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Second Graders′ Metacognitive Actions in Problem Solving Revealed Through Action Cards

Ana Kuzle

Despite the important role that metacognition plays in school mathematics, attention has only recently turned to primary grades. The aim of this exploratory qualitative study was to find out to what extent 6 second graders engage in metacognitive behaviors during mathematics problem-solving. The analysis was based on the adaptation of the multi-method interview approach, whose core idea lies upon using action cards consisting of metacognitive cues. The results show that even young children engage in different metacognitive actions. However, the use of action cards revealed some drawbacks with respect to studying young children's metacognition during mathematics problem-solving.

Over the years, metacognition has been linked to improved student outcomes (e.g., Hattie, 2009; Wang, Haertel, & Walberg, 1993), especially in the field of mathematics. Metacognition has been advocated as an important factor in student learning and as a driving force during problem solving (e.g., Depaepe, DeCorte, & Verschaffen, 2010; Schoenfeld, 1985b; Veenman, Van Hout-Wolters, & Afflerbach, 2006). For that reason, there has been a long-standing effort to increase students' ability to engage in problem solving by providing them with instruction rich with metacognitive activities (e.g., Boekaerts, 1977; Brown, 1978; Desoete, Roeyers, & Buysee, 2001; Garofalo & Lester, 1985; Goos & Galbraith, 1996; Kramarski, Mevarech, & Arami, 2002; Schraw, 1998).

More recently, there is increasing evidence that young children (birth to 8-year-olds) who are provided with proper tasks and enough time to work on them do exhibit metacognitive

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behaviors (e.g., Larkin, 2010; Whitebread, Coltman, Anderson, Mehta, & Pasternak, 2005). Yet, there is little research regarding (young) children's thinking, the level of their thinking, and their metacognitive abilities during problem solving (e.g., Alexander & Schwanenflugel, 1996; Desoete et al., 2001; Kuzle, 2018; Whitebread et al., 2005). To better understand (young) children's metacognition in mathematics, coherent and viable models of metacognition and accompanying rigorous, age-appropriate methods for analyzing young children's metacognition are needed.

In this article, I present the model of metacognition developed by Wilson and Clarke (2002, 2004) and their multi-method interview approach (MMI-approach). This method was developed to study student mathematical metacognition in the context of sixth graders' problem solving. In this study, I examined the usefulness of Wilson and Clarke's model of metacognition and the utility of the adaptation of the MMI-approach for the analysis of second grade students' metacognition during mathematics problem-solving. The results of the empirical study highlighted second grade students' metacognitive actions in problem solving, as well as the extent of the usefulness of the metacognitive model and the utility of the adaptation of the MMI-approach for the analysis of young children's metacognition.

Metacognition and Models of Metacognition

Many models or definitions of *metacognition* refer to Flavell (1976, 1979), who introduced the term. Metacognition is a form of cognition and a higher order thinking process that has been defined as any knowledge or cognitive activity that "takes as its object or regulates any aspect of any cognitive endeavor" (Flavell, 1976, p. 8). That is, metacognition has a managerial and a regulatory role in cognitive processes; it overlooks and governs the cognitive system, while at the same time is a part of the cognitive system (Veenman et al., 2006). Flavell (1979) identified various components of metacognition that occur through the actions of and interactions among four classes of phenomena: (a) *metacognitive knowledge*, which refers to acquired knowledge addressing cognitive matters and incorporates person, tasks, goals, actions, and experiences; (b) *metacognitive experiences*, which refer to conscious cognitive or affective experiences that accompany and pertain to any intellectual enterprise; (c) *goals* (or tasks), which refer to the objectives of a cognitive enterprise; and (d) *actions* (or strategies), which refer to behaviors employed to control one's own cognitive activities and to ensure that a cognitive goal has been met. For example, when confronted with a problem, a problem solver may feel that the task is easy to solve (i.e., a metacognitive experience) because he or she is aware of strategies that can be used to solve a given problem (i.e., metacognitive knowledge). As a result of a problem-solving process, the problem solver produces an answer to the problem (i.e., goal) which can then be evaluated for its correctness (i.e., action).

Building on Flavell's work, different models emerged (e.g., Boekaerts, 1997; Brown, 1978; Garofalo & Lester, 1984; Schoenfeld, 1985b). Wilson and Clarke (2002, 2004) critiqued the dual nature of these models because the models focused only on knowledge about and regulation of cognition. Rather, they argued for the importance of extended models of metacognition that would be more helpful in explaining how students use metacognition. Moreover, Wilson and Clarke conceptualized metacognition as having three functions: awareness of thought processes, individual evaluation, and regulation of these thought processes. This approach is especially fruitful with respect to metacognition during problem solving given that a problem solver requires a variety of metacognitive processes for completing any complex task, such as planning, monitoring, testing, revising, and evaluating (e.g., Kuzle, 2017; Schoenfeld, 1985b, 1992; Schraw, 1998).

Metacognitive awareness refers to "individuals' awareness of where they are in the learning process or in the process of solving a problem, their content-specific knowledge, and their knowledge about their personal learning or problem solving strategies" (Wilson & Clarke, 2004, p. 27). Moreover, metacognitive awareness entails knowledge of what has been done, needs to be done, and might be done in order to attain a specific goal related to problem solving. For example, a student may think about what helped him or her when solving a similar problem.

Metacognitive evaluation refers to judgments made with respect to one's thinking processes, capacities, and limitations (Wilson & Clarke, 2004). For instance, individuals could be evaluating the effectiveness of their thinking by judging the correctness of a solution or a solution path, as well as the effectiveness of the latter. Evaluatory behaviors also assume some kind of awareness of the individual's thinking processes and anticipate possible regulatory processes (Wilson & Clarke, 2004).

Metacognitive regulation "occurs when individuals make use of their metacognitive skills to direct their knowledge and thinking" (Wilson & Clarke, 2004, p. 27). It draws on individuals' knowledge about self, possessed strategies, and executive skills (e.g., self-correcting, setting goals) to optimize the use of their own cognitive resources (Wilson & Clarke, 2004). For instance, a student may develop a plan to solve a given problem before engaging in a particular cognitive action (e.g., drawing a figure, making a table).

