program level affordances and constraints; relationships and negotiations with school districts, teachers, and PTs; and associated influence of policy and vision documents such as the *Common Core State Standards for Mathematics* (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) play significant roles in the ways MTEs design and implement activities. MTEs are aware of the role of such factors in their work, yet reports tend to include little or no detail about the context and the negotiation of tensions that could occur to implement activities in methods courses. This second type of report makes the work of MTEs more transparent and allows for insight into the complexity of MTEs' development (Monroe, 2013; Tzur, 2001) of relational practice (Fletcher, 1998; Grossman et al., 2009) that current empirical studies do not. Such reports provide records of practice and serve as sources of professional development for MTEs seeking to build understanding of how to build practices in university and school contexts.

The existence of reports that describe the development of MTEs' activities and account for the role of context provide the research community with the opportunity to develop a third type of research report. Reports looking across descriptions of contexts over time can describe the historical evolution of MTEs' practices and can look for the impact of policies and historical events on such practices. It is currently difficult to track and develop a historical perspective on the influence of contexts and policy on MTEs' practices. Without the existence of reports zooming in on context, we have no hope of zooming out to understand the landscape of MTEs' practices.

Exploring the Learning Strand

To explore the potential of existing literature in mathematics teacher education to support MTEs' practices, we examined four mathematics education journals for articles focused on activities used in mathematics methods courses. We began by searching the *Journal for Mathematics Teacher Education* (JMTE), the flagship journal for research in mathematics teacher education. We later added three journals included in the Social Science Citation Index (*Journal for Research in Mathematics Education* (JRME), *Cognition and Instruction* (C&I), and *Educational Studies in Mathematics* (ESM)) in hopes of broadening our collection of reports. Our use of these journals was not intended to comprise an exhaustive review of the literature, but rather to sample from the most prominent mathematics education research journals. JRME, JMTE and ESM are three of the six Tier 1 journals in mathematics education as identified by Williams' (2008) venue study. *Cognition and Instruction* was also identified as a prominent and influential venue, though it was not as strongly associated purely with mathematics education. As our intent was to assess the status of research in mathematics teacher education, we assumed a survey of these journals would be sufficient.

We searched each journal (inception to March, 2012) for articles containing the term “preservice” or “prospective.” In addition, we read abstracts from all issues of articles in the identified journals. From these we selected only reports of findings from empirical studies of mathematics methods course activities. Articles reporting on activities used in content courses, student teaching, and in induction were not included. We then identified the activities described in the remaining articles. All activities designed to address PTs’ knowledge of mathematics were eliminated.

We defined *activities* as situations MTEs provide for PTs and *experiences* as “ways in which the preservice teachers internalized those activities” (Mewborn, 2000, p. 31). This relationship between activity and experience parallels the description of the relationship between task and activity provided by Watson and Mason (2007) in their overview of the complexity of mathematical task design. Watson and Mason claim that the task as designed and conceived by the teacher and the experiences of the student with the activity are different, but related. The intended purpose of the task may be enriched or diminished in students’ experiences. In mathematics teacher education, the purpose of an activity may also be enriched or diminished in PTs’ experiences. We call evidence of PTs drawing on or reconsidering an experience during methods the *impact* of an activity. We distinguish between PTs’ experiences that occur within the methods course and the *residue* of such experiences beyond methods courses. In a mathematics course, residue refers to the mathematics retained by students as a result of solving problems or completing a specific task (Davis, 1992). We define residue in mathematics teacher education as evidence of PTs drawing on and reconsidering experiences, developed during methods activities, after the conclusion of methods.

