What Do We Know About Secondary Mathematics Teacher Preparation in the United States?

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The structures of secondary mathematics teacher preparation (SMTP) programs in the United States are not well documented. This study sought to identify baseline data for SMTP programs. Survey data was collected from 86 different institutions regarding their SMTP programs. Results indicated that most SMTP programs are housed in mathematics departments, with programs often producing less than 10 teachers per year. Additionally, four-year SMTP programs were most common, with nearly all requiring a teaching methods course as well as clinical experiences. Tenure or tenure-track faculty did most student teaching supervision. Half of the participating institutions reported having a capstone course

In 2017, The Association of Mathematics Teacher Educators (AMTE) published *Standards for Preparing Teachers of Mathematics* (SPTM) and made recommendations for high school mathematics teacher preparation:

High school mathematics teachers must have strong content knowledge, knowledge of mathematics-specific pedagogy, and much more—including knowledge about their individual students and their cultural contexts, school policies, and how to collaborate with other teachers. Only with this knowledge, will mathematics teachers be able to

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With respect to strong content knowledge, SPTM notes that preservice high school teachers should have a thorough understanding of the content found in the high school curriculum, including knowledge of the mathematics that is taught before and after high school. Moreover, preservice high school teachers should acquire this content knowledge via courses that one would find in an undergraduate mathematics degree, including three courses that focus specifically on high school mathematics content. This recommendation might prompt questions about what specific content courses are necessary for preservice teacher to take in order to have a thorough understanding of high school mathematics. Furthermore, one might wonder who should be teaching the content courses for secondary preservice teachers (PSTs) in order to satisfy the recommendations.

SPTM also calls for preservice high school teachers to learn pedagogy relevant to teaching high school (AMTE, 2017). In order to do this, AMTE recommended that three teaching methods courses be included in the program of study for preservice high school teachers. Given the recommendations, one might wonder how to fit six classes (three teaching methods and three content courses) specifically tailored for preservice high school teachers into a degree plan that often has little extra space for classes. Furthermore, one might wonder if there are any teacher preparation programs that are currently providing preservice high school teachers the courses and experiences recommended by SPTM? Likewise, teacher educators may ask if recommendations, such as the ones from AMTE, will produce positive results with PSTs.

Given the questions that arise when new recommendations are proposed, it is helpful to have some idea of the current status of secondary mathematics teacher preparation (SMTP) programs to use as a starting point for future work with SMTP programs. Similar to Dossey's (1981) work to establish baseline data for the mathematics preparation of elementary PSTs, the purpose of this article is to establish baseline data regarding the preparation secondary mathematics PSTs undergo in university preparation programs in the United States. Specifically, this article seeks to identify baseline data regarding institutional demographic information of SMTP in the United States, the structure of preparation programs, methods coursework, specialized content coursework, clinical experiences, state licensure, and faculty involvement during preparation coursework. We believe the data provided in this article will provide a snapshot of SMTP programs in addition to serving as a catalyst for research focused on SMTP.

Literature Review

The Education of Mathematics Teachers

Mathematics education has a long history of committees governing bodies making recommendations and for mathematics teacher preparation (e.g. Conference Board of the Mathematical Sciences [CBMS], 2001, 2012; Committee on the Undergraduate Preparation of Secondary Teachers of Mathematics [CUPM], 1971; National Advisory Committee on Mathematical Education [NACOME], 1975; National Council of Teacher of Mathematics [NCTM], 1970, 1991). The recommendations focus on topics such as content preparation, pedagogical preparation, and clinical training (or professional experiences; NCTM, 1970; Wilson, Floden, & Ferrini-Mundy, 2002). Although this is not an exhaustive list of topics, it highlights some main concerns facing mathematics teacher preparation. Many of the questions we posed to faculty members in our study are rooted in the recommendations mentioned in the following sections. Then we will turn our attention to how different groups have examined mathematics teacher preparation through the lens of research.

Content preparation. Content preparation has often taken center stage in the recommendations for mathematics teacher preparation. For example, The 1923 Report, issued by the National Committee on Mathematical Requirements (NCMR) organized by the Mathematical Association of America (MAA), recommended the following courses for preservice high school teachers: plane and spherical geometry, plane analytic geometry, college algebra, differential and integral calculus, and synthetic projective geometry (NCTM, 1970).

Likewise, the Mathematical Education of Teachers II (CBMS, 2012) proposed a set of courses for secondary preservice mathematics teachers to study. They included single and multivariable calculus, introduction to linear algebra, statistics and probability, introduction to proofs, abstract algebra, real analysis, modeling, and differential equations, group theory, number theory, and the history of mathematics. Note that some of the above courses are listed as recommended elective courses. Moreover, CBMS (2012) recommended that PSTs take nine semester-hours of courses specifically designed to allow PSTs to gain a profound understanding of fundamental mathematics (Ma, 1999). One such course designed specifically for secondary PSTs was a capstone course (CBMS, 2001). The objective of the capstone course would be to examine "conceptual difficulties, fundamental ideas, and techniques of high school mathematics ... from an advanced standpoint" (p. 39).

In order to identify the content courses currently required of secondary PSTs, we asked faculty at colleges and universities to name and describe the content courses required of their students. Moreover, we asked faculty if the institutions had courses specifically designed for secondary PSTs, such as a capstone course recommended by CBMS (2012). If the faculty answered in the affirmative, we asked them to describe the goals of their capstone course. All questions sought to determine PSTs' opportunity to learn the mathematics recommended by CBMS (2001, 2012).