To solve any mathematical problem, the interplay of cognition and metacognition is fundamental (Adibnia & Putt, 1998; Kuzle, 2017; Schoenfeld, 1992). Hasselhorn (2000) claimed that there is not one metacognitive pathway for success, but rather, components of different metacognitive categories are responsible for initiating strategies for learning opportunities when engaged in a complex cognitive endeavor, such as problem-solving. Thus, the nature of cognitive processing is dual—cognitive and metacognitive. Aligned with Flavell's definition of metacognition (1976), Wilson and Clarke's (2002, 2004) definition emphasized the importance of the interplay between cognition and metacognition during problem solving by stating that "the objects on which metacognition acts are cognitive objects" (p. 33). Thus, we interact via cognitive behaviors purposefully with the world, and the overt actions are the results of cognitive activity, which itself is influenced by metacognitive activity (Wilson & Clarke, 2002, 2004). Figure 1 offers a graphic illustration of this understanding of cognitive processing and posits a multiplicity of metacognitive pathways, as suggested by Wilson and Clarke (2004). Specifically, the structure of the model of metacognition is a function of three metacognitive components, namely awareness, regulation, and evaluation. However, the process of problem solving involves a continual alternation between key aspects of student metacognitive and cognitive activity. The mediation between different metacognitive functions as well as between a particular metacognitive function and cognition is illustrated with two-way arrows. Here, it is important to stress that the arrows do not suggest any particular process or favored sequence (Wilson & Clarke, 2004). Rather, it is a graphical illustration of "the relationship between the key aspects of student metacognitive and cognitive activity, and a ready means to identify any patterns in that activity" (Wilson & Clarke, 2004, p. 33).

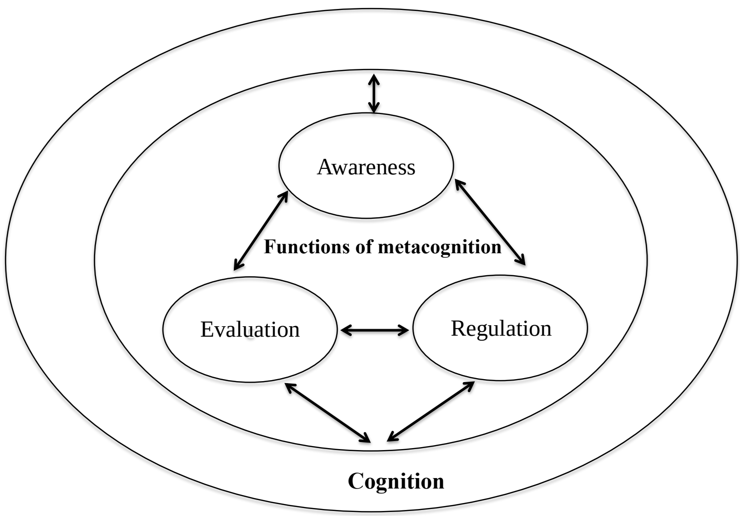


Figure 1. Wilson and Clarke's (2002, 2004) structure of a model of metacognition.

Literature Review

Research has shown that children as young as five years old start to develop metacognitive functions (e.g., Alexander, Carr, & Schwanenflugel, 1995; Whitebread et al., 2005), and by Grade 6, they have a more robust metacognitive development (e.g., Wilson & Clarke, 2004); however, less is known about children's metacognitive development during problem solving in mathematics. Furthermore, studying metacognition is difficult and has been approached with a variety of methods. Below I discuss literature related to children's metacognitive development and how children's metacognition has been studied.

Children's Metacognitive Development

In educational psychology, children's metacognition is described as still being incomplete. Metacognition starts developing early, at the age of 5 to 7, and continues to grow over time until reaching its full development at the age of 12. One's development of metacognition occurs parallel to the development of one's intellectual ability (Alexander et al., 1995) and becomes more powerful and effective as a result of years of accumulated experience in making thought the object of thinking (Brown, 1978). In relation to metacognition specific to children's mathematical problem-solving, these results have been confirmed by Veenman et al. (2006), who hypothesized that "metacognitive knowledge and skills already develop during preschool or early-school years at a very basic level but become more sophisticated and academically oriented whenever formal educational requires the explicit utilization of a metacognitive repertoire" (p. 8). For instance, Whitebread et al. (2005) have shown that the behavior of young children (5-year-olds) may reveal elementary forms of orientation, planning and reflection when engaged in a contextualized problem-solving task. In their study, more than half of metacognitive behaviors accounted for strategic control behaviors, such as articulating explanations, evaluating, and planning. Other strategic control behaviors, such as monitoring and applying existing knowledge to new problems, were present to a very limited extent. With respect to the latter, Focant, Grégoire, and Desoete (2006) in their study on arithmetical problem-solving similarly reported that certain metacognitive skills (e.g., monitoring, evaluation) appear to mature later than others (e.g., planning). Compared to Whitebread et al. (2005), Desoete et al. (2001) and Kuzle (2018) studied metacognition of older children, namely Grade 3 and 4 students respectively, during mathematics problem-solving. They showed that certain metacognitive skills (e.g., prediction, evaluation) differentiate between average or above-average and below-average Grade 3 students (Desoete et al., 2001) and Grade 4 students (Kuzle, 2018).

Although young children's behavior reveals elementary forms of metacognitive functions and not its complete repertoire, by the time students are in Grade 6, they have a more robust and complete set of metacognitive actions from which to draw (Wilson & Clarke, 2004). In Wilson and Clarke's (2004) study on mathematical problem-solving, Grade 6 students exhibited all functions of metacognition (i.e., awareness, evaluation, regulation), and all possible transitions were present. By using the model of metacognition (see Figure 1), Wilson and Clarke identified consistent sequences of metacognitive functions employed by the students. For instance, the students almost always began with awareness. Awareness was followed by evaluation and regulation (AER sequence) or by regulation first and then evaluation (ARE sequence). Although Hasselhorn (2000) suggested that metacognitive development cannot be attributed to a simple age grid, he emphasized that it was generally accepted that the knowledge of the function of metacognition increases with knowledge acquisition.

While we *do* know about children's general development, especially with respect to students older than 12 (Händel et al., 2013), studies on metacognition in the specific context of problem solving in mathematics with students younger than 12 are relatively limited (Veenman et al., 2006). Thus, we need to know more about children's metacognition during problem solving, such as what functions of metacognition they engage in during problem solving and what specific behaviors pertaining to these functions are available to primary grade students.