The description of an activity allowed us to develop a way of associating each report with an activity category from our survey of mathematics methods instructors. In reports that included multiple activities, we identified the central activity and used this description to classify the research reports. For example, Norton and Kastberg (2012) used letter witting with secondary PTs in order to study their development in posing cognitively demanding tasks for Algebra II students. The researchers studied PTs' analyses of tasks for levels of cognitive demand (LOCD) and abilities to design tasks with higher LOCD. The investigation identified understanding students' mathematical thinking as a potentially significant factor in designing and implementing cognitively demanding mathematical tasks. Although the goal for the researchers was to develop PTs' abilities to pose cognitively demanding tasks, we did not characterize this study as focused on task analysis. Instead we identified the PTs' dynamic interactions with students as the core activity, with levels of cognitive demand serving as the lens through which to examine the interactions between PTs and students.

As a result of this process of identification, we were able to associate the articles we identified with single activities. We recognize that this process may be a simplification of the design of the instructional approach used in a study, but we claim that this approach was necessary to identify activities, and associate them with experiences or impacts. Although we understand that the evidence of experience or impact reported in an article may have been the result of multiple activities, it was, in part, the result of the activity with which it was ultimately associated.

We read and summarized the reports using a common template including descriptions of activities, context, theoretical framework, experience, and residue. This careful exploration of the reports enabled us to associate activities in the reports with activity categories from our survey analysis. For example, a category of activities, described in research articles, we called "dynamic interactions" was aligned with the survey subcategory "Interviews/interventions with K–12 students about their mathematical thinking." Examples of dynamic interactions from the survey data include letter exchanges, interviews with learners, and work with small group of learners for one year. Only two activity categories from our survey contained more than five research reports: dynamic interactions and video and case studies. Next, we share our summary of reports from the dynamic interactions activity category to illustrate the potential of existing scholarly inquiry to address the learning and landscape strands of the suggested research program.

Dynamic Interactions Summary

Our search of the four research journals unearthed seven reports focused on the activity dynamic interactions. Four articles reported evidence of PTs' experiences resulting from face-to-face interactions with students, and three included evidence from letter writing. Table 2 includes the reference for each report and briefly describes the intervention, context, and findings. We follow the table with a summary of the interventions and findings in light of their contribution to the learning and landscape strands.

Table 2
Article Summaries: Dynamic Interactions Activities

Reference	Interventions, Context, and Findings
Face-to-Face Interactions	
Ambrose, R. C. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education, 7*, 91–119.	Elementary PTs worked in pairs to pose open-ended problem solving activities focused on whole number operations and fractions. The goal was to impact PTs' beliefs about teaching, potentially shifting their beliefs from teaching as explaining, by leveraging their current beliefs as caregivers. Ambrose concluded PTs developed new beliefs that were incorporated in existing belief structures.
Jenkins, O. F. (2010). Developing teachers' knowledge of students as learners of mathematics through structured interviews. *Journal of Mathematics*	Six middle grades PTs worked in pairs in alternating roles to pose open-ended tasks focused on patterns and proportions to students. Jenkins searched for evidence of PTs' "interpretive listening skills and awareness of the different ways that middle school students make sense of mathematics" (p. 147). Jenkins reported

Table 2

Teacher Education, 13, 141–154.	"the structured interview process fosters an interpretive orientation to listening and initial awareness of the variety of ways that middle school students think about mathematics" (p. 147).
Moyer, P. S., & Milewicz, E. (2002). Learning to question: Categories of questioning used by teachers during diagnostic mathematics interviews. *Journal of Mathematics Teacher Education, 5,* 293–315.	Forty-eight PTs used scripted protocols and rational number tasks to conduct interviews with children throughout a semester. PTs' final interview was recorded, transcribed, analysed, and reflected upon by the PTs and served as evidence of PTs' experiences and use of questioning. Analysis of the interviews resulted in a classification for the types of questioning: (a)"checklisting," asking the questions in the protocol, with little regard for student responses; (b) "instructing vs. assessing," in which PTs explained mathematics directly to the student with little regard for students' reasoning; and (c) "probing and follow up questions," characterized as PTs genuinely listening to student responses and generating follow-up questions meant to elicit further student thinking. The authors noted PTs rarely employed "probing and follow up questions."
Nicol, C. (1998). Learning to teach mathematics: questioning, listening, and responding. *Educational Studies in Mathematics*, *37*(1), 45–66.	Fourteen PTs were engaged in weekly interactions with small groups of 6th and 7th grade students. The PTs solved problems involving multiplicative reasoning in class and then posed adapted or extended versions of these tasks to students. Nicol examined PTs' abilities to question, listen and respond to students using prospective teachers' journal reflections as sources of evidence for these behaviors. Across the weekly implementations of the activity, PTs began to shift their approaches from those that focused on arriving at a correct answer toward an inquiry-based approach to teaching mathematics focused on eliciting and understanding student thinking. With this shift, PTs came to question their mathematical knowledge and began to view teaching mathematics as "…more complex, difficult, uncertain, and risky" (p. 63).