Pedagogical preparation. Wilson et al. (2002) note that pedagogical preparation can refer to such matters as teaching methods, assessment of students, theories of learning, and the psychological aspects of learning. The diversity of pedagogical preparation can be observed as early as The 1923 Report (NCTM 1970) where NCMR recommended that secondary teachers should take the following courses: history of education, principles of education, methods of teaching, educational psychology, and organization and function of

secondary education. More recently, the *Professional Standards for Teaching Mathematics* (NCTM, 1991) proposed standards for mathematics preservice teacher education that would develop facility in PSTs to employ and assess

- instructional materials and resources, including technology;
- ways to represent mathematics concepts and procedures;
- instructional strategies and classroom organizational models;
- ways to promote discourse and foster a sense of mathematical community; and
- means for assessing student understanding (p. 151).

From the examples above, it seems apparent that pedagogical preparation is broadly defined. As a result, we asked participants about the pedagogical preparation of their preservice secondary teachers. We asked about the length of the SMTP program (i.e., four vs. five years). We also asked about teaching methods courses, specifically if their courses were teaching methods courses aimed at the general population (e.g., of teachers English. History. Social Studies. Mathematics, and Science PSTs all take the same teaching methods course) or mathematics specific teaching methods courses (AMTE, 2017). We also asked if the teaching methods course was housed in the mathematics department or in the college of education.

Clinical preparation. Many recommendations for secondary preservice teacher preparation include the need for PSTs to participate in clinical experiences (AMTE, 2017; CUPM, 1971; NACOME, 1975; NCTM 1970, 1991). The recommendations for clinical include opportunities that PSTs should have in order to meet certain teaching standards (e.g. NCTM, 1991). There are often fewer recommendations about how long the clinical experiences should last. One extreme example by today's standards comes from The 1923 Report (NCTM, 1970). NCMR proposed that secondary mathematics PSTs have "satisfactory performance of the duties of a teacher of mathematics in a secondary school for a period of not less

than ten years" before they could be certified as a teacher (NCTM, 1970, p. 317)

In our study, we asked participants to describe the kinds of clinical experiences PSTs had before student teaching, including the number of required hours for the clinical experience. We also asked about who supervised PSTs during student teaching (e.g., tenure/tenure track faculty, adjunct faculty, retired teachers, graduate students, etc.), reasoning that the supervisors could potentially affect the student teaching experience.

Are the recommendations being met? Recommendations from stakeholders about teacher preparation are starting points for discussion and examination. For example, in the 1960s and 70s, due to the scarcity of information about mathematics teacher education programs and fueled by recommendations for teacher preparation made by several governing bodies, researchers (e.g., Aviv & Cooney, 1979; Dossey, 1981; Fey, 1979; Johnson & Byars, 1977) set out to investigate the status of mathematics teacher preparation programs.

The primary method researchers used for examining mathematics teacher preparation programs was to send surveys via mail to teacher preparation programs (Dossey, 1981; Fey, 1979; Johnson & Byars, 1977), asking about the courses offered to preservice mathematics teachers, the number of credit hour required of students, and the characteristics of field experiences offered to PSTs. Surveys allowed researchers to generate a larger snapshot of teacher preparation programs, instead of research conducted at a single institution, which may lead to overgeneralization of results (Wang, Spalding, Odell, Klecka, & Lin, 2010; Wilson et al., 2002). Johnson and Byars (1977) found that in response to the recommendations, universities were offering more mathematics courses, which provided a more in-depth study in of mathematics. Aviv and Cooney (1979) found that PSTs valued their field experiences the most.

Assessing Teacher Preparation

Calls to prepare teachers based upon a given set of criteria generate a need to study SMTP programs in the United States. Efforts have been made to examine mathematics teacher preparation nationally as well as internationally. For example, The Association of Public and Land-Grant Universities (APLU) in 2012 initiated the Mathematics Teacher Education Partnership (MTE-Partnership), which "provides a coordinated research, development, and implementation effort for SMTP programs to promote research and best practices in the field" (Hazelrigg, 2017, para. 1). The MTE-Partnership uses Research Action Clusters to examine and provide solutions to issues in SMTP (e.g., developing effective clinical experiences). The MTE-Partnership has developed a set of guiding principles for SMTP and sought to develop research-tested methods for preparing secondary mathematics teachers that can then be disseminated to SMTP programs nationwide.

One maior international effort to examine K–12 mathematics teacher preparation was the Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto et al., 2012). The TEDS-M had two driving forces, first to discover how teacher educators in various countries around the world prepare PSTs to teach mathematics. The second purpose was to examine the variability of the makeup of teacher preparation programs and the impact those programs have on pupil learning in schools. The TEDS-M identified five factors that could contribute to the variability of teacher preparation programs, two of which are "the nature of teacher education programs," (Tatto et al., 2012, p. 19) and "the content of teacher education programs" (Tatto et al., 2012, p. 20). Through data drawn from surveys, interviews, and case study reports provided by participating countries, TEDS-M worked to answer research questions such as,

What are the policies that support primary and secondary teachers' achieved level and depth of mathematics and related teaching knowledge?

- What learning opportunities, available to prospective primary and secondary mathematics teachers, allow them to attain such knowledge?
- What level and depth of mathematics and related teaching knowledge have prospective primary and secondary teachers attained by the end of their preservice education? (Tatto et al., 2012, p. 21).

These three questions of TEDS-M are related to recommendations mentioned earlier for subject matter preparation, pedagogical preparation, and clinical experiences.