Measuring Metacognition

In problem-solving research, no uniform, comparable measurement method of metacognition exists (Veenman, 2005), but rather a variety of verbal report methods are available, such as think-aloud, both individual (Ericsson & Simon, 1980) and pair protocols (Schoenfeld, 1985a, 1985b); clinical interviews (Ginsburg, Kossan, Schwartz, & Swanson, 1983); concurrent probing (Ericsson & Simon, 1980); retrospective probing (Ericsson & Simon, 1980); retrospective clinical interview (Schoenfeld, 1985a); and surveys (e.g., Baumert, Heyn, & Köller, 1992). Wilson and Clarke (2004) criticized the widely used think-aloud method for measuring metacognition during problem solving. They brought into question "the accessibility, veridicality, and completeness of verbal reports" (Wilson & Clarke, 2004, p. 28). Additionally, they criticized the problem solvers' ability to verbally report their thinking processes simultaneously with the problem-solving process, as this task is cognitively and verbally demanding for young children. Children's insecurity can lead to resorting to immediate experiences, rather than reporting on their strategies (Garner, 1988). In such cases, qualitatively and quantitatively incomplete strategy reports are to be assumed. One way to resolve this problem is to provide students with retrieval cues, such as action cards, so that they can more easily explain what they were thinking at a given time (e.g., Randhawa, 1994).

Wilson and Clarke (2002, 2004) developed a new method that combined multiple verbal report methods, taking into account the cognitive and linguistic development of the subjects in order to overcome the previously mentioned drawbacks. Their MMI-approach combined several different instruments: a problem-based clinical interview (a self-reporting process using *action cards*), audio and video recordings for stimulated-recall, and observations. The main feature of the clinical interview was a card-sorting procedure wherein students used action cards to reconstruct their thought processes after the problem-solving process. The term *action* reflects a purposeful activity captured in students' statements. Wilson and Clarke (2004) developed 14 action cards that individually listed metacognitive statements on cards, each associated with one of the three metacognitive functions (5 awareness, 5 evaluation, 4 regulation cards), as well as problem specific and general action cards listing cognitive behaviors (see Table 1). The use of both cognitive and metacognitive action cards allowed Wilson and Clarke (2004) to distinguish between the nature of cognitive processing (cognitive versus metacognitive) as well as the interplay between the two.

Table 1
Overview of Wilson and Clarke's (2002, 2004) Action Cards

Awareness	Evaluation
I thought about what I already know. I tried to remember if I had ever done a problem like this before. I thought about something I had done another time that had been helpful. I thought 'I know what to do'. I thought 'I know this sort of problem'.	I thought about how I was going. I thought about whether what I was doing was working. I checked my work. I thought 'Is this right?' I thought 'I can't do it'.
Regulation	**Cognition**
I made a plan to work it out. I thought about a different way to solve the problem. I thought about what I would do next. I changed the way I was working.	I drew a diagram. I added. I counted. I turned a shape over.

Wilson and Clarke's (2004) MMI-approach also used videos to stimulate students' reflections on the constructed card sequence (i.e., video-stimulated recall). In their work, "of particular significance was the consistency with which access to the video record prompted students to change their initial accounts of their problem solving attempts" (Wilson & Clarke, 2004, p. 43). Such a combination of verbal report methods increased the validity of the results, as nearly all the students made changes to their reported card sequences. Even though Wilson and Clarke demonstrated the effectiveness of the MMI-approach in the context of Grade 6 mathematics problem-solving, the general utility of their approach, especially when working with younger students, remained open.

Research Questions

Although researchers know more about Grade 6 or older students' metacognition, little is known about younger students' metacognitive abilities during mathematics problem-solving.

Knowing what models and methods allow insights into children's metacognition and consequently to what extent students engage in metacognitive behaviors when problem solving provides a filter for their problem solving in mathematics beginning from early grades. Moreover, identifying developmental trajectories using age-appropriate models and methods would be a first important step before identifying what practices, if any, would help young students to develop and enhance their metacognitive behaviors. With these assumptions in mind, I sought to study Grade 2 students' metacognition during problem solving drawing largely from the work of Wilson and Clarke (2002, 2004) by adapting their MMI-approach. The study focused on the following research question: What metacognitive structure sequences were exhibited by Grade 2 students using the adaptation of the MMI-approach after mathematics problem-solving?

Method

Research Design and Subjects

For this study, an exploratory qualitative research study design was chosen. The study participants were Grade 2 students from one urban school, who had experience in problem solving. Concretely, the teacher engaged the students in problem solving (i.e., numeric and combinatorial problems) at least once per month. During the problem-solving lessons, the students individually solved the problems which were then discussed in plenum. Students from one Grade 2 class were invited to participate in the study. From those students who volunteered, a group was identified that balanced both gender (two girls and four boys) and mathematical ability[1] (two below-average, two average, and two above-average students). In total, six Grade 2 students from one teacher's class participated in the study.

[1] Mathematical ability was assessed by the teacher on the basis of oral and written contributions during mathematics lessons.

Mathematical Tasks

The mathematical tasks included in the study were carefully selected to allow Grade 2 students to exhibit metacognitive behaviors (e.g., Larkin, 2010; Whitebread et al., 2005). The following criteria were considered in the selection of the mathematical tasks:

1. The participants are faced with an unfamiliar problem situation without an apparent solution path.
2. The problems provide students with opportunities to engage in metacognitive activity.
3. The problems deal with mathematical content of numbers and operations, measurement and combinatorics, which were covered in mathematics lessons (The Standing Conference of the Ministers of Education and Cultural Affairs of the Länder in the Federal Republic of Germany [KMK], 2005).
4. The tasks are short and can be solved in a short period of time to ensure that students' retrospective reconstruction of the problem-solving process is complete and accurate.

For the purpose of this study, a vast pool of tasks from elementary school resources (e.g., textbooks, websites) was examined, from which three problems were chosen that fit the above criteria (see Appendix for the mathematical tasks).