Table 2

Letter Writing Interactions

Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education, 3*, 155–181.	Crespo examined the ways in which elementary PTs listened to the responses of the fourth-grade students in a series of six interactive letter exchanges. PTs' initial interpretations of student work focused on correctness and tended to contain conclusive claims about student understanding based upon small samples of student thinking. Reflective journals were used by PTs to explore their interactions with students. PTs' interpretations began to focus on what the student intended or meant in a solution by the fifth week of the course.
Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers" practices. *Educational Studies in Mathematics, 52*, 243–270.	Crespo used the same letter writing activity and data to explore PTs' abilities to pose problems. Initially, PTs attempted to "make their problems less problematic and more attainable to their pupils" (p. 251). PTs' questions were worded to avoid student errors or confusion rather than to generate learning opportunities for students or themselves as teachers. Problems included in the last three letters "were puzzle-like and open-ended, encouraged exploration, extended beyond topics of arithmetic, and required more than computational facility" (p. 257). These questions were posed to challenge or extend student thinking and often asked for multiple solutions and explanations. Overall PTs' earlier approaches to problem posing tended to occur less frequently, yet Crespo noted PTs approaches did not change in the same way, along the same trajectory, or at the same rate across letters.
Norton, A., & Kastberg, S. (2012). Learning to pose cognitively demanding tasks through letter writing. *Journal of Mathematics Teacher Education, 15*, 1–22.	Norton and Kastberg used letter witting with secondary PTs to study their development in posing cognitively demanding tasks for Algebra II students. The researchers studied PTs' analyses of tasks for levels of cognitive demand (LOCD) and letter pairs to explore PTs' abilities to design tasks with higher LOCD. The investigation identified understanding students' mathematical thinking as a potentially significant factor in designing and implementing cognitively demanding mathematical tasks.

Contributions to the Learning and Landscape Strand

Our proposed learning strand involves examining the relationship between dynamic interactions as methods course activities and their influence on PTs' experiences, impact, and residue. The empirical evidence provided in the existing research on dynamic interactions suggests that involving PTs in dynamic interactions can (a) develop PTs' knowledge of students' mathematics and strategies (Ambrose, 2004; Crespo, 2000), (b) encourage PTs to shift their focus in working with students from attaining correct answers to eliciting and understanding student thinking (Crespo, 2000; Jenkins, 2010; Moyer & Milewicz, 2002), and (c) develop PTs' emerging abilities to use student thinking in crafting responses and posing new problems (Crespo, 2003; Norton & Kastberg, 2012). Although the reports provided evidence of experiences and impact, we did not find empirical research examining the residue of dynamic interactions. We see the exploration of residue from dynamic interactions as an area for future scholarly inquiry contributing to the learning strand.

Our proposed landscape strand has the potential to help MTEs make sense of the variety of contextual factors involved in enacting dynamic interactions, and provide insight into variation involved in different implementations of the same activity. Descriptions of rationales for including dynamic interactions in methods courses and design elements for this activity associated with PTs' experiences provide options MTEs can weigh as they adapt activities for their local contexts.