TEDS-M and MTE-Partnership represent rigorous efforts to describe and improve secondary mathematics teacher education. However, there is also a necessity to examine secondary mathematics teacher education from a ground-up perspective. Just as an open-ended survey question often provides data that is unexpected and informative, so too can research such as ours provide an added perspective to complement the work done in MTE-partnership and the TEDS-M. This ground-up perspective motivated the design and implementation of our research.

Research Questions

The aim of this research was to begin to develop an overview of the structure of SMTP programs in the United States. As we focused our energies two questions served as our anchor:

- 1. What institutional demographics serve as a context for SMTP programs?
- 2. What content and structures currently characterize secondary mathematics teacher preparation programs in the United States?

We were interested in the demographics of institutions with SMTP programs because of the changing landscape of teacher preparation. Demographics such as size of institution, number of students in SMTP program, number of faculty participating in SMTP, and the type of institution, (e.g. public or private) could have an effect on the type of SMTP that occurs. Our second research question helped us to focus on the recommendations for preparing PSTs to teach. As identified in the literature review, we were interested in the subject matter preparation, the pedagogical preparation, and clinical/practical preparation of secondary PSTs. Responses to our questions are intended to provide a snapshot of current SMTP programs in the United States. Having a snapshot of this baseline data not only provides a context to situate further research on areas of need in SMTP, but also provides context for future conversations about preparing secondary mathematics PSTs.

Methods

The motivation for creating a survey that would ascertain the structure of SMTP programs came from a work-group session at the 35th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA) held in Chicago, IL (Winsor et al., 2013). The focus of the work-group session was to promote research on the preparation of secondary mathematics teachers. The organizers of the work-group created a draft of a survey asking participants in the workgroup sessions to share information on the structure of the SMTP program at their institution. The survey was then piloted at the work sessions. Organizers were surprised at the variety, and variability, of the responses that were submitted by the work-group participants. Based upon this variety and variability, a subset of the organizers of the work-group decided it would be helpful to have a clearer snapshot of SMTP programs to continue the conversation about research on SMTP.

Given that the first survey was crafted at the PME-NA meeting, the authors decided to revisit the survey to refine the tool to capture information most relevant to SMTP. A revised version of the SMTP survey was sent to colleagues for review. Based on colleagues' suggestions for more clarity of certain questions as well as other potential survey questions not included, we revised the survey and sent it to colleagues at the first author's university for final comments. We incorporated

colleagues' suggestions and sent the surveys to SMTP programs.

The Survey

The survey consisted of 25 questions. The first eight questions focused on the demographics of the institution participating in the survey. Questions asked for such data as institution size, type of university (public or private), and number of secondary mathematics PSTs that graduate from the SMTP program. We also asked how graduates are certified or licensed upon graduation, though this is more descriptive of the state's licensure than the institution. The next three questions elicited responses concerning the nature of the pedagogy courses at the participating intuitions. The survey then turned its focus to the content preparation of PSTs. Five questions asked about the types of content courses that PSTs were required to take in the SMTP programs, including whether or not PSTs participated in a capstone course as recommended by the Mathematical Education of Teachers (MET; Conference Board of the Mathematical Sciences [CMBS], 2001) and MET II reports (CMBS, 2012). The next four questions asked participants to describe the clinical experiences that PSTs have in their SMTP program. Finally, the remaining four questions asked how many faculty were directly involved in the SMTP program.

Method for Survey Solicitation

In order to obtain a diverse sample of colleges and universities in the United States, we mined several sources for potential study participants. We first turned to the National Council for Accreditation of Teacher Education's (NCATE) website to identify potential participants. We identified 245 institutions that housed nationally recognized programs for preparing secondary mathematics teachers (NCATE, 2017). We gathered contact information for faculty who were involved with the SMTP programs and e-mailed an invitation to participate to all identified faculty. A total of 36 out of the 245 institutions found in the NCATE website responded to the survey.

To expand our sample, we extended the invitation to participate to members of professional organizations in the education including of mathematics field TODOS: Mathematics for All, the MAA's Project NExT, and AMTE. In total, we sent out 1,200 e-mails soliciting participation in the survey. It should be noted that because of the diverse membership of Project NExT and AMTE, several of the emails soliciting participation went to faculty members that were not involved with SMTP. Moreover, the solicitation emails went to multiple faculty members from the same institution, which would reduce the number of surveys completed. Also, the surveys were sent out during the summer when faculty members might not regularly check their e-mails.

We received 101 completed surveys. Out of the completed surveys, we eliminated any surveys that did not identify the university being discussed. We asked for the name of the participating university in case we needed to clarification of a response. The process narrowed the number of usable surveys to 92 responses. We also found six universities that were represented twice (i.e. two different faculty members completed the survey). After eliminating the superfluous surveys, we had 86 viable sets of responses to the survey.

Data Analysis

We analyzed the qualitative data using the constant comparative method (Glaser & Strauss, 1967) to develop categories and associated descriptions for those categories. We first categorized a subset of the qualitative data individually. Next, discussed our categories and associated descriptions to develop an initial coding scheme. We then individually applied this scheme to categorize another subset of the data, and then met to discuss our coding and revise the categorization scheme and/or descriptions to account for any data that did not fit within the current scheme. We iterated this process until the coding scheme stabilized and no new categories appeared. We then applied this coding scheme to the entire set of data, and

then met together to review the categorization and address any discrepancies in our individual coding of the data. We reached consensus by discussing any discrepancies found in our coding.

Survey Findings

We have separated the findings from our survey into two main sections described below. We first provide demographical information about the institutions surveyed. Next, we focus on the structure and content of the SMTP programs in place at these institutions. Lastly, we share the distribution of the bodies administering content exams as part of teacher's state certification.

