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Mathematics Teacher Developers' Views of a Demonstration Class

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This article examines the professional vision of mathematics teacher developers during a professional development experience that featured observations of a content course for elementary teachers. The researchers examined whether these mathematics teacher developers viewed the demonstration class as an analysis class serving as a site for reflection and analysis, or a model class serving as an example of teaching to be emulated. Results indicated participants could hold either view and, in some cases, both. Each view provided opportunities for professional reflection, but particular aspects of the experience promoted an analysis class view.

Reports have cited an urgent need for improving both the quality and size of the mathematics teacher workforce in the United States (e.g., National Science Board, 2007). Widespread adoption of the Common Core State Standards (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) and its emphasis on engaging students in mathematical practices has heightened this need. Undertaking a task of this complexity and scale requires a cadre of professionals who prepare and provide ongoing professional education of mathematics teachers (Sztajn, Ball, & McMahon, 2006). Several definitions of *teacher educator* have emerged in the literature (John, 2002).

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Mathematics teacher developers (MTDs) are a diverse group of professionals working in a variety of settings as teachers of prospective and practicing mathematics teachers. We call these professionals MTDs rather than mathematics teacher educators and include those who may not identify themselves as teacher educators. Although some MTDs do not have degrees in education or consider themselves teacher educators, they teach courses designed for teachers (e.g., mathematicians teaching content courses specifically designed for prospective teachers). MTDs include community college and university faculty members from mathematics and education departments, privately practicing professional developers, and school district leaders offering professional development.

Despite some research on MTDs' practices and professional learning (e.g., Doerr & Thompson, 2004; Superfine & Li, 2014; van Zoest, Moore, & Stockero, 2006), literature about MTDs' professional learning is limited (Sztajn et al., 2006). Though some work on the mathematical knowledge needed for teaching teachers (Castro Superfine & Li, 2014) has been undertaken, we do not fully understand what comprises the knowledge base of teacher educators, or how they can systematically learn to prepare teachers (Knight et al., 2014). Lee and Mewborn (2009) have called for research efforts to focus on the design and development of MTEs' practices which would support the development of scholarly practices for MTEs that are informed by "empirical studies of the teaching and learning of mathematics and the preparation of mathematics teachers" (p. 3). This study makes an initial foray into understanding the ways that a group of MTDs with diverse professional backgrounds experience and understand their own professional development.

Given the work of MTDs is to support teachers' professional learning, it is surprising the professional learning of MTDs is often neglected (McGee & Lawrence, 2009). The body of research on effective professional development for MTDs is less developed than that for teachers but there is some attention focused on teacher educators (Knight et al., 2014). MTDs may participate in a variety of professional development experiences, including conferences, workshops, and collaboration with colleagues (Van der Klink, Kools, Avissar, White, & Sakata, 2017). Professional development for MTDs should be purposefully designed and implemented, yet, other than self-studies, there is not much literature about professional development of MTDs. As such, MTDs need a professional development strategy that provides agency and expanded scholarship (Loughran, 2014). Like teachers, MTDs need to "undergo shifts in their knowledge, beliefs, and habits of practice that are more akin to a transformation than to tinkering around the edges of their practice" (Stein, Smith, & Silver, 1999, p. 262). These shifts require MTDs to reconsider their teaching practices, adopting an *inquiry stance* (Cochran-Smith, 2003), "a way of learning from and about the practice of teacher education by engaging in systematic inquiry on that practice within a community of colleagues over time" (p. 8).

There are similarities between the work of mathematics teachers and the work of MTDs. Both teachers and MTDs teach mathematics and they face challenges in how to support their students' understanding of mathematics (Castro Superfine & Li, 2014). There are also distinctions that set MTDs' and teachers' work apart. One such distinction is related to the specific content addressed and the prior knowledge and experiences of their students (Nipper & Sztajn, 2008). K-12 students need to learn and understand mathematics, while teachers need to learn mathematics in ways that helps them teach their students. Bass (2005) defined mathematical knowledge for teaching (MKT) as "the mathematical knowledge, skills, habits of mind, and sensibilities that are entailed by the actual work of teaching" (p. 429). Castro Superfine and Li (2014) noted that the knowledge MTDs need to develop is their own MKT and the ways to connect teachers' mathematical learning to the practice of teaching K-12 students. Some of the content teachers learn in professional development is how to teach mathematics whereas MTDs learn how to teach teachers to develop pedagogical content knowledge (Muir, Fielding-Wells, & Chick, 2017). These differences imply the work of K-12 teachers and MTDs are different.