Reports of studies of dynamic interactions provided two rationales for including the activity in methods courses. Dynamic interactions (a) provided PTs opportunities to engage with an "authentic audience" (for example, Crespo, 2003, p. 265) that would respond to the problems and questions posed, and (b) afforded PTs the opportunity to engage in a simplified form of a complex practice by focusing on one student's mathematical thinking. These rationales are consistent with professional practices identified by Grossman et al. (2009), in particular the implementation of approximations of practice and deconstructions of practice that have as their central goals the utility suggested in the dynamic interactions literature.

MTEs seeking to implement dynamic interactions in their methods courses may want to include two design elements researchers have identified as positively influencing PTs' experiences: opportunities to solve new mathematics problems and to reflect on experiences interacting with their student. PTs should be provided with opportunities to solve challenging and unfamiliar mathematical problems, prior to posing variations of them to students. MTEs considering the use of this activity should begin by asking PTs to pose the same or similar problems to students. This approach allows MTEs to facilitate discussions of the PTs' experiences with students' approaches to the same task, students' representations of their thinking, and struggles PTs may have had when interacting with their students.

PTs' development may also hinge on opportunities they have to reflect on and revisit experiences they had with students. Whole-class and small-group discussions built from common experiences provide one opportunity for reflection. MTEs should also provide PTs with opportunities to generate a written reflection after each interaction with the students. MTEs sustain and provoke alternative interpretations of PTs' experiences by responding to their reflections. PTs should be encouraged to reflect across the totality of the interaction experience to provide additional opportunities for learning. Having a digital or written record of interactions, allows PTs to revisit their actions and decisions they make at various points in the course. The culminating reflection can be powerful as it allows PTs to examine their experiences from a different perspective. Through the process of revisiting their prior decisions, struggles and successes, PTs can see evidence of their own development, and their growth as teachers.

Aside from the commonalities found across these studies, there is also considerable variation in the ways dynamic interactions were structured for PTs. Beyond grade level differences such as use with elementary, middle grades or secondary PTs, the design of these dynamic interactions also varied in mathematical content, PTs' autonomy within their interaction, and intended impact on PTs' knowledge, beliefs, skills, and dispositions. Each of these variations highlights elements of activity design, which should be investigated in future research.

Researchers asked PTs to utilize tasks from a variety of topics from number and operation in their interactions with students. Topics ranged from whole number problems utilizing the four operations, to rational number, to multiplicative and proportional reasoning. Some researchers left the choice of content open to PTs, further increasing the variety of mathematics emphasized in these experiences. If one takes the stance that mathematics is not monolithic, and that PTs can hold different knowledge and beliefs for different content areas, it becomes paramount to systematically examine the benefits and drawbacks of utilizing different content with PTs preparing for different grade bands.

The autonomy PTs were afforded within dynamic interactions also varied across the studies we investigated. In some activity designs, PTs were given complete control over the mathematical topic, the selection and wording of individual tasks, and were free to guide the interaction with their student as they saw fit. In other activities PTs were given less autonomy, as mathematical topics were pre-determined, with PTs modifying or selecting problems from list of potential tasks. In some cases, autonomy was minimal, as diagnostic interview protocols outlining the tasks as well as potential follow up questions were provided for and utilized by PTs. In the field of mathematics teacher education, we are largely unaware of the affordances and constraints of such choices on PTs' development.

Finally, MTEs acting as teacher/researchers in the research we investigated sought to develop different aspects of PTs' practice using dynamic interactions. Activities were designed to impact PTs' abilities to listen, to question, to respond, or to pose problems with increased levels of cognitive demand. Although each of these aspects of practice is important, how would a MTE select the most appropriate one to address considering the limited amount of time in a methods course?

The answer to such a question can be explored through further investigation within our landscape strand.

Discussion

MTEs' practices identified in existing studies (Harder & Talbot, 1997; Taylor & Ronau, 2006; Watanabe & Yarnevich, 1999) and in our survey suggest that lesson planning is a commonly used activity in methods courses, but beyond that there is substantial variation in activities MTEs use in their courses. We interpret this research as suggesting that scholarly inquiry into activities and the variation across and within activities may have the potential to support the development of MTEs' scholarly practices.