Institution Demographics

Regional representation. We collected data from all the regions of the United States and adopted the demarcation of the regions (Figure 1) as set by the National Assessment of Educational Progress (NAEP, 2007).



Figure 1. NAEP Regional demarcation for the United States (NAEP, 2007, Census-defined Regions).

Our analysis shows that 17% (15 institutions) were from the Northeastern region; the same held for the Western region. On the other hand, the Midwest and Southern regions each contributed 32% (28 institutions each) to the survey. This is important to note as the findings described in this study were more influenced by the institutions from the Midwest and Southern regions of the United States—56 of the 86 (65%) of the reporting institutions were from these regions.

Department affiliation. Data indicated that of the 86 programs surveyed, 45 (52%) house their SMTP programs in a Mathematics Department, 38 (45%) house their programs in a department in the College of Education (CoE), and three programs (3%) belonged to a Department of Mathematics Education. Although the distribution between housing programs in Mathematics Departments and a department in a CoE is skewed only slightly towards Mathematics Departments, programs housed in Departments of Mathematics Education are rare.

Public or private institution. Out of the 86 institutions responding to the survey, 60 (70%) identified themselves as public universities and 26 (30%) identified themselves as private universities. It should be noted that the numbers reported are similar to a finding that 73% of students attend all types of public colleges and universities whereas 16 % of students attend private nonprofit universities (O'Shaughnessy, 2011).

Additional institutional information. In addition to identifying themselves as public or private universities, participants were asked if there was any other pertinent demographic information that they would like to share. The question was an open response and participants were not required to answer this question. Forty-five institutions offered additional demographic information about their institutions. We read through and classified the responses provided by the participating institutions. The following is a summary of the classifications of the various statements. We only include classifications that had more than one university being a member of that classification:

a. Small rural university (11 institutions)

- b. Many first-generation college students (5 institutions)
- c. Land–grant universities (2 institutions)
- d. Hispanic serving institution (3 institutions)
- e. Originally a normal college (2 institutions)
- f. Institutions founded and run by religions (3 institutions)
- g. Liberal arts institutions (2 institutions)
- h. Historically Black College/University (2 institutions)
- i. Serves large population of Native Americans (2 institutions)

This information demonstrates diversity in the institutions that answered the survey. The demographics of the participating institutions showed an abundance of diversity that covers a wide range of the expected variation found in colleges and universities across the country.

Student population. Data about student enrollment at each university was also collected and is displayed in Figure 2.



Figure 2. Participant institutions student population.

Student enrollment ranged from 900 to 82000 students across all institutions participating, with the lowest 75% of the institutions identifying a total student population of at most 24,250 students (i.e., minimum to the third quartile). The universities surveyed had mean population of 16,807 students. On the extreme side, two universities had a student population of more than 60,000 students. This data informs us that the majority of participating institutions with SMTP programs have student populations of no more than approximately 24000.

Number of secondary teacher graduates. Table 1 below provides the statistics on the number of secondary majors,

regardless of teaching field, graduating from the participating institutions.

	Institutional Number of Graduates in
Statistic	Secondary Education
Mean	84
Median	60
Standard deviation	81
Maximum	510
Minimum	5

Number of Secondary Teacher Graduates

It should be noted that there was a large difference between the maximum value of 510 graduates and the minimum value of 5 graduates. There was also a notable difference between the mean value of 84 graduates and the median value of 60 graduates. The data seems to indicate that the prominence of secondary teacher education at different universities varied.

Number of secondary mathematics teacher graduates. As identified in Table 2, the average number of secondary mathematics teacher graduates was 12 and the median number of secondary mathematics teacher graduates was 10. The range was between 1 and 60.

Table 2

Table 1

Secondary Mathematics Teachers Graduating from the Eighty-Six Institutions

	Number of Secondary Mathematics Education
Statistic	Graduates
Mean	12.2
Median	10
Standard deviation	10.8
Maximum	60
Minimum	1

Note that 56 responding institutions reported ten or fewer graduates on average while nine responding institutions reported 30 or more graduates on average. Looking at the percent of total graduates, the 56 institutions accounted for 32.8% of the total number of reported graduates (1053) while

the nine institutions accounted for 32.1% of the total number of reported graduates.

Who are the teacher educators? It is important to know who are teaching our secondary mathematics PSTs. We found that teacher educators of secondary mathematics teachers are differentiated according to the institutional department they belong to and the discipline they teach. Analysis of the responses indicates that the faculty working directly with the secondary mathematics PSTs were either housed in the CoE, Department of Mathematics, or both. Table 3 details the number of faculty working directly with the secondary mathematics PSTs in the CoE and Table 4 shows how related faculty are distributed in the mathematics departments.

Faculty in CoE working Directly with Mathematics FS1s				
	No. of institutions with <i>n</i> faculty			
No. of	categoriz	cu as.		
faculty	Tenure-track math	Clinical / Part		
(n)	education faculty	time faculty		
0	14	27		
1	11	11		
2	14	8		
3	7	6		
4	2	4		
5	3	2		
6	2	1		
7	2	0		
8	1	0		
10	0	1		
More than 10	4	0		
No Response	26	26		
Total	86	86		

 Table 3

 Faculty in CoE Working Directly With Mathematics PSTs

In Table 3, the data from the second row shows that 11 of the surveyed institutions' CoE only have one tenure/tenuretrack faculty working with their secondary mathematics PSTs, whereas Table 4 indicates that 10 of the institutions have a single faculty member in the math department who is tenured/tenure track working with secondary mathematics PSTs. At the extreme end of Tables 3 and 4, four institutions' CoE have more than 10 tenure/tenure-track faculty working ⁸⁸ with secondary mathematics PSTs, whereas 12 institution's departments of Mathematics have more than 10 tenure/tenure-track faculty working with secondary mathematics PSTs. Barring this category, the distribution between the CoE and mathematics department's number of faculty working with secondary mathematics PSTs were relatively the same.

