The Mathematics Educator
2018 Vol. 27, No. 2, 3–32

The Impact of Math Teachers' Circle Participation: Case Studies

Brianna Donaldson, Michael Nakamaye,
Kristin Umland, and Diana White

The Math Teachers' Circle (MTC) model is a widely implemented form of professional development that invites teachers to become practitioners of mathematics, thereby developing their fluency with mathematical practices. This article presents comparative case studies investigating the self-reported experiences of several MTC-participating teachers representing a range of mathematical backgrounds. Teachers' reflections were examined through the lenses of three themes: views of mathematics, approaches to pedagogy, and professional engagement and leadership. Cross-case comparisons suggest several ways MTCs may be able to improve their support of teachers' professional growth, for example through discussing mathematical mindsets and incorporating additional leadership opportunities.

> Math Teachers' Circle complements my other more curricular and pedagogic activities. Math Teachers' Circle is an activity I do for me. It is not an activity that translates directly to my classroom, but it is the most important influence on the climate I set in my classroom. Each session gives me renewed energy, enthusiasm, and

Brianna Donaldson *is Director of Special Projects at the American Institute of Mathematics. Her scholarly interests include mathematical identity, professional learning communities, and K-12 and higher-education collaborations in mathematics.*

Michael Nakamaye *is Mathematician in Residence at Illustrative Mathematics. His scholarly interests include teacher professional development and K-12 mathematics curriculum development.*

Kristin Umland *is Vice President for Content Development at Illustrative Mathematics. Her scholarly interests include mathematical knowledge for teaching, teacher professional development, and K-12 mathematics curriculum development.*

Diana White *is Associate Professor of Mathematical and Statistical Sciences at the University of Colorado Denver. Her scholarly interests include mathematics professional development for K-12 and higher-education faculty, and Math Circles for students and teachers.*

> empathy for the task of educating the mathematicians of the future. (Math Teachers' Circle survey respondent)

Mathematical practices such as perseverance in problem solving, constructing and critiquing arguments, and looking for and making use of structure are at the heart of college- and career-readiness standards (National Governors Association, Council of Chief State School Officers, & Achieve, Inc., 2009; Partnership for 21st Century Skills, 2008). Yet, in international assessments, U.S. students struggle with higher-order mathematical thinking, modeling, and reasoning (Organisation for Economic Co-operation and Development, 2013), and evidence suggests that there is still too much focus in U.S. classrooms on procedural rather than conceptual learning in mathematics (National Council of Teachers of Mathematics [NCTM], 2014).

Math Teachers' Circles (MTCs) are a model of professional development that engages teachers as practitioners of mathematics as a scientific discipline (Taton, 2015). Through experiential learning, teachers develop fluency with, and understanding of, mathematical practices. In addition to serving as professional development related to the practices of mathematics, MTCs provide teachers and mathematicians with a long-term, ongoing forum for collaboration, thereby addressing the national need for stable collaborative structures and sources of professional development that combat the all-too-frequent professional isolation of teachers (NCTM, 2014) and of mathematicians (Conference Board of the Mathematical Sciences, 2012). The MTC model began in 2006 at the American Institute of Mathematics (AIM), which is one of six U.S. National Science Foundation-supported mathematics research institutes. Currently implemented by more than 100 sites in 39 states, MTCs reach approximately 2,000 K–12 teachers and 350 mathematicians and mathematics educators. The MTC Network, hosted by AIM, provides centralized mathematical and planning resources for the MTC community, including training and mentoring for new MTCs.

Although the specifics of implementation vary, all MTCs share several core features: (a) regular meetings (6 to 8 per

year); (b) involvement of mathematics professionals from both K–12 and higher education; (c) a problem-solving component with significant time (1.5 hours or more per meeting) devoted to collaborative investigation of non-routine, rich, low-threshold, high-ceiling problems; and (d) additional time for reflection and social interactions to build community. Many local sites launch with an intensive summer workshop, during which teachers participate in 18 to 30 hours of MTC activities, with 6 to 8 follow-up meetings of 2 to 3 hours each held during the academic year. Sites typically hold academic-year meetings for multiple years, with intensive workshops as needed to sustain or increase membership. Most MTCs average 15 to 20 participants per meeting (Silverstein, 2014). The model was initially developed for middle school teachers, but national surveys indicate that participation extends across the K–12 range, with 50–60% of MTC participants teaching in middle schools, 10–20% in elementary schools, and 20–30% in high schools (Silverstein, 2014).

The problem-solving component of each MTC session is intended to support teachers as practitioners of mathematics (Taton, 2015), while modeling high-leverage instructional practices such as supporting learners in productive struggle, posing purposeful questions, and encouraging meaningful mathematical discourse (NCTM, 2014). A common pedagogical model for meetings—although none is prescribed—is the "you-y'all-we" format (Green, 2015). A facilitator (typically a mathematician, a master teacher, a mathematics educator, or a teacher-professor pair) poses a problem, question, or situation, allows participants to puzzle over the challenge on their own, and then asks them to collaborate with peers. The facilitator serves as a "more knowledgeable other" (Vygotsky, 1978), who circulates and offers critical feedback, but also respects and validates participants' own, creative approaches. Occasionally, the facilitator explains a particular foundational concept, but the majority of the time is highly interactive.

Although the problem-solving component is fairly similar across MTCs, the reflection component varies based on the needs and interests of local teachers. Many MTCs have a

"debriefing" after the problem-solving component, during which participants reflect on what they have learned as practitioners of mathematics and of teaching. The participants may consider connections with the content or practice standards they teach (Manes, 2015) or discuss which of the NCTM high-leverage teaching practices they observed the facilitator using (Karakok, 2016). Some MTCs also provide more structured opportunities for teachers to incorporate their experiences into their teaching. For example, teachers might design MTC-related classroom lesson plans or help facilitate the problem-solving component of a meeting. In addition, MTCs often serve a complementary role with other professional development and degree programs.