Our exploration of research focusing on dynamic interactions allowed us to illustrate the potential of scholarly inquiry to support the development of MTEs' scholarly practices. Interactions took different forms and were used to impact different facets of PTs' development including: beliefs, listening orientations, awareness of cognitive demand of tasks, and questioning. Evidence from the research reports suggests that PTs' interactions with K–12 students as a methods course activity has the capacity to influence a variety of dimensions of PTs' development as a teacher. For MTEs planning methods courses, this evidence suggests that methods courses should require PTs' interactions with students as a scholarly practice. The evidence further suggests that experiences from and impacts of dynamic interactions may rest on MTEs' goals in selecting and structuring activities as well as methods used in implementing these activities with PTs. Given the number of context factors and decision points in the development of approximations of practice such as dynamic interactions, further research is needed to investigate the additional facets of the landscape strand.

Existing research on dynamic interactions illustrates the power and information already available to serve as the basis for developing practice, while highlighting the need for reports that dig into the contexts of MTEs' work. A research program that supports MTEs seeking to develop practices in the context of methods courses would contain strands focused on exploring learning and the landscape. The development of the learning strand could include studies linking MTEs' practices and the PTs' development. Such studies would provoke questions about MTEs' practices and provide tools that could be used to defend and advance claims that mathematics methods courses are a critical component in teacher development.

The call for a learning strand in a mathematics teacher education research program is consistent with the recommendations of Kazemi, Franke, and Lampert (2009) and Simon (2008). Although the views and suggested directions of these authors differ, they share a common rationale for their efforts: building PTs' practices that support children's learning of mathematics. As researchers conduct scholarly inquiry designed to inform MTEs' practices and contribute to existing scholarly inquiry, the need to hypothesize about facets of development, such as building expert performance in the form of instructional routines, must drive the design of research. For example, such instructional routines might include "posing a sequence of… related computational problems" (Kazemi, et al., 2009, p. 14). MTEs must further identify the concepts needed for mathematics teaching. As Simon (2008) suggests, MTEs have not yet made significant progress describing what he calls "pedagogical concepts" (p. 20) such as "understanding of classroom norms and their negotiation" (p. 20) and "understanding assimilation" (p. 21). Without continued exploration of specific goals and impacts of methods instruction, the design and implementation of activities is likely to continue to produce a broad array of impacts on PTs' development as has been reported in empirical studies.

Additional studies of existing activities such as lesson planning are also needed and will likewise contribute to the learning strand. Our search unearthed few reports on lesson planning (Jansen & Spitzer, 2009; Ricks, 2010). The number of MTEs using lesson planning in methods suggests that much more scholarly inquiry is needed to inform the field on the structure and impacts of this activity. Questions about the use of planning have emerged from other reports in educational research in general (John, 2006; Jones & Vesilind, 1996) and in mathematics education (Zazkis, Liljedahl, & Sinclair, 2009) in particular. Jones and Vesilind (1996) found that perspectives on planning and what might be considered planning shifted significantly across PTs' experiences. John (2006) and Zazkis et al. (2009) suggested alternatives to lesson plans that may support PTs to further understand content and gain insight into the possibilities for the enactment of a lesson. These articles suggest that further inquiry regarding the impacts of planning activities used in methods is needed in contribution to the learning strand.

For reports contributing to the landscape strand, we would envision a structure different, yet no less rigorous, than reports included in the learning strand. In particular, to gain understanding of an activity in context, far more would be shared about the development of the activity, the decision points in the process, and a discussion of school and university affordances and constraints than are traditionally included in empirical studies. For example, the use of letter writing might involve substantial negotiations with a school district or teachers and multiple iterations of implementation across which materials for PTs are developed. Identifications and discussions of these important decision points are critical elements MTEs might use to inform their own implementations of activities. Additional elements of such reports might include the context for the activity, view of teacher development/learning held by the MTE implementing the activity, and the facets of teacher development the MTE plans to impact. Journal articles that provide the details described above have the potential to support MTEs' understanding of activities and reduce the number of cycles of implementation a MTE might need to produce the impacts reported in empirical studies. Reports of scholarly practices and practical knowledge could further enable explorations of the variation and evolution of MTEs' practices that would yield characterizations of the field and its development.