Table 4

Faculty in Math Department Working Directly With Mathematics PSTs

	No. of institutions with <i>m</i> faculty categorized as:		
	Tenure-	track	
No. of faculty		Math ed.	
(m)	Math faculty	faculty	Clinical/Part time
0	12	33	39
1	10	8	9
2	7	7	6
3	6	5	2
4	3	2	2
5	2	4	0
6	2	0	0
7	1	1	0
8	2	0	1
9	1	0	0
10	3	0	2
More than 10	12	1	0
No Response	25	25	25
Total	86	86	86

Institutional Demographics Summary

The institutional demographic information described indicated that the majority of SMTP programs participating in the study were located within the South or the Midwest of the United States. SMTP programs housing were relatively balanced between Mathematics Departments and Colleges of Education, with a slight skew towards Mathematics Departments. However, though the number of teacher educators of secondary mathematics PSTs are distributed fairly equally between Colleges of Education and Mathematics Departments, there was a slight skew towards Colleges of Education. The majority of SMTP programs are located at public institutions, with the average annual number of secondary mathematics teacher graduates being 10-12.

Structure and Content of SMTP Programs

In addition to gaining an understanding of the institutional context in which SMTP programs were being carried out, we also wanted to gain insight into the content and structure of SMTP programs at different institutions. The sections below expand upon characteristics of the content and structure of SMTP programs.

Program duration. To provide a more complete picture of the structure of SMTP programs, we collected data about the duration of the programs. A total of 78 institutions responded to this survey question. Of these, 68 (87%) of the surveyed institutions offer a four-year SMTP program, whereas 10 (13%) have a 5-year program. This data indicates that four-year SMTP programs appear to be the norm in the United States.

Mathematics courses required by participating institutions. Given the recommendations from the CBMS (2001, 2012) for required mathematics courses in the secondary mathematics teacher degree plan, we gathered data on the mathematics courses required at the participating institutions via the participating institutions' websites. We classified the courses offered by 85 participating institutions by course description and arrived at 31 different course classifications. Table 5 provides the courts of the fourteen most common course classifications (sorted from larger to smaller), and a summary description of each course classification. Note that each description is an amalgam of the descriptions of similar courses; it is not an exhaustive description of all the content in each course.

Description.	S	
		Number of Institutions Requiring
Course	Descriptions	Course
Statistics and Probability	Study of sampling, descriptive statistics, probability, distributions, central limit theorem, confidence intervals, hypothesis testing, and regression.	85
Calculus 2	Study of techniques for integration, improper integrals, sequences and series, analytic geometry, and elementary differential equations with transcendental and algebraic functions of a single variable.	84
Linear Algebra	Examination of matrices, vector spaces, linear transformations, eigenvalues and eigenvectors, and applications.	81
Calculus 1	Study of limits, continuity, differentiation, introduction to integration, and applications, with algebraic and transcendental functions of a single variable.	80
Calculus 3	Examination of multivariable functions, vectors, partial derivatives, and multiple integrals.	79
Abstract Algebra	Examination of mathematical structures (e.g., group, ring, field) and morphisms (e.g., homomorphism theorems).	71
Geometry	Examination of Euclidean and non-Euclidean Geometries.	70
Discrete Mathematics	Study of discrete mathematical structures (e.g., graph theory, combinatorics, set theory, recursion and induction, number theory) in contrast to continuous (e.g., real number system).	46
Exploration and Proof	Study of logic (e.g., propositional statements, quantifiers, truth tables) and proof structures (e.g., direct or indirect, induction) used in writing proofs. Topics often considered within the context of elementary number theory or set theory.	39

Table 5Most Common Mathematics Content Course Frequencies and
Descriptions

		Number of
		Institutions
		Requiring
Course	Descriptions	Course
Analysis	A more rigorous study of Calculus concepts— real numbers and real-valued functions. Topics include sequences and series, limits and convergence, continuity, differentiation and integration,	37
IT Course/ Mathematics using Technology	Study of programming language (e.g., C++), or technologies requiring a specialized language (e.g., Mathematica), with a focus on technique or algorithm development. Secondary school topics sometimes used as context for the technology.	34
Differential Equations	The course focuses on linear differential equations (DEs) of first order (mainly) and higher order, solutions to systems of linear DEs, Laplace transform, numerical solutions, series solutions, and nonlinear DEs.	32
History of Mathematics	Survey of mathematics history and development, including famous mathematicians (e.g., Euclid, Descartes, Gauss) or cultural groups (e.g., Greeks, Babylonians, Egyptians).	30
Capstone Course	The course focuses on developing a thorough understanding of the high school mathematics content. This usually includes connecting the high school content to be taught to content from college level mathematics courses.	28

After aggregating the data in Table 5, we noticed that common courses to SMTP programs were Statistics and Probability, Calculus, and Algebra. These findings were consistent with the recommendations made by CBMS (2012). However, we also noticed that there was a sharp decline between the number of institutions requiring courses in Geometry (70 institutions) to those requiring courses Exploration and Proof (39 institutions) and IT Course/Mathematics using Technology (34 institutions). This stood out to us because CBMS (2012) also recommended that "experiences with reasoning and proof" (p. 56) and "experiences with technology" (p.57) occur across all

experiences learning mathematics. Given the reduced number of institutions requiring Exploration and Proof, and IT Courses/Mathematics using Technology, we wondered to what extent these experiences recommended were actually present in the SMTP programs. This also agreed with the low frequencies associated with these two categories (i.e., experiences with reasoning and proof, experiences with technology) in special courses for preservice SMTs (see Table 7).