Research Context: The Impact of MTCs on Teacher Participants

Due to its wide implementation and potential effectiveness as a model of professional development, it is important to understand the impact of MTCs on participating teachers. Previous research suggests that there are four primary areas of impact: (a) views about mathematics, (b) classroom practices, (c) professional engagement and leadership, and (d) mathematical knowledge for teaching.

Views About Mathematics

The MTC Network conducted national surveys of MTC participants in 2012, 2013, and 2014, with 172, 220, and 405 respondents, respectively (Donaldson, Nakamaye, Umland, & White, 2014; Silverstein, 2014). The surveys requested information about demographics, educational and teaching background, and nature of participation in MTCs, and included several open-response questions focused on teachers' motivations for participating and any perceived impact on their views about mathematics and teaching. The most striking survey responses concerned teachers' perceptions of themselves, their students, and what it means to do mathematics. A recurring theme was that participating teachers

came to view themselves as mathematicians (for example, "Math Teachers' Circle moved me from being a math teacher to feeling like I am a mathematician") and as part of a community of mathematics professionals (58% of all respondents identified belonging to such a community as motivation for participating). This shift in mathematical identity suggests that MTC participation may be related to teachers' sense of belonging to a mathematical community. Other research has associated sense of belonging in academic disciplines with motivation to persist in those disciplines (e.g., Good, Rattan, & Dweck, 2012; Walton & Cohen, 2007).

Other responses suggested changes in teachers' mindsets about the nature of intelligence. Specifically, teachers came to see mathematics as a skill to be developed through hard work and effort rather than as a fixed trait. For example, one teacher wrote, "It has been neat to see such high level math become 'reachable' and 'doable' for me. It has made me feel that, maybe I really CAN do it!" Teachers' mindsets about the nature of intelligence—as either a fixed trait or a malleable quality—are known to significantly affect their pedagogical practices, with ramifications for student learning (Anderman et al., 2001; Rattan, Good, & Dweck, 2012). Some respondents reported that their changing views of what is required to succeed in math made them more willing to challenge their students (e.g., "I have increased my level of expectations for all students") and to encourage them in productive struggle (e.g., "I feel much more confident letting my students struggle and take time to solve problems. I use this concept with most lessons and my students have grown because of it").

Classroom Practices

Converging evidence from mathematics education and cognitive science indicates that mathematics learning is an active process (NCTM, 2014). A core feature of MTCs is to provide an active mathematics learning environment for teachers. A key question is whether teachers are then able to translate their MTC experiences into the creation of active mathematics learning environments for their students. In a

follow-up analysis of the Silverstein (2014) survey data, 84% of the 220 respondents self-reported at least one change in their classroom practices that they attributed to their MTC participation:

- 49% of respondents reported using more tasks promoting reasoning and problem solving (e.g., "Less rote math. More hands-on activities, word problems, and multiple representations").
- 29% described ways they now facilitate more meaningful mathematical discourse (e.g., "Extending on one another's thinking when questions are asked is the method I use to teach students how to problem solve collectively").
- 23% reported ways they support students in productive struggle (e.g., "I ask more questions to lead students to get the solutions for themselves rather give the solutions away").

Marle and his colleagues (2012) reported that after one year of MTC participation, teachers' math teaching self-efficacy (Enochs, Smith, & Huinker, 2010) increased significantly, and classroom observations using the Reformed Teaching Observation Protocol (RTOP; Sawada et al., 2002) provided evidence of significant increases in the use of inquiry-oriented teaching practices. These results have potential implications for student learning, as teaching self-efficacy and reform-oriented practices are both linked with improved student motivation, attitudes, and learning (cf. Stipek et al., 1998; Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998).

Professional Engagement and Leadership

Fifty-seven percent of respondents across the 2012–2014 national surveys reported that their experience with MTCs had increased their participation in professional activities outside the classroom (Silverstein, 2014). Reported activities included (a) collaborating more with other teachers, (b) participating in decisions about curriculum for their school or district, (c)

attending professional conferences, and (d) leading extracurricular math activities for students.

Mathematical Knowledge for Teaching

White, Donaldson, Hodge, and Ruff (2013) studied mathematical knowledge for teaching among MTC participants, which they measured using assessments developed by the Learning Mathematics for Teaching project (Hill, Schilling, & Ball, 2004). Among participants of three summer MTC institutes, the mean increase from pretest to posttest was 0.31 standard units ($p < .001$), indicating that, on average, participants experienced a statistically significant gain in this measure of mathematical knowledge for teaching. This increase in mathematical knowledge for teaching has potential implications for student learning since previous research by Hill and her colleagues established an association between higher mathematical knowledge for teaching and higher student achievement (Hill, Rowan, & Ball, 2005).

Although White and her colleagues (2013) found a significant increase of 0.31 standard units in teachers' mathematical knowledge for teaching after participating in a weeklong MTC workshop, it is noteworthy that average standardized scores varied by about 0.5 units across sites, with standard deviations around 1 unit within each site. This suggests that teachers across and within sites had considerable variation in their mathematical knowledge for teaching. In addition, teachers' educational backgrounds in mathematics varied considerably across sites as well. For example, the percentage of teachers with master's degrees ranged from 7% to more than 60% across sites. This evidence suggests that there is considerable variability in MTC participants' mathematical knowledge for teaching and educational backgrounds. Given this variability, we wanted to understand at a finer level of detail how teachers with different mathematical backgrounds experience their participation in MTCs. More specifically, we investigated the following research questions using a comparative case study approach:

1. How do teachers perceive their participation in MTCs in relation to their views about mathematics, their classroom practice, and their professional engagement and leadership?
2. To what extent does the participants' mathematical background play a role in their self-reported experience of MTCs?