Data Collection Instruments

I collected data during three problem-solving sessions. The data consisted of (a) audio and video data of the problem-solving sessions, (b) a retrospective clinical interview, (c) students' written artifacts, and (d) researcher field notes. To collect audio and video data, two video cameras recorded the sessions, one focusing on the child and the other on the child's work. Audio data consisted of the student's unprompted verbal reports during problem solving and prompted verbal reports after the problem-solving process. Retrospective clinical interview comprised two parts, retrospective reconstruction using action cards and retrospective interview questions. The action cards were developed through a pilot study with one Grade 2 and one Grade 4 classroom. The students solved the mathematical tasks in groups and discussed their solutions as they solved the tasks. Their verbal data were used to create the statements on the action cards. Afterwards, the actions cards were tested in an individual setting with five Grade 2 and five Grade 4 students, and then revised and prepared for the main study. As a result, 12 metacognitive cards (4 cards per metacognitive function), and 3-5 general and problem-specific cognitive cards (varied depending on task) were developed for the main study (see Table 2). Compared to the MMI-approach of Wilson and Clarke (2002, 2004), fewer cards were developed, and the statements were shorter and phrased in a student-friendly way using the language of the students from the pilot study to match the level of the cognitive and linguistic development of the subjects, as suggested by Brown et al. (1983) and Patrick and Middleton (2010).

Table 2
Action Cards with Cognitive and Metacognitive Statements

Awareness	Evaluation
I thought about math I know. I knew what math would help me. I had an idea what I could do. I thought about a similar problem.	I thought about whether what I was doing was working. I checked my last step. I checked my solution. I wasn't sure if I could do it.
Regulation	**Cognitive cards**
I made a plan in my head. I thought about another way. I thought about my next step. I decided to do something else.	I drew a figure. (general card) I made a table. (general card) I calculated how many meters a snail makes in a day. (problem-specific card)

In addition to the video recorded sessions and retrospective clinical interviews, I also collected students' written work produced while students solved the three problems. Finally, I took field notes during the problem-solving sessions. Researcher field notes contained protocols of students' problem-solving actions and helped validate their reconstructed metacognitive sequences. As such, they were used to negotiate the problem-solving process with the student, especially when discrepancies between what the student said and what the researcher saw the student do were observed. Multiple data sources were used to increase the validity of the results.

Data Collection Procedure

The interviews[2] occurred in a one-to-one setting between the student and the researcher in a school environment at the beginning of the second school semester. Each student completed a familiarization task at the beginning of the first session. By solving an exemplary problem, the student was introduced to the procedures (i.e., concurrent verbalization, reconstruction of the problem-solving process using action cards, retrospective interview). Special attention was given to the action cards. Each card was discussed to make sure that the student had the same understanding of a particular statement as the researcher. If this was not the case, the researcher explained the card to the student. During each session, a student solved one problem, which lasted about 7-10 minutes. The researcher asked each student to verbalize his or her own thinking processes during the problem-solving process. The session started with a student receiving a problem and a blank piece of paper. During the problem-solving process the students either verbalized their thinking processes or solved the problem in silence. The retrospective reconstruction using action cards took place immediately after completion of the problem. Each student was given the actions cards,[3] which he or she then used to chronologically reconstruct his or her problem-solving process. This occurred in two phases. In the first phase, students put aside the action cards they thought did not align with their problem-solving behavior. If the provided action cards did not align with their problem-solving behavior, then the researcher provided them with a blank card and the students described that specific behavior or action in their own words. If a student had difficulties reading the statements on the actions cards or writing

[2] The interviews were conducted in German and the results (e.g., students' problem-solving processes, action cards) were translated into English.

[3] No cues were present on the action cards, so that the student did not know the nature of the cards.

them down, then the researcher helped the student by reading and/or writing down the statements. In the second phase, the student reconstructed the problem-solving process using the remaining action cards and additional actions cards if necessary.

In order to validate the reconstructed sequence made by the student, the problem-solving process was elaborated on retrospectively. The student first verbally described his or her own problem-solving behavior using the piece of paper with the problem solution. During this phase, the reconstructed sequence was removed from sight to avoid influencing the student's retrospective elaboration of the problem-solving process. At the same time, the researcher took careful notes of each student's report. This was followed by a discussion of the card sequence taking into consideration the researcher field notes to validate or question the reconstructed sequence made by the student. If discrepancies (e.g., nature of actions, order of actions, solution processes) between the student's narrative and the reconstructed sequence occurred, then this was discussed until an agreement was reached and the reconstructed sequence was newly arranged. The same procedure was used for each subsequent session. Each student was interviewed three times, once for each mathematical task within a one-week period.

Data Analysis

To analyze the students' reconstructed sequences, I first color-coded the cards (i.e., awareness–yellow, evaluation–red, regulation–green, cognition–blue) and divided the action cards used in the reconstructed sequences into three groups pertaining to the three metacognitive functions. The action cards that were developed by the students were likewise assigned to one of the three metacognitive functions based on its nature. Here each action card was additionally assigned a letter, namely A (awareness), R (regulation), E (evaluation), and C (cognition). In order to identify a metacognitive sequence, repeated actions as well as cognitive actions were omitted. For instance, a sequence of behaviors A, R, E, C, E, C, C, E, was coded as an ARE sequence, as suggested by Wilson and Clarke (2002, 2004). Repeated actions, both cognitive and metacognitive, were omitted in order to identify any patterns in the activity. In addition, each reconstructed sequence was visualized using Wilson and Clarke's structure of a model of metacognition as was shown in Figure 1. Then, each metacognitive cluster was refined by taking into account the statements on the action cards. Finally, descriptive statistics were calculated to analyze the distribution of cognitive and metacognitive behaviors.

Results

In this section, I first illustrate one problem-solving session of two students who worked on a numeric task (see Appendix for the mathematical tasks). Then I compare the results of six Grade 2 students on the same task. Lastly, I offer an overview of the students' solutions to three mathematical problems with respect to exhibited metacognitive functions and metacognitive structure sequences.

Metacognition during Mathematics-Problem Solving: The Snail in the Well Task

In this section, I discuss the problem-solving sessions of Jana (above-average student) and Mike (average student). Both students engaged in all metacognitive behaviors, exhibited consistent metacognitive patterns, and approached the task in a similar manner. However, only Jana solved The Snail in the Well Task correctly (see Appendix for the mathematical tasks).

Jana's problem-solving process. Immediately after reading the problem, Jana knew she had to draw an informative figure. She marked "9 m" at the top of the informative figure and then started at the bottom of the figure, marking the distance the snail crawled during the day and slid back during the night. After each step (+3, –1), she added "1T" (1 day) onto her figure (see Figure 2). She repeated these steps until she reached the 9 m marker. She hesitated briefly whether the snail would slide back again, before stating that this does not happen, and that the snail reaches the top on the fourth day.