Teaching methods course. We collected data about the structure of the teaching methods courses that preservice secondary mathematics take at participating institutions and what departments are responsible for administering these courses. Seventy-eight secondary teacher training institutions responded to both survey questions. Fifty-four percent of the respondents reported either having their methods courses specifically designed for secondary mathematics PSTs (32 institutions), or for all content areas (10 institutions). Forty-six percent of the respondents reported that their PSTs take both general methods courses and mathematics-specific methods courses (36 institutions).

Data showed over half of the responding institutions offer them in the CoE. The number of responses indicating the distribution is shown in Table 6 below.

Table 6

Department	Number	Percent
Mathematics (Math)	16	20%
Education (Ed.)	48	62%
Both (Math & Ed.)	13	17%
Mathematics Education	1	1%
Total	78	

Departments Offering the Secondary Mathematics Methods Course

We note that our data indicated that the majority (62%) of methods courses were taught in the CoE. However, Ball & Bass (2000) noted that "the gap between subject matter and pedagogy fragments teacher education by fragmenting teaching." (p. 85). We wondered in what ways the department (and associated faculty) teaching the methods course

93

influenced secondary PSTs developing an interconnected understanding of mathematics and pedagogy useful for teaching mathematics.

Special courses for secondary mathematics PSTs. We asked participants, "Do you have mathematics courses specifically designed for preservice secondary mathematics teachers? (e.g., a capstone course as described in the MET and MET II reports, etc.)." Of the 78 viable responses, 42 participating institutions responded "yes." The institutions that responded yes to this question were then asked to, "... briefly describe the course goals and the content covered in your mathematics course(s) specifically designed for preservice secondary mathematics teachers." To classify the open responses given by the various institutions, we first looked at the MET and MET II (CBMS, 2001, 2012) report to identify recommended goals and characteristics of courses specifically designed for secondary mathematics PSTs. Table 7 includes the criteria for categorizing responses based on the MET and MET II report. We added pedagogy emphasis criteria because several responses focused on pedagogy (though pedagogy is not mentioned in the MET and MET II reports). Additionally, we note that 75 total courses were described—three institutions that responded "yes" did not include a description of their courses.

Table 7

Reported Gould and	counts for capsione courses	
Criteria	Description	Counts
Experiences with	Being able to reason with mathematics.	6
reasoning and proof	Understanding and being able to produce logically sound proofs. <i>The course</i>	(8%)
	goals/content specifically mention proof.	
Experiences with	Being able to use technology to examine	10
technology	and make sense of mathematics.	(13%)
Treat High School	"emphasize the inherent coherence of	19
Mathematics from an Advanced Standpoint	the mathematics of high school, the structure of mathematical ideas from which the high school syllabus is derived." <i>Connects high school content to college</i> <i>content.</i>	(25%)

Reported Goals* and Counts for Capstone Courses

Criteria	Description	Counts
Take up a particular mathematical terrain related to high school mathematics and develop it in depth	"For example, a course might develop the mathematics necessary to prove the fundamental theorem of algebra or the impossibility of the classical straight-edge and compass constructions." <i>In-depth examination of topic</i> .	57 (76%)
Mathematical Habits of Mind	"All courses and professional development experiences for mathematics teachers should develop the <i>habits of mind</i> of a mathematical thinker and problem- solver, such as reasoning and explaining, modeling, seeing structure, and generalizing. Courses should also use the flexible, interactive styles of teaching that will enable teachers to develop these habits of mind in their students." <i>This is similar to the standards for</i> <i>mathematical practice mentioned in the</i> <i>Common Core.</i>	12 (16%)
Pedagogy emphasis	Mentions that in <i>addition to mathematical goals</i> there are pedagogical goals.	12 (16%)
Note: *Soveral part	icinating institutions montioned more f	han ono

Note: *Several participating institutions mentioned more than one goal for their capstone course, which is why the percentages do not add to 100%.

Note that the goal of developing a particular part of mathematics in depth, is by far the most mentioned goal of the courses described. The second most mentioned goal was to treat high school mathematics from an advanced standpoint, which seems closely related to the first goal. For example, in order for students to connect the concept of groups to solving equations of the form $a \cdot x = b$, they must take the time to not only understand groups but also to understand how equations are treated in the high school curriculum. However, it is important to note that capstone courses have variation in their interpretation (Winsor, 2009), so though we see this as an example of viewing high school mathematics from an advanced standpoint, such an interpretation may not be ubiquitous across all capstone courses.

Mathematics education elective courses. Two questions in our survey asked if preservice mathematics teachers had the

option of taking any elective mathematics education courses and of the elective mathematics education courses were a required part of the preservice mathematics teachers' degree plan. Only ten respondents said that they had elective mathematics education courses. Upon inspection of the various institutions' websites, only one institution had courses that were mathematics education focused (as opposed to an elective mathematics content course) that students could choose.

Clinical Experiences. Participants were also asked three questions aimed at understanding how much time secondary mathematics PSTs spend interacting with high school students. The interaction experiences included observations, teaching lessons, working with groups, tutoring students, or other instances of interacting with secondary students. We used "practicum" and "observations" in these questions to describe this potential for interaction with high school students.