Methods

Participants

As part of a larger study focused on the classroom practices of MTC participants, data were collected between August 2012 and May 2014 from nine teachers at three MTC sites who had varying levels of previous experience participating in MTCs. Data consisted of video classroom observations with accompanying lesson plans. Teachers then completed a semi-structured clinical interview (e.g., Bernard, 1994) as soon as possible after the completion of each set of video classroom observations. Teachers were compensated in the amount of $100 for completing each set of two recordings and an associated interview.

From the larger data set of nine teachers, the researchers selected four teachers to analyze using a comparative case study approach (Merriam, 1998). The goal of the comparative case study was to increase our understanding of how mathematical background relates to teachers' perceptions of their MTC participation vis à vis their views about mathematics, classroom practices, and professional engagement and leadership. The cases were selected to represent a variety of mathematical backgrounds, considering their educational, teaching, and professional development (PD) experiences (Table 1).

Table 1
Educational, Teaching, and Professional Development Background of Case Study Participants

Teacher	Education	Teaching Experience	MTC Experience	Other Math PD
Pam	Graduate degree, STEM field	12 years, all in middle school math	4 years, multiple week-long summer workshops and academic-year meetings	Significant previous participation in local, national, and online opportunities.
Lisa	Graduate degree, STEM field	3 years, all in middle school math	3 years, multiple week-long summer workshops and academic-year meetings	Participated in a summer research experience for teachers one year prior to joining MTC. No content-focused PD since.
Jason	Undergraduate education major with a math minor	7 years, 6 in middle school math	1 week-long summer workshop just prior to study	Ongoing participation in online courses and district-run PD on understanding the Common Core.
Sharon	Undergraduate humanities major, master's degree in elementary education	3 years, all in elementary school enrichment and math enrichment. Began teaching 5th grade math content at beginning of study.	1 week-long summer workshop just prior to study	None.

Notes. All teacher names are pseudonyms. "MTC Experience" indicates participation in MTC activities prior to the beginning of the study. Similarly, "Other Math PD" indicates content-focused professional development or coursework that was undertaken prior to the beginning of this study.

Procedure

Video observations. Three sets of video observations were collected from the participants' classrooms: at the beginning and end of Year 1 (2012–2013 school year) and at the end of Year 2 (2013–2014 school year). The purpose of these observations was to provide additional context for in-depth interviews with the teachers, as well as to develop richer background profiles illustrating what they are like in the classroom. During each set of observations, a single class of students was observed over two days of the teacher's choosing. Observation days were consecutive when possible, but always within a week of each other, and teachers were requested to provide copies of lesson plans and any relevant classroom materials, which were used to help support the research team's analysis of the depth of mathematical work the students were engaged in.

During the videotaping, the single camera focused primarily on the teacher but also included footage of students interacting with the teacher. As only the teacher wore a microphone, there were many utterances by students during group or partner work time that were not captured. Video data were transcribed by project research assistants, with an estimated 99% of teacher utterances captured audibly. During whole class discussions, and when teachers interacted with individual students, approximately 80 to 90 percent were captured audibly.

The analysis of the videos was guided by three sets of *a priori* hypotheses about the ways in which MTC participants negotiate the following activities during their classroom teaching:

1. Nature of the mathematical work that students are expected to do. Drawing on the work of Hill and her colleagues (2008) on Mathematical Quality of Instruction, the researchers hypothesized that MTC participants would assign non-routine problems to their students, give students sufficient time to work on these problems, expect students to explain mathematical

ideas rather than just steps, and expect students to use precise mathematical language.

2. Nature of how the teachers present the mathematics they want their students to learn. Again, drawing on Hill et al. (2008), the researchers hypothesized that MTC participants would go beyond demonstrating steps to explain "why," make connections between mathematical ideas, use precise mathematical language themselves, explore mathematical ideas or directions that students suggest, and explicitly describe problem-solving strategies for their students (e.g., do a smaller/simpler problem, look at special cases, draw a picture, or organize your work).
3. Nature of "meta" comments about mathematics or mathematicians. The researchers hypothesized that MTC participants would seek opportunities to talk with students about mathematics as a discipline, for example referring to what it means to "do mathematics," who is a "mathematician," and how mathematicians approach doing mathematics.

In order to analyze the videos, the entire research team watched each video unless they personally knew the participant who had been filmed. In addition, each set of videos was assigned to two of the researchers for detailed analysis. The assigned researchers collaboratively prepared a summary of the observed lessons and any evidence pertaining to the hypotheses, including relevant excerpts from the video transcripts and the provided classroom lesson materials. These video summaries were presented to the rest of the research team for feedback during weekly calls. During the preparation of the case studies, excerpts from the video summaries were then used to complement teachers' experiences as reported during the interviews.

Interviews. After the completion of each set of video classroom observations, the first and third author conducted semi-structured phone interviews, transcribing teacher comments in real time and using a template with the interview questions to produce notes that were shared with the rest of the research team. Interview 1, in Fall of Year 1, gathered

information on educational background; current and past teaching experiences; mathematical and professional activities outside the classroom, including participation in MTCs and other professional development; classroom teaching philosophy; classroom practices; and how participants perceived that MTCs had or had not impacted their attitudes, dispositions, and practices. Interview 2, conducted in Spring of Year 1, focused on teachers' MTC activities over the year and their perception of the impact MTCs had on their teaching and collaborations with other teachers. Teachers were also asked to describe a lesson they felt was particularly successful and why. Interview 3, conducted in Spring of Year 2, included the same questions as Interview 2, but also asked teachers to reflect on how they saw MTCs fitting in with their other professional development experiences and what they felt was most important about their MTC experience.