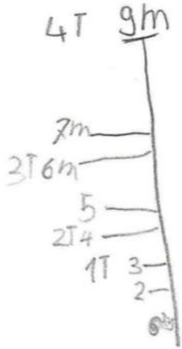
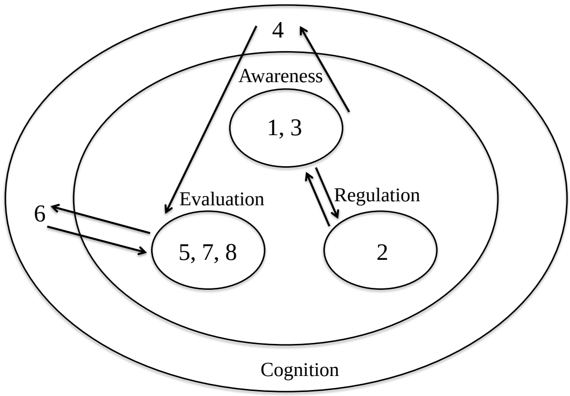


Figure 2. Jana's solution.

After the session, the student reconstructed the problem-solving process using action cards in combination with a retrospective interview. As a result, eight action cards remained on the table. Following this, Jana rearranged the action cards together with the interviewer in order to fit her train of thoughts. In terms of the three functions of metacognition—awareness, regulation, and evaluation—in combination with cognitive actions, the sequence A, R, A, C, E, C, E, E was reconstructed as shown in Figure 3. In Figure 3, the numerals in the ovals show the order of the actions (1-awareness, 2-regulation, 3-awareness, 4-cognition, 5-evaluation, 6-cognition, 7-evalaution, 8-evaluation), whereas the arrows illustrate transitions between different (meta-) cognitive behaviors (e.g., arrow between numerals 1 and 2 illustrates transition from awareness to regulation).

Jana's metacognitive structure sequence reveals all metacognitive functions as well as the interrelation between cognitive and metacognitive processes. She immediately knew how to approach the task (i.e., awareness), which was followed by making a plan (i.e., regulation), namely drawing an informative figure (i.e., cognition). She also made use of her knowledge gained by reading the task (i.e., awareness). While supporting her calculations with a visual, she was thinking about the correctness of her approach (i.e., evaluation). This was followed by calculations (i.e., cognition), checking the solution (i.e., evaluation) and evaluating the result with respect to the task requirements (i.e., evaluation).



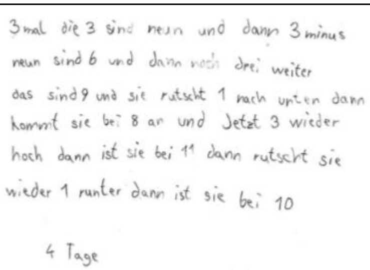
Action card 1: I had an idea what I could do. [awareness]
Action card 2: I made a plan in my head. [regulation]
Action card 3: I thought about math I know. [awareness]
Action card 4: I drew a figure. [cognition]
Action card 5: I thought about whether what I was doing was working. [evaluation]
Action card 6: I calculated until I reached 9 m. [cognition]
Action card 7: I checked my solution. [evaluation]
Action card 8: I thought about whether the snail slips back or not. [evaluation]

Figure 3. Jana's metacognitive structure sequence.

Jana's metacognitive structure sequence corresponds to the results of Wilson and Clarke (2002, 2004); she commenced at awareness, ended with an evaluatory action and engaged in a regulatory action in between. Children in primary school often encounter difficulties in problem solving since they hardly possess knowledge of tasks, strategies and goals (metacognitive knowledge; Flavell, 1976, 1979). However, this was not the case with Jana, who showed a high quality of metacognitive processes (e.g., flexibility in thinking processes, reflexive ability) as a Grade 2 student.

Mike's problem-solving process. After reading the task, Mike repeated the task in his own words. After a few moments, he uttered that the snail reaches the top on the sixth day. Mike explained his solution as follows: He multiplied 3 by 3 to reach

9 m and subtracted 3 from it (he reckoned –3 for three nights during which the snail slid back 1 m each time) reaching 6 m. Following that, he took into account that the snail advances 3 m a day and slides down 1 m during the night. He added then +3 and subtracted 1 from it. The snail was then at 8 m. Here, he did not recognize that the snail had already reached the top at 9 m, which led him to subtract 1 m and then again to add +3 and subtract 1 reaching 10 m. He calculated the number of days, and he wrote down—contradicting what he stated at the beginning of the problem-solving process—that the snail needs 4 days to reach the top of the well (1 day for the first line, 1 day for the third and fourth line, 1 day for the fourth and fifth line, and 1 day for the sixth line as shown in Figure 4). Thus, it was difficult for him to understand what a day meant. His answer, though correct, was an error in adding, because according to his solution the snail would have needed five days.


3mal die 3 sind neun und dann 3 minus
neun sind 6 und dann noch drei weiter
das sind 9 und sie rutscht 1 nach unten dann
kommt sie bei 8 an und Jetzt 3 wieder
hoch dann ist sie bei 11 dann rutscht sie
wieder 1 runter dann ist sie bei 10

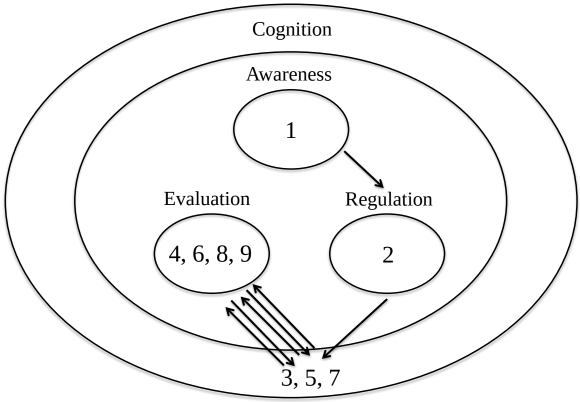
4 Tage

3 times 3 are nine and then 3 minus
nine are 6 and then three more
are 9 and she slides 1 down then
she is at 8 and now 3 up again
then she is at 11 then she slides
1 down again then she is at 10

4 days

Figure 4. Mike's solution and translation into English.