Despite efforts to support the growth of research on mathematics teachers by introducing and sustaining publication outlets such as *Journal for Mathematics Teacher Education* (JMTE) and *Mathematics Teacher Educator* (MTE), there is precious little publication space available for the reports we have described. Jaworski's (2008b) insights from her experiences as editor of JMTE suggests that MTEs who share research findings from their practices may not be disposed to share "how their findings from research had influenced their own thinking and impacted their own practice" (p. 351). Yet it is these reflections on the complexity of MTEs' work that can be recognized by colleagues seeking to make sense of and build productive practices. Research reports that contain descriptions of MTEs' practices and the development of such practices in context are needed to serve as the basis for the development of scholarly practices and understanding of the complexity and evolution mathematics teacher education.

Conclusion

Understanding MTEs' practices, variations, and impacts of such practices have been identified (Arbaugh & Taylor, 2008; Lee & Mewborn, 2009; Taylor & Ronau, 2006) as critical areas of future study. We have proposed two strands of research essential to MTEs developing curriculum and pedagogy for methods courses. Research in mathematics teacher education has already begun to contribute to what we have called the learning strand of a mathematics teacher education research program. Continued efforts to build and summarize literature exploring methods instruction and its impacts are needed. We suggest that researchers focus on summarizing or contributing to understanding of activities such as lesson planning commonly used in methods courses. Researchers could also focus on learning goals, such as understanding the development of socio-mathematical norms as suggested by Simon (2008) and explore activities used to achieve these goals. We further suggest a landscape strand needed in a research program focused on informing MTEs' practices. Reports that contribute to the landscape strand will provide detailed descriptions of the development and contexts of scholarly practices and practical knowledge. These descriptions will allow the field of mathematics teacher education to explore the variation in MTEs' practices and to understand the evolution of such practices. As MTEs develop their practice they need additional research on understudied elements of MTEs' practices, such as the role of contexts on practice.

Although the focus of our proposed research program is methods courses, this line of inquiry is situated in the larger exploration of the impact of teacher education programs on PTs' development. Scholarly inquiry into the use of activities and PTs' experiences with the activities is only one facet of MTEs' efforts to support PTs. PTs can develop new understandings of experiences they have had prior to methods courses, such as in mathematics courses, and from experiences they have had in methods courses as they engage in student teaching and beyond. Historic studies suggest that a focus on methods activities and their impact is but one step. For example, Cooney's (1985) account of Fred's commitment to problem solving, initiated in methods and developed through student teaching, began eroding as he faced the challenges of teaching all children. Exploration of the residue of such experiences is needed. MTEs are positioned to develop scholarly inquiry investigating the evolving meanings of methods experiences as PTs engage in other facets of teacher education programs and in practice. In addition, MTEs' roles in teacher education programs uniquely position them to advocate for the construction of a network of academic and field experiences based on findings generated through scholarly inquiry of such experiences and their evolution.

Scholarly practices that are defensible and productive are needed as the field of mathematics teacher education continues to develop. Different methods instructors will always rely on different frameworks and have different goals, but a literature base that provides insights into the work of MTEs in context, variation in scholarly practices MTEs develop, and experiences activities afford for PTs is needed. Continued development and sanctioning of publication outlets for such reports will support MTEs as they continue to create and seek out research that informs their scholarly practice.

References

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education, 7,* 91–119.

Arbaugh, F., & Taylor, M. (2008). Inquiring into mathematics teacher education. In F. Arbaugh & M. Taylor (Eds.), *Inquiry into mathematics teacher education* (Vol. 5, pp. 1–10). San Diego, CA: Association of Mathematics Teacher Educators.