Analysis

To compare outcomes across cases, the researchers used the method of structured, focused comparison, which considers each case using the same set of general questions that focus on the research objective (George & Bennett, 2005). Our analysis was primarily based on evidence from the interviews, with complementary illustrations from video observations. Each teachers' reflections were examined through the lenses of three themes: views of mathematics, approaches to pedagogy, and professional engagement and leadership.

Results

Results are presented below, with the cases ordered from strongest to weakest initial mathematical background. For each case, we first provide an overview to contextualize findings, and then focus more deeply on the themes of how each teachers' views of mathematics, approaches to pedagogy, and professional engagement and leadership are reflected in the video observations and interviews.

Pam: Learning the Art of Teaching Problem Solving

Pam was a 12-year veteran middle school mathematics teacher who had been deeply involved with her local MTC for four years at the beginning of the study. She had a strong academic background and a rich and diverse professional life which included work with students preparing for national mathematics competitions, professional development outside of the state, and a leadership role at her school, helping her colleagues develop their classroom skills and prepare for the transition to the Common Core State Standards for Mathematics (CCSSM).

From the beginning of the study, Pam relished the challenge of working on open-ended MTC questions and valued being placed in the role of student and active learner, noting in Interview 1 that "I'm the expert in my classroom, and then I get to go to something else where I'm the student." She wanted her students to experience mathematics in the same way. At the same time, Pam was uncertain how to bring this excitement and engagement into her own classroom. As she noted, "It is backward from how we learn to teach, how we think about teaching in general" (Interview 1).

By the time of Interview 2, Pam reported experiencing what she called "a huge philosophical shift" that stemmed from taking Jo Boaler's online course about mathematical mindsets. She reported that the most important thing she got from the course was that "we need to learn from mistakes, we shouldn't be penalized for making mistakes while we are learning." She further explained, "MTCs are fun and I've enjoyed doing it, but the big question I've had and I know other teachers have had is how to do it in our own classroom—we were never able to figure that out. This class helped me do that." By the time of Interview 3, she felt she had been able to make this transition to some extent: "I love it. The whole discovery of how to do that. It has taken me years. I see myself, well there's always room for growth, but I finally have an idea for how I want to be teaching."

The videos from Pam's classroom provided supporting evidence that she was trying to integrate interesting,

challenging problems into the classroom, which paralleled the types of problems given during her MTC sessions. For example, in the first set of observations, students worked on trisecting an angle with appropriate tools, while in the second set of observations, students estimated the radius of a large sphere in a picture. In both of these cases, however, Pam provided significant guidance in the form of suggested solution methods or extra information. Overall, all three sets of video observations suggested that students were engaged, appeared comfortable testing different strategies, and worked in an atmosphere of mutual respect and support. Pam circulated around the classroom, providing consistent encouragement and guidance when necessary. Both Pam and her students also appeared to value precise language, noted by Hill and her colleagues (2008) as a characteristic of high mathematical quality of instruction. In a typical exchange from the third set of video observations, Pam says, "When we take an isosceles triangle and make it the lateral face of a pyramid, this height has a new name. It is no longer just the height, it is the…" and a student chimes in: "Slant height."

Throughout the interviews, Pam returned several times to the central question of how to bring the open-inquiry format of MTCs into her classroom: "What is my role now? Should I just stand off to the side? Should I have them write their answers on the board so I'm not the answer book? I don't ever want to be the teacher who goes in and pulls the lesson out of the file cabinet" (Interview 2). By the end of the study, a picture had emerged of a teacher who valued letting students struggle toward making sense of mathematics, while still often succumbing to the natural instinct that all teachers have to intervene and provide guidance.

Pam's views about mathematics. In Interview 1, Pam reported, in large part due to her MTC participation and her other PD experiences, that she views mathematics as a challenging pursuit whose goal is reasoning through difficult problems:

> I need to move them to a place where they can be confused, to the edge of understanding, but not to the level

> of frustration. I was like that in school, but I wasn't pushed enough. Learning how to learn: what do you do when you don't understand. That whole idea of perseverance. You are going to do hard problems that you don't get, you are going to stick with it until you understand it. Learning is working through confusion, they never had that experience, they are bored to tears.

Asking questions also appeared to be key to mathematics for Pam, and she stated in Interview 3 that she was asking her students more questions than in previous years.

Pam's classroom practices. In her interviews, Pam repeatedly stressed how important it was for her to give students the experience of being actively engaged in problem solving, the same experience she said she personally valued in her MTC. Pam also openly recognized that it takes a lot of time and effort to bring the inquiry-oriented environment of MTCs to the classroom: "A lot of teachers want to go to a workshop and then take it straight back to the classroom—that is not what happens at an MTC. It's more how to think about math and math teaching—that whole shift" (Interview 1). Pam reported that she continues to work on making this transition to help empower her students.

Pam's professional engagement and leadership. Pam exhibited leadership on several levels. For example, within her own local MTC, she had attended meetings regularly for multiple years and had co-led several sessions with mathematicians. She lamented that she had not been able to get her colleagues more enthusiastic about attending, but was also happy to have convinced two of them to attend a summer workshop, noting that she "really enjoy[s] networking—well, sitting down and talking with other teachers" (Interview 3). As indicated above, she also had taken advantage of a large array of professional development opportunities, including summer programs in Colorado Springs and Vermont. In addition, she took a leading role at her school in helping teachers prepare to implement the CCSSM. Pam also worked with students preparing for national math competitions. While MTCs may not have been the primary catalyst in this professional

engagement, Pam reported that they are an important and ongoing part of staying engaged with mathematics and a local mathematics community.