Table 8

Clinical	Hours	Preservice	Secondary	Mathematics	Teachers	Spend
Prior to	Student	Teaching	-			-

Statistic	Number of hours
Number of viable responses	55
Maximum hours spent with students	345 hours
Minimum hours spent with students	0 hours
Mean	84.9 hours
Median	74.5 hours
Standard deviation	60 hours

From the responses given, it was evident that practicum and observations have different meanings at different universities. Given the variety of answers, we focused on the total number of hours that secondary PSTs spend interacting (in whatever manner) with high school mathematics students. The hours had to be a required part of the secondary mathematics degree program. preservice teacher's We contacted representatives from universities for clarification when survey responses were not clear. Table 8 represents the statistics for the number of hours secondary mathematics PSTs spend interacting with high school mathematics students. It is

interesting to note the large range of hours required of students. We wonder if there is a saturation point at which PSTs become overwhelmed with the hours required to interact with high school students so that additional hours do not yield any benefit.

Who supervises student teachers? We also asked participants, "Who supervises student teachers – tenure-track faculty, graduate students, or other (please describe)?" We summarize the responses in Table 9.

	Number of universities using
Type of supervisor	this type of supervisor
Tenure or tenure-track faculty	41 (62%)
Adjunct faculty	22 (33%)
Adjunct faculty who are retired educators (as distinguished from the category above where the faculty work in the department)	25 (38%)
Graduate students	14 (21%)
Other	2 (3%)
Total	66 (100%)

Table 9Student Teacher Supervisors

Note that many universities had more than one type of student teacher supervisor. Although 62% of the institutions reported that tenured or tenure-track faculty supervise secondary mathematics PSTs, we wondered if these were mathematics educators, or faculty with a different area of expertise. Given recommendations for meaningful clinical experiences (AMTE, 2017) and research identifying the separation of teaching and learning to teach (Ball, 2000; Ball & Bass, 2000), we wondered if having faculty who were mathematics educators might help secondary mathematics PSTs make useful connections between their university coursework and secondary classrooms to help bridge the gap between theory and practice. Moreover, are graduate students qualified to serve as student teacher supervisors when the graduate students may not have much experience with teaching?

Teacher certification. We also asked participants which test developer was responsible for administering mathematics content certification tests used within their state, or if this was a certification. requirement for state Although teacher certification is not directly involved in the preparation of secondary mathematics PSTs, we wanted to include this information as it is required for secondary mathematics PSTs who plan to teach in public schools and may add to the basis being established of how PSTs are being prepared. It is common and expected that PSTs must demonstrate readiness for teaching, but there is not a standardized certification exam across all states in the United States. For example, the American Association of Colleges for Teacher Education (AACTE) claims that teacher preparation programs in 40 states are using edTPA to assess the readiness of their teacher candidates (AACTE, 2018).

All 86 responding institutions reported having certification tests in mathematics in their respective states as a requirement to officially teach mathematics in a K-12 classroom. Data indicated that the testing bodies followed by different states for secondary PSTs contained some variation. Although tests developed by ETS were the most commonly used, Table 10 also notes that tests developed by Pearson were other commonly used assessments. With few exceptions, all teacher assessment certifications are based upon tests administered by ETS or Pearson.

Mathematics Certification Test - Development and Administration		
Test Developer	Number	Percent
Pearson	25	29%
ETS	49	57%
ETS and edTPA	2	2%
Respondent not sure	1	1%
State	6	7%
NES by Pearson	3	4%
Total	86	

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Note: ETS = Educational Testing Service, edTPA = Educational Teacher Performance Assessment, NES = National Evaluation Series

SMTP Content and Structures Summary

The content and structures of SMTP programs described indicated that common courses of Statistics and Probability, Calculus, Algebra, and Geometry are the foundation of nearly all programs surveyed, with limited numbers of programs having courses in Proof, Mathematics with Technology, or Capstone courses. Mathematics education specific elective courses were nearly absent in the programs surveyed, and most methods courses were taught in schools of education. Although SMTP programs had wide variation in the opportunities provided to secondary mathematics PSTs to interact with secondary students at clinical placements, tenured or tenuretrack faculty were most often performing observation at these sites. ETS and Pearson were used by nearly all reporting institutions as part of the teacher certification process.

Summary of Findings

Our study set to address two research questions. The first question was, "What institutional demographics serve as a context for SMTP programs?" This study found that SMTP programs are primarily located within public institutions. These programs typically graduate 10–12 students per year. SMTP programs are housed in a balance between Departments of Mathematics and Colleges of Education, with the teacher educators in these programs being distributed relatively evenly between Mathematics Departments and Colleges of Education.

Our second research question was "What content and structures currently characterize secondary mathematics teacher preparation programs in the United States?" Our survey informed us that common courses of Calculus, Algebra, Geometry, and Probability and Statistics are present in nearly all SMTP programs, but these programs vary in terms of requiring courses in exploring mathematics with technology, proof, and capstone experiences. With one exception, SMTP programs did not have mathematics education specific electives as part of the program. We also observed that tenured or tenure-track faculty were most frequently performing

observations for secondary mathematics PSTs at clinical placements, though the number of clinical hours varied widely across the programs.