After the session, the problem-solving process was reconstructed using action cards in combination with a retrospective interview. At times Mike was not sure if the process stated on the cards took place. When this happened, the process was discussed to help him recall his thinking. Some actions did not fit the action cards, so three extra cards were written in his own words (action cards 3, 4, 5). As a result, nine cards remained on the table. Mike then rearranged the action cards together with the interviewer to align with the order of his thoughts. In terms of the three functions of metacognition in combination with cognitive actions, the sequence A, R, C, E, C, E, C, E, E was reconstructed as shown in Figure 5. Again, the numerals in the ovals show the order of the actions (1-awareness, 2-regulation, 3-cognition, 4-evaluation, 5-cognition, 6-evaluation, 7-cognition, 8-evaluation, 9-evaluation), whereas the arrows illustrate transitions between different (meta-) cognitive behaviors.



Action card 1: I had an idea what I could do. [awareness]
Action card 2: I made a plan in my head. [regulation]
Action card 3: I calculated 3 times 3 until I was at 9. I subtracted three because of the sliding back. [cognition]
Action card 4: I checked the last calculation. [evaluation]
Action card 5: I calculated always +3, and then –1. [cognition]
Action card 6: I thought about what I was doing. [evaluation]
Action card 7: I counted till I reached 9. [cognition]
Action card 8: I thought about whether what I was doing was working. [evaluation]
Action card 9: I checked my solution. [evaluation]

Figure 5. Mike's metacognitive structure sequence.

Mike's metacognitive structure sequence reveals all metacognitive functions as well as interrelation between cognitive and metacognitive processes. He began his problem-solving process with awareness by using his existing knowledge. He made a rough plan (i.e., regulation) before engaging in calculations (i.e., cognition), which were then evaluated. This was followed by more calculations (i.e., cognition) and a check of the solution with respect to task requirements (i.e., evaluation). He then engaged in alternating cognitive and evalautory actions. Even though Mike said he often evaluated his work (action cards 4, 6, 8, 9), this is questionable since he added the days incorrectly. It may be that he lacked content-specific criteria to use in his evaluatory actions. In a similar way to Jana, his metacognitive structure sequence was also aligned with results of Wilson and Clarke (2002, 2004). He commenced at awareness, ended with an evaluatory action and engaged in a regulatory action in between.

Summary of all students' results with respect to The Snail in the Well Task. Table 3 offers a summary of the Grade 2 students' metacognitive actions and problem-solving processes when working on The Snail in the Well Task. Surprisingly, only Jana arrived at the correct solution. Niko and David used the same idea of adding +2 (2 m; 4 m; 6 m; 8 m; 10 m), while Ben added −2 to 9, concluding, however, that the snail needs 5 days. Thus, three children composed the snail's upward and downward movements into a single daily rate of 2 m. Lara and Mike, on the other hand, showed evidence of grouping the snail's upward progress into multiple days and then compensating for the slide. Even though all these mathematical approaches were reasonable, the students came to solutions of three or five days, revealing difficulties in connecting upward and downward movements of the snail with the number of days.

In the reconstructed metacognitive sequences, all functions of metacognition—awareness, evaluation, regulation—were reported (see Table 3) with 63.4% of all reported actions (n = 41) being of a metacognitive nature. Evaluation was the metacognitive behavior the students engaged in the most (31.7%). Evaluatory actions were exhibited by five out of six students. The two action cards used most often were, "I thought about whether what I was doing was working" and "I checked my solution." All the students engaged in awareness (21.9% of reported actions), but once or twice at most. Predominantly students chose the action cards "I had an idea what I could do" and "I thought about math I know." They never thought about a similar problem which is rather surprising since the students had experiences with numeric tasks. Regulation was reported the least (9.8%), where only three students (one average and two above-average students) engaged in regulation. In three cases the students used the card "I made a plan in my head" and in one case "I thought about my next step." With respect to reconstructed metacognitive sequences, only three revealed all functions of metacognition, namely ARAE and ARE metacognitive sequence. Other sequences contained one (A) or two metacognitive processes (A, E), which was often seen in below-average students' solutions.

Table 3
Overview of Wilson and Clarke's (2002, 2004) Action Cards

Student	Ability	Metacognitive sequence	Problem-solving process	Correctly solved
Ben	below-average	A, C, C	for each day −2 was added to 9	No
Niko	below-average	C, A, C, C, E, E	for each day +2 got added	No
Lara	average	A, A, C, C, E, E	3 got added 3 times until reaching 9, after which −3 got added to 9	No
Mike	average	A, R, C, E, C, E, C, E, E	3 got multiplied by 3, after which −3 got added to 9, then +3, −1 got added	No[4]
David	above-average	A, A, R, C, R, C, E, E, C	for each day +2 got added	No
Jana	above-average	A, R, A, C, E, C, E, E	sequential steps (+3, −1) using an informative figure	Yes

In summary, Grade 2 students displayed all the functions of metacognition when working on The Snail in the Well Task, but there were differences in their frequency and in the spectrum of different actions. Even though students' metacognitive behaviors showed consistent patterns, it seems that they lack content-specific criteria (i.e., coordinating snail's upward and downward movement in relation to elapsed time) to use in their evaluating and regulating activities.

[4] Even though Mike's answer was correct, it was due to an error in adding up, because according to his solution the snail would have needed five days.

Metacognition during Mathematics-Problem Solving of Grade 2 Students

Here, I outline the results of this study across all three problems (see Appendix for the mathematical tasks). All functions of metacognition—awareness, evaluation, regulation—were observed (see Table 4) with about 2/3 of all reported actions being of a metacognitive nature. The students engaged most in the metacognitive behavior of evaluation (27.4%). Two action cards were used by students independent of mathematical achievement in 82% of cases: "I checked my solution" and "I thought about whether what I was doing was working." The second most frequently reported metacognitive behavior was awareness (22.6%). The action card "I had an idea what I could do" was used in 50% of cases. Two other action cards "I thought about math I know" (28.6%), and "I thought about a similar problem" (21.4%) were used mostly by the above-average, and average and below-average students, retrospectively. This shows that even Grade 2 students can develop awareness behaviors, but these behaviors differ in nature. Regulation was the least reported metacognitive function (14.5%). In 66.7% of cases the students used the action card "I thought about my next step", whereas the action card "I made a plan in my head" was only used in 27.8% of cases. Developing a plan sequentially rather than all at once seemed to be more accessible to below-average and average Grade 2 students. The action card "I thought about another way" was used one time only. It may be that this function of metacognition is cognitively too demanding for young children or that they are not used to regulating their actions.