In summary, Pam had a strong mathematical background supported by rich and diverse professional development experiences. She reported constantly experimenting to find effective ways to engage students in the creative struggle associated with the deep mathematical thinking exemplified by MTC problems. MTCs appeared to serve as a source of mathematical enrichment, a motivation for striving to improve her teaching, and a supportive structure for continued professional engagement.

Lisa: Developing Leader

Lisa taught science courses at a large state university before moving to the middle school classroom in 2009. Part of the reason for this change was to help students cultivate good learning habits at a younger age, habits which will help them thrive throughout their lives: "Then when they grow up, they will know what to do" (Interview 1). She started attending her local MTC in 2010, and she valued this experience from the beginning: "I love MTCs because it is fun and helps me see math as problem solving" (Interview 1). Lisa sought to teach in a way that she believed would empower her students as capable problem solvers: "I...let the kids do it. I prepare the food, but it is their job to cut it and eat it. The chef doesn't feed the patrons. I let them do as much as possible" (Interview 1).

Lisa discussed in her interviews that she wanted students to be in charge of making sense of mathematics. To accomplish this, Lisa strived to cultivate a classroom where students worked together on problems while she asked questions and helped them reflect on their thinking. She also helped them reflect on the consequences of their work habits. "When I'm walking around, I know it is successful when the students aren't frantically asking for help—instead they are frantic to show off what they have done" (Interview 1). The classroom video observations suggested an environment of student intellectual engagement and creative problem choices by Lisa.

For example, in the Fall Year 1 lesson, she used a debate format to engage students in using mathematical reasoning to defend a position. In the Spring 2014 lesson, although the problems were relatively routine (translating story problems into equations and vice versa), Lisa pushed students to explain their reasoning.

Lisa's views about mathematics. Lisa's views of mathematics were not a point of emphasis in the interviews overall, but in Interview 2, she described how being a mathematical learner has affected her:

> MTCs is a xeric plant that thrives and grows in this desert of no support. I have been to countless PDs that had no impact on me whatsoever. I got nothing out of them. Whereas MTCs really put me in the learner's seat, then let me put the teacher's hat on and reflect about how I could take this back to my students.

Based on the interviews and video observations, she appeared to make great efforts to find contexts to engage her students so that they would be excited and willing to work hard to make sense of the mathematics at hand. One interesting example of this emerged from Interview 2, where she described her students as being interested in supporting a teacher who had breast cancer: "I asked my students if they wanted to fill her room with pink balloons. I asked them, 'What's your plan? You have six weeks to pull it off.' " In this open-ended problem, students had to understand constraints, make assumptions, and check answers for reasonableness. In short, she was calling on her students to be mathematicians.

Lisa's classroom practices. Lisa stressed repeatedly how her own experiences as a learner in MTCs strengthened her commitment to actively engage her students in problem solving. In Interview 3, she described her experience as follows: "I discovered that who was doing the doing was doing the learning. I came back to my classroom and had my students do the doing... Now I have self-efficacy, now I have the skills." As the CCSSM became a focus across the U.S., Lisa also

reported attempting to integrate both the content standards and the mathematical practices into classroom activities.

Lisa's professional engagement and leadership. At the beginning of the study, Lisa was a co-organizer of her MTC. Over the course of the study, she took on a number of new leadership roles at her school: "I'm department chair, instructional leadership for the school, data coach for the school" (Interview 3). In addition, Lisa took on national leadership by reviewing items for the PARCC assessment and consulting for the National Education Association. She attributed these increased responsibilities to the gains shown by her students since beginning to participate in her MTC:

> I owe my success to MTC—my only source of content related PD. Anything I do I feel comes from my math circle. My principal sees my students grow on tests better than any of the other cohorts in the building. 200% growth (two times expected) when other teachers struggle for any growth. He sees I'm increasing rigor—engaging in mathematical conversations—interacting with the math, making sense of it, real world or not, no pseudo concepts—delving in deep and getting dirty. He sees the students' projects—what they produce, and that it is superior to what other students are producing. Even parents are calling in noticing a difference. (Interview 2)

Ironically, her increased leadership roles were indirectly responsible for her no longer participating as actively in her local MTC in Year 2 of the study.

In summary, Lisa reported that her experiences as a learner in MTCs helped change the focus of her classroom teaching to put greater emphasis on active student engagement. She attributed improvements in her students' achievement to her new style of teaching, and also reported that this improved student achievement led to significant new opportunities for professional engagement.

Jason: Problem-Solving Skeptic

Jason had been teaching middle school mathematics for six years at the beginning of the study. His first MTC experience was during the summer just prior to the first fall videotaping. In Interview 1, he said he enjoyed being challenged by the mathematics in MTCs, and expressed an interest in developing problem-solving materials for his students: "I also get ideas for how to challenge my students. I'd like to know how can I develop these problem-solving activities for them?" Jason described the CCSSM and the greater demand that they place on problem-solving skills as a motivation for participating in MTCs: "Now we need to go in more depth. With Common Core, math is more than just a set of steps" (Interview 1). Jason indicated that he believed in the importance of doing problem solving in his classroom, but that the district requirements for content coverage left little time for it:

> Now the challenge is to find the time to do problem solving.... [In our district,] we are moving to more performance-based assessments. It does have connections to MTCs, but it takes us out of the classroom. I don't trust a substitute; when I'm gone, I feel it could have been a day of problem solving, but when I return I have to get back to our pacing guide. (Interview 1)

Jason's first and second set of classroom video observations suggested a passionate and caring teacher who was focused on having students work through routine problems. He had an elaborate system of rewards and routines to encourage his students, although the focus was often on finding the right answer. Students were not given enough time to explore ideas in much depth, and Jason often started discussing the answer to problems without giving time for students to think on their own. Across videos, the majority of class time was spent on procedural problems, and little or no time was devoted to exploring student ideas or providing clear arguments based on careful analysis of definitions or consideration of cases.