Discussion

We conducted this study to identify a snapshot of the current preparation contexts used to prepare secondary mathematics teachers. Based upon the findings reported, we found the diversity represented in the data useful for better understanding teacher preparation. The data collected indicated that there is variation from institution to institution in the number and types of mathematics courses taken, where education courses are taught and if there are mathematicsspecific teaching methods courses being taught, as well as the structure of the field experience, including the number of hours spent in the field and who supervises the field experiences. Darling-Hammond (2006) noted that creating more effective teacher preparation programs requires "tight coherence and integration among courses and between course work and clinical work in schools" (p. 300). We recognize that programs at universities in different states may have different demands (e.g., accreditation, teacher certification, partnerships with local schools), but the variation from institution to institution made us question the extent to which this integration and coherence may be present in these programs.

Connections to TEDS-M

As mentioned earlier, the data in this paper can help to amplify the findings from other studies such as TEDS-M. For example, TEDS-M reported that there are differences in the lengths of the various teacher preparation programs. Our data confirms that eighty-seven percent of respondents offer a fouryear SMTP whereas 13% of respondents offer a five-year SMTP.

TEDS-M found that SMTP programs varied in the number of mathematics and mathematics education courses required. Our data delineates the specific mathematics courses that are required for PSTs, as well as the focus of the teaching methods courses offered (i.e., mathematics-specific methods courses vs. general methods courses taken). Moreover, our data notes that the majority of teaching methods courses are taught in a CoE (62% of respondents), followed by being taught in a Mathematics Department (20% of respondents). We wondered if teaching methods courses in a CoE would lean towards being more general in nature so that they could cater to a more varied teacher preparation population (e.g. history teacher, English, teachers etc.). It seems plausible that a methods course taught in a Mathematics Department would be focused on teaching mathematics.

Furthermore, the findings in TEDS-M noted that teacher preparation programs require clinical experiences but do not provide future teachers the opportunity to become familiar with the practical aspects of working in a school (e.g., how the school is organized, daily tasks such as taking attendance and discipline). Our data, as mentioned earlier, showed variability in the number of clinical hours required of PSTs. For example, PSTs in the programs that require few hours may not have enough contact time with high school students in order to gain the practical experience necessary to be effective first year teachers. On the other hand, is there a threshold where too many hours in clinical experiences fails to add value to the PSTs' education? This wondering reinforces the question as to how many hours are necessary for PSTs to become familiar in the practical aspects of teaching.?

Implications

We conducted this study with the hope that the snapshot provided by the data would serve as a catalyst for further research on SMTP. We looked for common themes and variation within data to help us generate questions that can help improve SMTP. What follows are three questions we had as we examined the data. We note that this is not an exhaustive list of questions. It is our hope that this article will help the reader generate many more questions about SMTP to be examined in the future. The data showed that the number of clinical hours required by programs ranges from 0–345 hours. This led us to wonder, "How many clinical hours does a preservice teacher need to be prepared to enter the classroom?" Is there a point of diminishing returns, or a minimum number of clinical hours needed to have an effect on PSTs?

Moreover, a dataset such as the one in this study allows SMTP programs to compare themselves to a national sample. Data indicated that many institutions graduate no more than 10–12 secondary mathematics teachers a year. Although the number of graduates from each university was roughly proportional to the size of the university, we wondered if effective teacher preparation practices at a small institution would not be effective at a large institution? The variation within our collected dataset led us to wonder if there were multiple models of SMTP that would achieve the recommendations found in AMTE (2017) depending on the size of the institution.

CBMS (2001, 2012) recommended that institutions implement a capstone course to be taken by secondary mathematics PSTs. Yet, when we considered our data, only 42 of the 78 reporting institutions (54%) indicated that they had such a course. Moreover, only 28 of these 78 institutions (36%) listed their capstone course as a requirement in their course catalogue. Cox et al. (2013) noted in their study on capstone courses that 42 out of the 73 (57.5%) participating institutions in their study reported having a capstone course. One question that the data has raised is, "What institutional factors/structures promote the implementation of a capstone course to be taken by secondary mathematics PSTs as described by CBMS?" If the consistent, multi-decade recommendation has been to include capstone courses in SMTP, then what factors have been inhibiting this progress?

Closing Remarks

Standards for Preparing Teachers of Mathematics (AMTE, 2017) calls for the following:

Mathematics educators and researchers must make these standards a focal point for research that will guide the improvement of mathematics teacher preparation. By identifying research questions and carrying out studies to support the development of mathematics teacher preparation programs, the research community will provide compelling ideas and evidence to ensure that program graduates are well-prepared beginning teachers of mathematics. (p. 166)

The findings in this paper serve as a starting point for examining SMTP and the recommendations made by the governing bodies of mathematics teacher education for SMTP. The snapshot we have provided allows researchers to identify research questions based on the current context of SMTP in the United States instead of focusing solely on their own institution's SMTP program. Moreover, the data in this study may serve as an impetus for institutions from various parts of the United States with similar demographics to partner to research SMTP. The MTE-Partnership (Hazelrigg, 2017) is an example of land-grant institutions uniting to study SMTP.

A united, systematic study of the preparation of secondary mathematics PSTs aligns with what Hiebert, Morris, and Glass (2003) described as "teacher educators engaged in learning and how to study their teaching of prospective teachers" (p. 218) in an effort to contribute to a "shared knowledge base for teaching" (p. 211). Systematic inquiry into the ways in which secondary mathematics PSTs are being prepared is needed to assess the extent to which our practices and support structures align with the field's standards, which allows for adjustments to these practices and structures. Such inquiry also provides space for future research studies, such as the questions identified in the Implications section above. Our hope is that mathematics teacher educators will take the findings presented in this paper, generate additional interesting and important research questions and move further in the directions suggested by AMTE.

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