Table 4
Absolute and Relative Frequencies of the Reported Cognitive and Metacognitive Actions Across All Three Problems

	Metacognition			Cognition	Total
	Awareness	Evaluation	Regulation		
Grade 2 students	28 (22.6%)	34 (27.4%)	18 (14.5%)	44 (35.5%)	124

Similar to results of Wilson and Clarke (2004), ARE and AER metacognitive sequences were evident in the empirical data and were the most frequently reported sequences (66.7%). These two sequences were almost always embedded in longer sequences, and contained repeated similar actions, such as A, A, R, C, R, C, E, E, C and A, C, R, C, E, E. The ARE sequence was more evident in the data (66.7%) than the AER sequence. Besides the AER and ARE sequences, longer models were also observed, such as ARAE, AERE, and ARERE, especially in the case of The Snail in the Well Task and The Sports Equipment Task. Additionally, in 1/3 of cases the reported sequences did not include all functions of metacognition, but only one or two, such as A, AE, AR, RE, ER, which was most often reported by below-average and average students.

Discussion

Primary Grade Students' Model of Metacognition

The results of the study showed that the second graders' problem-solving process was a non-linear, dynamic interplay between cognitive and metacognitive actions as was also reported by other researchers (e.g., Adibnia & Putt, 1998; Hasselhorn, 2000; Kuzle, 2017, 2018; Wilson & Clarke, 2004). Hypothetically, the tasks could have been solved without metacognition as there were enough cognitive action cards to create a plausible start-to-finish solution sequence. For instance, in the case of The Snail in the Well Task, a student could have chosen the following action cards:

- Action card 1: I made a table. [cognition]
- Action card 2: I calculated always +3, and then –1. [cognition]
- Action card 3: I calculated until I reached 9 m. [cognition]
- Action card 4: I added the days. [cognition]

Nevertheless, in this study, each student reported engaging in at least one metacognitive behavior.

Metacognition acted upon about 1/3 of cognitive actions and the products of cognition (see Table 3, and Figures 3 and 5). Evaluation was the most frequently reported metacognitive function, which is consistent with the studies of Kuzle (2018) and Wilson and Clarke (2002) with Grade 4 and Grade 6 students, respectively. Here, most often the students reported using the action cards "I checked my solution" and "I thought about whether what I was doing was working." This result is not surprising as a lot of emphasis is placed on evaluative behaviors (i.e., evaluating progress, reflecting on the solution, checking the result and its plausibility) in the German mathematics curriculum for primary education (KMK, 2005). These different evaluatory behaviors are explicitly stated as a competence expectation at the end of primary school, which in that manner, acknowledges the promotion of metacognition as an integral topic of classroom conversations. Desoete et al. (2001) similarly reported that Grade 3 students often evaluated their problem-solving process. On the other hand, the least commonly reported metacognitive function was regulation; however, the reported behaviors pertaining to regulation using action cards differed among levels of students. While the below-average and average students chose the action card "I thought about my next step," only above-average students reported using the action card "I made a plan in my head." Thus, developing a plan sequentially rather than all at once seemed to be more accessible to average and below-average Grade 2 students. This result is supported by Desoete et al. (2001), who reported that the ability to plan distinguishes above-average students from average and below-average students. Since not all participants engaged in regulatory activities, it may be that regulatory activities are cognitively too demanding for Grade 2 students as was suggested by researchers (Desoete et al., 2001; Kuzle, 2018) or they may not be used to regulating their action since this behavior is not explicitly anchored in the German mathematics curriculum for primary education (KMK, 2005).

The reconstructed metacognitive sequences revealed several consistent patterns. According to Wilson and Clarke (2002, 2004) A, R, E and A, E, R sequences of metacognitive functions with cognitive activities between the individual components when problem solving are plausible when solving a mathematical task. Given that a problem solver requires a variety of metacognitive processes for completing any problem-solving task (e.g., Kuzle, 2017; Schoenfeld, 1985b, 1992; Schraw, 1998), at best, all components of metacognition (awareness, regulation, evaluation) are present and intertwined with cognitive activities. Concretely, if transferred into the language of action cards a metacognitive structure sequence for The Snail in the Well Task may look as follows:

- Action card 1: I had an idea what I could do. [awareness]
- Action card 2: I drew a figure. [cognition]
- Action card 3: I made a plan in my head. [regulation]
- Action card 4: I calculated how many meters a snail makes in a day. [cognition]
- Action card 5: I thought about whether what I was doing was working. [evaluation]

or

- Action card 1: I thought about a similar problem. [awareness]
- Action card 2: I drew a figure. [cognition]
- Action card 3: I calculated how many meters a snail makes in a day. [cognition]
- Action card 4: I thought about whether what I was doing was working. [evaluation]
- Action card 5: I thought about my next step. [regulation]

This model of behavior was only partially reported by Grade 2 students.

As illustrated in Table 3, the sequence A, R, E was observed in students' reconstructed sequences of The Snail in the Well Task, whereas the A, E, R sequence was not. Given that "regulation occurs on the basis of retrieval procedures" (Wilson & Clarke, p. 38), the latter may be due to students' unfamiliarity with this type of task. In that sense, "regulation is a consequence of the need for a decision on the part of the problem solver as to how best to proceed" (Wilson & Clarke, p. 38). The A, E, R sequence was, however, reported in the two other problems. Their longer models were also observed, namely ARAE, AERE and ARERE, especially in the case of the average and above-average students with respect to all three problems. These sequences are consistent with the general notion of ARE and ARE sequences as was reported by Wilson and Clarke (2002, 2004) in their study with Grade 6 students. Although Wilson and Clarke found these sequences to be most often associated with unsuccessful problem solving, this study shows the opposite. Those students who reported on longer sequences were successful in their problem solving which was mostly observed in average and above-average students. However, given the heterogeneity of participants it seems more plausible that the students who lacked mathematical skills or problem-solving experience, or experienced difficulties during problem solving due to an unfamiliar task, need to employ different metacognitive behaviors in order to attain the solution to a given problem for a prolonged period of time. Furthermore, average and especially below-average Grade 2 students more often reported shorter metacognitive structure sequences (e.g., A, AE, AR, RE, ER). This result aligns with Desoete et al. (2001) who reported that average and below-average students have less developed metacognition, which could explain the shorter sequences. Shorter sequences were reported on all three tasks but most often in The Snail in the Well Task. Since the students had experience with numeric and combinatorial tasks similar to The Sports Equipment Task and The Clothes Closet Task, it may be that the familiarity of the tasks influenced the reconstructed sequences. With the development of intellectual ability and with increasing knowledge acquisition metacognition becomes more sophisticated, robust and complete (e.g., Brown, 1978; Desoete et al., 2001; Hasselhorn, 2000; Veenman et al., 2006). Therefore, a larger repertoire of metacognitive actions of the average and above-average students may have contributed to longer metacognitive sequences and vice versa.