By Interview 2, Jason seemed to be aware of some key features of MTCs that could be valuable for him in the classroom. He indicated that his experience in his local MTC gave him some idea of "how to ask good questions of students. To push them forward in thinking" (Interview 2). He also indicated, "with the math circles, there's a lot of open-ended problems that caused us to think, persevere. With CCSSM we want students to persevere. As I do it more, I see better how to have my students do it" (Interview 2). This showed an awareness of the connection between MTCs and classroom practices. However, Jason seemed to struggle with moving beyond procedures in his classroom teaching. By Year 2 of the study, he no longer participated in his local MTC, and he chose not to participate in the third interview or set of observations. Overall, Jason's case study reveals the themes very differently from the others.

Jason's views about mathematics. Jason appeared to be conscious of a shift in his own thinking about mathematics teaching, remarking in Interview 1 that MTC participation and his online classes "are both pushing more for problem-solving and conceptual understanding. They can help me change the way I ask students questions, push them to go further. In the online class, I'm learning to present concepts tangibly, in a concrete, spatial way." This quote also revealed an awareness of the value of problem solving in mathematics and a desire to bring this to the classroom. There was less evidence that his views of the nature of mathematics had fundamentally changed.

Jason's classroom practices. Although Jason expressed interest in giving his students more opportunities for problem solving, there was little evidence that he attempted to implement this in his classroom. In particular, in Interview 2, he noted that he struggled with using a problem-solving approach in a 50-minute class period, explaining further: "They didn't directly connect with my standards—I struggled with that."

Jason's professional engagement and leadership. Jason did not hold any leadership roles, although he did participate in

other content-focused professional development, such as online classes.

In summary, while Jason reported that he recognized the value of open-ended problem solving and that MTC participation had some effect on his own views of mathematics, he seemed to struggle with engaging students in non-procedural thinking. In addition, there were no observable links between his MTC participation and increased professional engagement.

Sharon: Transition to Math Teaching

Sharon was a career-changer who was beginning her fourth year of teaching and first year of teaching math content when she enrolled in this study. She had previously taught 4th and 5th grade enrichment, including math enrichment, at a public school. Knowing that she would begin teaching a math content course in the fall, she was persuaded by a friend to participate in a summer MTC workshop. It was her first experience with MTC, and she described it as follows in Interview 1: "I felt like the dumbest person in the room, but I learned so much. I felt like a light bulb went on… . In the past, I identified as a language arts kind of person. Now I'm passionate about math."

Sharon described student-centered investigation as integral to her approach to teaching. This philosophy seemed to predate her involvement in MTC, although perhaps MTC participation reinforced it: "I want the class to be interactive and hands-on, to involve lots of experimentation. Especially for the gifted kids, I want them to experience being frustrated and to stretch them, to help them understand that it's OK to be wrong, that that's what allows you to continue investigating" (Interview 1). The first and second set of classroom video observations both reflected Sharon's stated philosophy of allowing students to investigate and make discoveries. For example, in the first set of observations, the lessons spanned a project where students were asked to use polygons and polyhedra made from Zometool manipulatives to build the tallest structure they could that would support three textbooks. Other projects described in her interviews were also non-routine and invited investigation.

For instance, one lesson used investigation to develop a formula for the sum of the interior angles of a polygon. On the other hand, in the video observations, although there was ample time for student discovery and the students seemed to be engaged throughout, there was less emphasis on developing a shared mathematical understanding during students' investigations.

In Interview 2, conducted just after school let out for the summer, Sharon noted that she felt she had been successful in instilling a mindset in which students want to figure out *why*: "I was trying to rush through area and perimeter before their test. I gave them the formula, and the kids kept saying, 'Why? Why does it work?' Not asking 'What will get me an A?' It's an indication of how the classroom culture is." In both sets of classroom observations, Sharon rarely answered a question directly, but instead responded by trying to get students to think things through themselves, which they attempted quite readily.

According to Sharon, she had become more engaged in learning mathematics since beginning to participate in MTC. For example, she had begun taking some online courses, including a content course on algebra. She had also joined NCTM and said she read all the articles in the NCTM publications she received. She was also becoming more involved in extracurricular math activities for students, for example running an after-school math program. In addition, she reported having "nerd hour" with the instructional coach after school, when they would discuss math they had been reading about and worked on problems at the whiteboard together.

Sharon also reported that since becoming more excited about math, she had gotten involved in developing materials and collaborating with other math teachers at her school. For example, in Interview 1, she said that she did lots of research on different ways to approach mathematical concepts and shared this information with her school's instructional coach as well as with receptive teachers. In Interview 2, she mentioned developing a database of lessons from NCTM for general education teachers at her school. By the time of Interview 3, in Spring 2014, Sharon was spending more and more of her time

"pushing into" other teachers' classrooms, frequently doing math lessons.

Sharon's views about mathematics. Sharon reported a dramatic positive transformation in her views about mathematics, and in particular her own self-concept with regard to mathematics. In Interview 2, she expressed that "Math Teachers' Circle teaches how to understand math. It's possible to teach it because we're investigating together. I strive to get students excited about math and to work on figuring it out together." This quote suggests that Sharon had a new view of what constitutes doing mathematics that involves collaborating to make progress.

Sharon's classroom practices. MTCs also seemed to have been transformative for Sharon in terms of her approach to teaching mathematics. Many quotes from the interviews illustrate this, for example, "If you can get kids excited and confident about math, this will make them successful" (Interview 1), and "I want the class to be interactive and hands-on... I want my students to argue with me" (Interview 1). In Interview 3, she provided a detailed description of a mechanics problem involving Egyptian pyramids, emphasizing the lengths she had gone to with respect to having students figure out the answer themselves with only some scaffolding from her.