Atlas.ti. (2011). The Knowledge Workbench (Version 6). [Computer Software]. Cologne, Germany.

Bergsten, C., & Grevholm, B. (2008). Knowledgable teacher educators and linking practices. In B. Jaworksi & T. Wood (Eds.), *The interational handbook of mathematics teacher education: The mathematics teacher educator as a developing professional* (Vol. 4, pp. 223–246). Rotterdam, The Netherlands: Sense.

Chauvot, J. (2009). Grounding practice in scholarship, grounding scholarship in practice: Knowledge of a mathematics teacher educator-researcher. *Teaching and Teacher Education, 25*(2), 357–370.

Cooney, T. J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education, 16,* 324–336.

Cooney, T. (1994). Research and teacher education: In search of a common ground. *Journal for Research in Mathematics Education, 25*(6), 608–635.

Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education, 3*(2), 155–181.

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics, 52*(3), 243–270.

Davis, R. B. (1992). Understanding "understanding". *Journal of Mathematical Behavior, 11*(3), 225–241.

Fletcher, J. (1998). Relational practice a feminist reconstruction of work. *Journal of Management Inquiry, 7*(2), 163–186.

Grossman, P. (2008). Responding to our critics: From crisis to opportunity in research on teacher education. *Journal of Teacher Education, 59*(1), 10–23.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record, 111*(9), 2055–2100.

Harder, V., & Talbot, L. (1997). *How are mathematics methods courses taught?* Paper presented at the Annual Meeting of Association of Mathematics Teacher Educators, Washington, DC. http://www.eric.ed.gov/PDFS/ED446936.pdf

Jansen, A., & Spitzer, S. (2009). Prospective middle school mathematics teachers' reflective thinking skills: Descriptions of their students' thinking and interpretations of their teaching. *Journal of Mathematics Teacher Education, 12*(2), 133–151.

Jaworski, B. (2008a). Development of the mathematics teacher educator and its relation to teaching development. In B. Jaworsju & T. Wood (Eds.), *The international handbook of mathematics teacher education: The mathematics educator as a developing professional* (Vol. 4, pp. 335–361). Rotterdam, The Neatherlands: Sense.

Jaworski, B. (2008b). Mathematics teacher educator learning and development: An introduction. In B. Jaworski & T. Wood (Eds.), *The interational handbook of mathematics teacher education: The mathematics teacher educator as a developing professional* (Vol. 4, pp. 1–11). Rotterdam, The Netherlands: Sense.

Jenkins, O. (2010). Developing teachers' knowledge of students as learners of mathematics trhough structured interviews. *Journal of Mathematics Teacher Education, 13*(2), 141–154.

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483–498.

Jones, M. G., & Vesilind, E. (1996). Putting practice into theory: Changes in the organization of preservice teachers' pedagogical knowledge. *American Educational Research Journal, 33*(1), 91–117.

Kazemi, E., Franke, M., & Lampert, M. (2009). *Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious teaching*. Paper presented at the Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia. http://sitemaker.umich.edu/ltp/files/kazemi_et_al_merga_proceedings.pdf

Lamon, S. (1993). Ratio and proportion: Connecting content and children's thinking. *Journal for Research in Mathematics Education, 24*(1), 41–61.

Lee, H., & Mewborn, D. (2009). Mathematics teacher educators engaging in scholarly practices and inquiry. In D. Mewborn & H. Lee (Eds.), M. E. Strutchens (Monograph Series Ed.) *Scholarly practices and inquiry in the preparation of mathematics teachers* (Vol. 6, pp. 1–6). San Diego, CA: Association of Mathematics Teacher Educators.

Lobato, J., & Ellis, A. (2010). *Developing essential understanding of ratios, proportions & proportional reasoning*. Reston, VA: National Council of Teachers of Mathematics.