In this report, we examine the professional vision of MTDs as they participated in a professional development experience that featured the observation of a mathematics content course for preservice elementary teachers. To focus our inquiry, we asked the following question: How do MTDs view and interpret a mathematics class for prospective elementary teachers? To focus and support this inquiry, we examined MTDs' observations of key features of the mathematics class and whether these features supported or constrained their views.

Background of the Professional Development

During the summer of 2004, the Center for Proficiency in Teaching Mathematics (CPTM) hosted an 8-day residential institute, entitled "Developing Teachers' Mathematical Knowledge for Teaching," to provide professional development for MTDs. The institute was called "the professional development of professional developers" (Tyminski, Ledford, & Hembree, 2010, p. 2). The institute organizers selected 65 MTDs from 140 applicants, guided by the goal of assembling a group representing diverse backgrounds in terms of current work and prior experiences (Sztajn et al., 2006). The design of the institute focused the MTD attendees on teachers' learning of MKT and how to incorporate it into their work with K–12 teachers. Institute attendees participated in a variety of activities, and the central feature included observing a mathematics content course for preservice elementary teachers, hereafter referred to as the *lab class*.

Purpose of the Lab Class

The lab class was a mathematics content course entitled Mathematical Content and Applications for the Teaching of Elementary School Mathematics. Sixteen students attended daily two-hour sessions taught by Dr. Deborah Ball, an experienced and well-known MTD with 15 years of elementary teaching experience and 20 years of experience teaching and researching mathematics teachers. The lab class content focused on meaning and representations of fractions and drew on NCTM (2000) process standards with a particular emphasis on the mathematical practices of explaining, representing, recording, and communicating. The lab class served as “a shared specimen for observation and manipulation” (Sztajn et al., 2006, p. 156).

The lab class embodied Loucks-Horsley, Love, Stiles, Mundry, and Hewson’s (2003) *demonstration lesson,* which included the cycle of prediscussion, observation, and postdiscussion, all with a clear purpose. Loucks-Horsley et al. further described two purposes of demonstration lessons. The first purpose, the *model class,* involves a master teacher presenting “an exemplary model of teaching that other teachers observe and then discuss” (p. 212) and provides observers with insights and ideas to adapt to a personalized classroom context. The second purpose, the *analysis class,* offers observers a site for analysis, critique, and reflection on mathematics teaching and learning. Although the lab class embodied features of a model class, such as having a distinguished MTD as the instructor of the class, the organizers of the institute intentionally designed the lab class activities to help the MTDs view it as an analysis class. Below we discuss the prelesson-observation-postlesson structure and provide details of the intentional design intended to situate the lab class as an analysis class.

Prelesson discussion. Prior to each lab class session, the MTDs reviewed and discussed its instructional goals and planned activities. Although the instructor had already planned the lesson, she considered the MTDs’ suggestions. The lesson plans did not significantly change due to the MTDs’ contributions but included additions, such as including a manipulative in a lesson or asking specific questions. To prepare for the observation, the MTDs solved and discussed the mathematics problems to be used and hypothesized potential student strategies and difficulties in solving these problems. Beginning with the second observation, the institute leaders created three MTD groups and assigned each group an observation focus based on the instructional triangle in *Adding it Up,* (National Research Council, 2001): students (preservice teachers), teacher (lab class instructor), or mathematics. The groups rotated, enabling each MTD to spend one lab class session on each of the three foci.

Lesson observation. During each lab class session, the MTDs’ roles were that of observers. They had no teaching responsibilities and were instructed not to interact with the students. For unobstructed observation, the MTDs sat in elevated rows on two sides of the lab class, which was arranged with tables in a U-shape opening towards a large projector screen. Microphones placed between every two or three preservice teachers ensured that the observers could clearly hear whole-class discussions.

Postlesson discussion. After each lab class session, the MTDs reconvened to discuss their observations, usually first in small groups and then as a whole group. The small group discussions were organized by their assigned focus. The institute leaders intended, in part, for MTDs to revisit their hypotheses made about student solutions and struggles during the prelesson discussion using the data they collected during the observation. These discussions spanned a wide variety of topics, including the definition of a fraction, the justification for multiplying by the reciprocal to divide fractions, and specific student solutions. To support an analysis view, during the fourth postlesson discussion experienced MTDs shared their observations and reactions to model this process.