Sharon's professional engagement and leadership. As described above, Sharon not only became more professionally engaged (e.g., joining NCTM), but also developed into a mathematical leader in her school, for example by starting an after-school math program and visiting other teachers' classrooms to help with math lessons.

In summary, Sharon is an example of a career-changer without a formal math background, whose experience with MTC appears to have been a catalyst for her deeper involvement with mathematics, both personally and professionally. After coming to see herself as a "math person," she became motivated to learn how to teach in a way that would get her students excited about math. To help herself change her own classroom practices, she began becoming more professionally engaged, and she eventually began to share her

new expertise with other teachers at her school as a mentor and coach.

Discussion

The four teachers whose stories we have shared above (i.e., Pam, Lisa, Jason, and Sharon) illustrate a broad range of the mathematical backgrounds we find among MTC participants. Taking all four cases together, perhaps the most notable finding to emerge is that it appears that teachers with a range of mathematical backgrounds can benefit from MTC participation, whether in their views of mathematics, classroom teaching, professional engagement and leadership, or some combination of these areas. A comparison of the cases in each of the focus areas follows below.

Views about Mathematics

Among the four cases considered, the teacher with the weakest initial math background (i.e., Sharon) showed the most dramatic change in her views about mathematics. The teachers with the strongest backgrounds, Pam and Lisa, seemed mostly to have their views of the nature of mathematics reinforced, although there was evidence that both Pam and Lisa experienced changes in their views about how mathematics should be taught. Jason, whose math background fell somewhere in the middle, seemed to change somewhat in his view of how mathematics should be taught, without necessarily changing his views of what mathematics is, suggesting that initial background in mathematics is not necessarily predictive of how teachers will perceive their MTC experience.

Classroom Practices

Again, math background alone did not appear to explain whether teachers reported making changes in their classroom practice as a result of their MTC experiences. Indeed, the two teachers who reported making the most dramatic changes to their classroom practice, Lisa and Sharon, began with very

different levels of math background, as well as very different lengths of time participating in MTCs (e.g., Lisa had participated for more than two years, and Sharon for only a few months at the beginning of the study). Interestingly, neither Lisa nor Sharon had access to other significant sources of content-focused professional development prior to the beginning of the study. In contrast, Jason and Pam both regularly took part in other content-focused courses or professional development opportunities. Jason noted that he had trouble seeing how taking a "MTC approach" to classroom teaching could fit with his district's expectations. So did Pam, initially, but after taking a course that gave her additional insight into productive mindsets about mathematics, she reported being better able to make this connection in her own teaching. For Lisa and Sharon, the fact that they regarded MTCs as their primary source of PD may have paradoxically empowered them to experiment more freely and to focus primarily on trying to implement what they were experiencing in their MTCs.

Considering all four cases together suggests that in order to make changes in their classroom practice, teachers had to feel that the inquiry-oriented approach of MTCs was consistent with their own beliefs about mathematics teaching and learning, as well as with the expectations of their schools and districts. This is consistent with Desimone's (2009) framework, which suggests that coherence among professional development, teachers' beliefs, and school expectations is a key component that leads to changes in instruction.

Professional Engagement and Leadership

Pam, Lisa, and Sharon all developed greater professional engagement and leadership during the course of the study. Lisa and Sharon both appeared to attribute their increased engagement to increased self-efficacy as math teachers, which in turn they credited to their experience with MTCs. While Sharon's leadership opportunities seemed to originate with her school, for Pam and Lisa, MTCs themselves provided

opportunities for leadership, including organizing activities and co-leading meetings.

Conclusions

> The difference between MTCs and other kinds of PD, [is that] the other things claim to be a silver bullet. MTC doesn't make that claim. MTC is a place for you to go and you to work hard and you to think and bring that thinking back to your classroom to work hard to develop lessons. It was transformative, but you had to put the work in to be that way. (Lisa, Interview 3)

Although the case studies suggest that MTCs can benefit teachers with a variety of mathematical backgrounds, the findings also have implications for how MTCs can be implemented so as to foster the type of transformation described by Lisa in the quotation above. In particular, to better support teachers' shifting views about mathematics or how mathematics should be taught, it may be worth devoting time during the reflection component of MTC meetings to active discussions about these ideas. For example, drawing on Pam's experience with the course on productive mathematical mindsets, it may be helpful to explicitly discuss how views about the nature of mathematics can connect with these mindsets, including sense of belonging (Good et al., 2012), the nature of intelligence (Anderman et al., 2001; Rattan et al., 2012), and the value of grit and persistence (Duckworth, Peterson, Matthews, & Kelly, 2010). Another possible avenue for supporting teachers in shifting both their beliefs about mathematics and their instructional practice is to connect MTCs with complementary PD that focuses more explicitly on pedagogy.

In addition to supporting teachers in developing productive mindsets about mathematics and implementing more inquiry-oriented, student-centered classroom practices, MTCs could also consider ways to incorporate leadership opportunities, such as organizing and leading sessions or designing connected curricular materials, into their own organizational structure.

Also, as a gathering place where at least some teachers are likely to be professionally engaged already, MTCs might consider providing time at their meetings for teachers to share information about relevant professional opportunities.

Taken together, evidence from these case studies suggests that for some teachers, participating in a MTC can serve as a catalyst for changing their reported views of mathematics, classroom practice, level of professional engagement, or even all three. In fact, as suggested by Desimone (2009) in her framework for studying effective PD, an experience such as a MTC, which allows teachers to change their ideas about what mathematics is, may be a necessary first condition for teachers to change their practice. Providing additional support for the development of productive teacher mindsets about mathematics, connecting with other PD programs that focus on pedagogical skills such as inquiry-oriented teaching, and incorporating opportunities for professional engagement are ways in which MTCs, and perhaps other content-focused PD models, may be able to support teachers more effectively in their professional growth.