To summarize, the model of metacognition was theoretically coherent and largely consistent with the data generated in this context, with all functions of metacognition being reported. Since these metacognitive sequences and behaviors have emerged in prior studies (e.g., Desoete et al., 2001; Kuzle, 2018; Wilson & Clarke, 2002, 2004) in older children, namely in studies with Grade 3, 4 and 6 students, this research confirmed the hypothesis that different metacognitive behaviors are likely to develop during early school years (Veenman et al., 2006), and across different student performance levels.

Limitations and Future Directions

Limitations of the Study

The data reported by Grade 2 students showed limitations to the adaptation of the MMI-approach. Unlike in Wilson and Clarke's (2002, 2004) study, video data were not used for stimulated-recall in order not to overburden Grade 2 students. By using videos, the problem solving would have lasted longer, which is problematic when working with young children because they have issues concentrating for a long period of time. As a consequence, this could have influenced the validity of the participants' claims about the action cards. Instead, the data was triangulated by combining the retrospective interview with the researcher field notes. During the retrospective interviews, some students' reported processes that were not available on existing action cards so that additional action cards were produced. In that manner, the students' reported metacognitive sequences were expanded and reflected their complete problem-solving process. Thus, the second graders' reconstructions of problem solving needed to be discussed; otherwise, some of the processes would not have been reported. Nevertheless, some particularities in the data occurred. Specifically, cognitive actions were not reported after each metacognitive action and were reported less frequently than metacognitive actions overall, which seems rather unreasonable. Thus, it has to be acknowledged that omissions and incompleteness of data occurred, which is often the case when assessing metacognition. As such, "the protocols provide only an incomplete record of the process" (Ericson & Simon, 1980, p. 235).

A further methodological limitation of the study is the action cards. First, the below-average and average Grade 2 students were not able to work with so many cards; having 12 cards at their disposal and then reconstructing their problem-solving process was a challenge for them. Second, some students had difficulties reading the statements on the action cards. In other words, it may be that some actions did not get reported or were not understood by the students. The level of both linguistic and cognitive development could explain the challenges that the below-average and average Grade 2 students had when employing the MMI-approach (e.g., Schneider & Artelt, 2010). A possible development of the instrument could include reverting to a non-verbal diagnosis of cognitive and metacognitive strategies, such as images (Garner, 1988). Specifically, statements on the action cards could be supported with an appropriate visual. For instance, the card "I had an idea what I could do" could be supported with a visual of a light bulb.

Lastly, the goal of this study was not to develop a prototype of a problem solver nor was the goal to prescribe particular practices for all problem solvers. I used a small sample of participants, so not every metacognitive sequence was exhibited nor would the reported sequences be representative of a large population. Furthermore, I reported here on the results with respect to three different mathematics problems. The results may depend on the characteristics of these particular tasks.

Implications and Future Directions

Despite the limitations of this study, the results have practical implications for elementary mathematics teachers. The study participants had experience with problem solving and reflection on the problem-solving process. Moreover, some metacognitive behaviors, such as evaluation, are anchored in the German mathematics curriculum for primary education (KMK, 2005) and are reported by all Grade 2 students. It may be that the mathematics curriculum contributed to the development of different metacognitive functions through teacher's teaching practices. However, more research is needed to answer the question about what is being promoted and what factors (e.g., nature of the task) influence this development. Further studies may focus on answering the question of what teaching practices, if any, would help young children to develop different metacognitive functions.

Moreover, future studies could evaluate the possibilities for classroom implementation of the MMI-approach and the practicability of it as a classroom tool for discussing metacognition in the context of mathematics problem solving. In this manner, students' metacognitive behaviors would be more visible to teachers, allowing both students and teachers to gain insights into metacognition in the classroom. Ultimately, the teachers would be able to modify their teaching practices with respect to promoting metacognition within the mathematics curriculum.

The study also showed that whether students successfully solved a problem was not characterized by any particular metacognitive function sequence. Furthermore, cognitive actions did not reveal any stable patterns, but rather varied among different levels of students and across problems. Future research may need to examine what metacognitive functions are likely to lead to success. Moreover, examining the importance of cognitive actions and what specific other factors within the sequences lead to success by taking into account also the performance level of students, may provide explanatory power for student metacognition.

Additionally, a study could be conducted with a larger data sample over a longer period of time using a variety of different tasks in terms of context, content and approach to create a more thorough description of young children's metacognition in a problem-solving context. This would allow answering the question of whether metacognitive styles (i.e., prototypes) for students' problem-solving ability exist. If such metacognitive styles do exist, these could then inform effective teaching of elementary teachers and learning practices of young students.

Lastly, the study showed that reconstruction of the processes using just action cards does not suffice to report on young children's metacognition. Even though field notes allowed the researcher to negotiate the problem-solving process with the student, additional or alternative instruments may be worthwhile to increase the accuracy and efficiency of results, such as the development of a validating protocol for the interviewer or using a video-stimulated recall technique. Bearing in mind the age of the students, it would seem appropriate to focus on only a few selected video sequences with respect to the latter. Further development and examination of the MMI-method in both clinical and teaching settings may help the field of mathematics education understand better the development of metacognition in primary school, as well as the possibilities for its promotion within the mathematics curriculum.

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Appendix
Mathematical Tasks

Sports Equipment
David would like to buy some new sports equipment. He has €30 in total. In the sports store he decided to buy two new items:

- Tennis racquet, €16
- Soccer gloves, €9
- Soccer ball, €14
- Gym shoes, €15

After paying for the two items, he had €6 left. What two items did he buy?

Clothes Closet
Lisa has 3 t-shirts that she likes to wear. They are red, green, and yellow.

She also has 3 pairs of jeans: brown, black, and blue. And 2 pairs of shoes: sneakers and boots.

She likes to dress up and look different each day. What are the options for her daily look?

The Snail in the Well
A snail in a 9 m deep well wants to go up and reach the top. The snail crawls 3 m up during each day and slides 1 m down in the night.

On what day does the snail reach the top of the well?