Lobato, J., Orrill, C., Bruken, B., & Jacobson, E. (2011). *Middle school teachers' knowledge of proportional reasoning for teaching*. Paper presented at the American Educational Research Association, New Orleans, LA. http://www.kaputcenter.umassd.edu/downloads/products/workshops/AERA2011/Lobato_Orrill_Druken_Erikson_AERA_2011.pdf

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco, CA: Jossey-Bass.

Mewborn, D. (2000). Learning to teach elementary mathematics: ecological elements of a field experience. *Journal of Mathematics Teacher Education, 3*(1), 27–46.

Mewborn, D. (2005). Framing our work. In G. M. Lloyd, M. R. Wilson, J. L. M. Wilkins & S. L. Behm (Eds.), *Proceedings of the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1–9). Virginia Tech, Blacksburg, VA: All-Academic.

Monroe, E. (2013). Being and becoming a mathematics teacher educator in and for differing contexts: Some lessons learned. *Studying Teacher Education, 9*(2), 96–107.

Moyer, P., & Milewicz, E. (2002). Learning to question: Categories of questioning used by preservice teachers during diagnostic mathematics interviews. *Journal of Mathematics Teacher Education, 5*, 293–315.

National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington DC: Author.

Nicol, C. (1998). Learning to teach mathematics: questioning, listening, and responding. *Educational Studies in Mathematics, 37*(1), 45–66

Norton, A., & Kastberg, S. (2012). Learning to pose cognitively demanding tasks through letter writing. *Journal of Mathematics Teacher Education,, 15*(2), 109–130.

Orrill, C., & Burke, J. (2012). Proportions, relations, and proportional reasoning: One teacher's navigation between professional development and personal knowledge. In L. Van Zoest, J. J. Lo & J. L. Kratky (Eds.), *34th annual meeting of the North American Chapter of the International Group of the Psychology of Mathematics Education* (pp. 539–542). Kalamazoo, MI: Western Michigan University.

Ricks, T. (2010). Process reflection during Japanese lesson study experiences by prospective secondary mathematics teachers. *Journal of Mathematics Teacher Education, 14*(4), 251–267.

Roth McDuffie, A., Drake, C., & Herbel-Eisenmann, B. (2008). The elementary mathematics methods course: Three professors' experiences, foci, and challenges. In B. Jaworksi & T. Wood (Eds.), *The interational handbook of mathematics teacher education: The mathematics teacher educator as a developing professional* (Vol. 4, pp. 247–264). Rotterdam, The Netherlands: Sense.

Sanchez, V., & Garcia, M. (2008). What to teach and how to teach it. In B. Jaworksi & T. Wood (Eds.), *The mathematics teacher educator as a developing professional* (Vol. 4, pp. 281–297). Rotterdam, The Netherlands: Sense.

Simon, M. (2008). The challenge of mathematics teacher education in an era of mathematics education reform. In B. Jaworksi & T. Wood (Eds.), *The international handbook of mathematics teacher education: The mathematics teacher educator as a developing professional* (Vol. 4, pp. 17–29). Rotterdam, The Netherlands: Sense.

Taylor, M., & Ronau, R. (2006). Syllabus study: A structured look at mathematics methods courses. *AMTE Connections, 16*(1), 12–15.

Tzur, R. (2001). Becoming a mathematics teacher educator: Conceptualizing the terrain through self-reflective analysis. *Journal of Mathematics Teacher Education, 4*(4), 259–283.

Watanabe, T., & Yarnevich, M. (1999). *What really should be taught in the elementary methods course?* Paper presented at the Annual meeting of the Association of Mathematics Teacher Educators, Chicago, IL. http://www.eric.ed.gov/PDFS/ED446931.pdf

Watson, A., & Mason, J. (2007). Taken-as-shared: A review of common assumptions about mathematical tasks in teacher education. *Journal of Mathematics Teacher Education, 10*(4), 205–215.

Zazkis, R., Liljedahl, P., & Sinclair, N. (2009). Lesson plays: Planning teaching versus teaching planning. *For the Learning of Mathematics, 29*(1), 40–47.