References

Anderman, E. M., Eccles, J. S., Yoon, K. S., Roeser, R., Wigfield, A., & Blumenfeld, P. (2001). Learning to value mathematics and reading: Relations to mastery and performance-oriented instructional practices. *Contemporary Educational Psychology, 26,* 76–95.

Bernard, H. R. (1994). *Research Methods in Anthropology: Qualitative and Quantitative Approaches.* Walnut Creek, CA: AltaMira Press.

Conference Board of the Mathematical Sciences. (2012). *The Mathematical Education of Teachers II.* Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher, 38*(3), 181–199.

Donaldson, B., Nakamaye, M., Umland, K., & White, D. (2014). Math Teachers' Circles: Partnerships between mathematicians and teachers. *Notices of the American Mathematical Society, December 2014, 61*(11), 1335–1341. Available online at https://www.ams.org/journals/notices/201411/rnoti-p1335.pdf

Duckworth, A. L., Peterson, C., Matthews, M. D., & Kelly, D. R. (2007). Grit: perseverance and passion for long-term goals. *Journal of Personality and Social Psychology, 92*(6), 1087.

Enochs, L.G., Smith, P.L. & Huinker, D. (2010). Establishing factorial validity of the mathematics teaching efficacy beliefs instrument. *School Science and Mathematics*, *100*, 194–202.

George, A. L., & Bennett, A. (2005). *Case Studies and Theory Development in the Social Sciences.* Cambridge, MA: The MIT Press.

Good, C., Rattan, A., Dweck, C. S. (2012). Why do women opt out? Sense of belonging and women's representation in mathematics. *Journal of Personality and Social Psychology, 102,* 700–717.

Green, E. (2015). *Building a Better Teacher: How Teaching Works (and How to Teach It to Everyone).* New York, NY: W.W. Norton & Co., Inc.

Hendrickson, K. A. (2016). *Math Teachers' Circles: The effects of a professional development community on mathematics teachers' identities* (Unpublished doctoral dissertation). Ohio University, Athens, OH.

Hill, H. C., Blunk, M., Charalambous, C., Lewis, J., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An exploratory study. *Cognition and Instruction, 26*, 430–511.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42,* 371–406.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal, 105,* 11–30.

Karakok, G. (2016). Good beginnings: Supporting "Principles to Actions." *MTCircular, Winter/Spring 2016,* pp. 8–9. Retrieved from http://www.mathteacherscircle.org/newsletter/

Manes, M. (2015). Modeling with mathematics: Developing a common language. *MTCircular, Summer/Autumn 2015,* p. 6. Retrieved from http://www.mathteacherscircle.org/newsletter/

Marle, P. D., Decker, L. L., & Khaliqi, D. H. (2012). *An inquiry into Math Teachers' Circle: Findings from two year-long cohorts.* Paper presented at the School Science and Mathematics Association Annual Convention, Birmingham, AL.

Merriam, S. B. (1998). *Qualitative Research and Case Study Applications in Education. Revised and Expanded from "Case Study Research in Education."* San Francisco: Jossey-Bass Publishers.

National Council of Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: Self.

National Governors Association, Council of Chief State School Officers, & Achieve, Inc. (2009).

Benchmarking for success: Ensuring U.S. students receive a world-class education. Washington, D.C.: National Governors Association.

Organisation for Economic Co-operation and Development. (2013). *Lessons from PISA 2012 for the United States, Strong Performers and Successful Reformers in Education*. Paris: Self. Retrieved from http://dx.doi.org/10.1787/9789264207585-en

Partnership for 21st Century Skills (2008). *21st century skills, education & competitiveness: A resource and policy guide.* Tucson, AZ: Author.

Rattan, A., Good, C., Dweck, C. S. (2012). It's Ok--Not everyone can be good at math: Instructors with an entity theory comfort (and demotivate) students. *Journal of Experimental Social Psychology, 48,* 731–737.

Sawada, D., Piburn, M. D., Judson, E., Turley, J., Falconer, K., Benford, R., et al. (2002). Measuring reform practices in science and mathematics classrooms: The reformed teaching observation protocol. *School Science and Mathematics, 102*(6), 1–20.

Silverstein, H. (2014). *The state of the Circles: National survey shows scope of MTC impact.* MTCircular, Summer/Autumn 2014, pp. 4–9. Retrieved from http://www.mathteacherscircle.org/newsletter/

Stipek, D. J., Salmon, J. M., Givvin, K. B., Kazemi, E., Saxe G. & MacGyvers, V. L. (1998). The value (and convergence) of practices suggested by motivation research and promoted by mathematics education reformers. *Journal for Research in Mathematics Education, 29,* 465–488.

Taton, J. A. (2015). Much more than it's cooked up to be: Reflections on doing math and teachers' professional learning. *Perspectives on Urban Education, 12*(1). Retrieved from https://www.urbanedjournal.org/archive/volume-12-issue-1-spring-2015/much-more-it%E2%80%99s-cooked-be-reflections-doing-math-and-teachers%E2%80%99-

Tschannen-Moran, M., Woolfolk Hoy, A. W., & Hoy, W. K. (1998). Teacher efficacy: Its meaning and measure. *Review of Educational Research, 68,* 202–248.

Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes.* Cambridge, MA: Harvard University Press.

Walton, G. M., & Cohen, G. L. (2007). A question of belonging: Race, social fit, and achievement. *Journal of Personality and Social Psychology, 92,* 82–96.

White, D., Donaldson, B, Hodge, A., & Ruff, A. (2013). Examining the effects of Math Teachers' Circles on aspects of teachers' mathematical knowledge for teaching. *International Journal for Mathematics Teaching and Learning.* Published online September 26, 2013, at http://www.cimt.org.uk/journal/index.htm