Related literature. Prior literature about the Summer Institute described how MKT was incorporated in a professional development experience for MTDs (Sztajn et al., 2006). Specifically, the authors outlined the purposeful design of the professional development experience and provided an example that highlighted features of the design. Tyminski, Ledford, and Hembree (2010) detailed a study that provided insights into the way a subset of MTDs, identified as mathematics content specialists, viewed mathematical knowledge and the work of preservice teachers. The results stated the content specialists focused on mathematical correctness and the clarity of student explanations when they discussed the mathematical knowledge of students. Also, the content specialists focused on mathematical language, representations, and explanations when they discussed the mathematical work of students. Our study expands this work by focusing on the ways MTDs viewed and interpreted the mathematics class and how key features of the observation experience supported or limited those views.

Framework: Professional Vision

Professional vision is "socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (Goodwin, 1994, p. 606). Professional vision is role-specific. In the context of teaching, *professional vision* refers to the ways teachers notice and interpret significant features of classrooms (Sherin, 2001) and can be a useful lens for examining teacher learning (Sherin & van Es, 2009). Sherin (2001) noted the differences in the visions of an educational researcher and a classroom teacher. The teacher was focused on what the teacher should do in the classroom while the researcher was focused on interpretation rather than action. Researchers have studied the effect of professional vision on teachers' interpretations of video lessons and student artifacts (Jacobs, Lamb, & Philipp, 2010; Sherin & van Es, 2005, 2009; Star & Strickland, 2008; van Es & Sherin, 2002). Teachers use their knowledge of content, how students think about content, and their classroom context to reason about classroom events (van Es & Sherin, 2002). There are three interrelated noticing skills: (a) attending to children's strategies, (b) interpreting children's understandings, and (c) deciding how to respond based on children's understandings (Jacobs et al., 2010). It is important to note that these processes may not happen chronologically, but instead are dynamic and complex (Castro Superfine, Fisher, Bragelman, & Amador, 2017). In making sense of what they notice, teachers might take on an *interpretative stance* or an *evaluative stance*. Teachers take an interpretive stance when they seek to understand the mathematical thinking underlining students' actions, why an event occurred, or how the event affected student learning (van Es & Sherin, 2002) and provide robust evidence to support their claims (Jacobs et al., 2010; Jansen & Spitzer, 2009; van Es & Sherin, 2002, 2008). For example, a teacher might provide an interpretation supported by robust evidence:

> The first pair [of students] understands the problem is a [subtraction problem] by writing a number sentence that showed $19 - 7 = \square$. They did not need to count out 19 and

> take away 7 to get 12. They simply used their fingers to count backwards from 19. They seem to have good number sense. (Jacobs et al., 2010, p. 185)

In this quote, the teacher went beyond describing the students' work to explain what the students understood based on the approach the students used to find the subtraction. Although some researchers classify teacher descriptions as a type of interpretation (e.g., Wallach & Even, 2005), Jacobs et al. (2010) consider description and interpretation to be related, but different. They make this choice to highlight how student strategies can be complex and attending to and describing the details of the strategies is an important skill. Teachers describe student actions or talk when they provide direct quotations or portray student ideas. Teachers with expertise in noticing will be able to provide more details when describing student thinking. The descriptions do not contain an analysis of student thinking or provide evidence for how the student is thinking. In this paper, a *descriptive stance* is when a teacher describes events by stating what happened during the class or providing nearly direct quotes of the teacher or students. In the descriptive stance, the teacher does not provide additional information such as how the students understood the mathematical ideas, connections between teacher actions and student learning, or whether the teacher made the appropriate choices.

Teachers take an evaluative stance when they express a judgement about their observations. Evaluative statements include whether an event was good or bad, what went well or poorly, or how they might modify the activity observed (van Es & Sherin, 2002). Providing robust evidence to support, explain, or justify that judgment is not required to be classified as an evaluative stance. For example, when a teacher observed a video of herself teaching, she stated, "I appeared rushed, and too loud. I did not give the students enough 'wait time' to think and respond. I should have been more 'silent' and listening" (van Es & Sherin, 2002, pp. 582–583). Unlike the interpretive stance example, this statement lacks evidence and does not demonstrate an attempt to justify her actions or how these actions might have affected student learning. The statement also does not describe what happened during the episode. We rely on this work to examine the stance (descriptive, evaluative, or interpretative) that our MTD participants took as they observed a content class for preservice elementary teachers that was embedded in a professional development.

Research Design

Of the 65 MTDs who attended the institute, the institute organizers selected 16 that represented of the group's diversity, including years of experience and type of job held, for further study. The 16 MTDs included four university mathematicians, three school district personnel, two community college instructors, and six university mathematics education faculty. In this report, *participants* refer to these 16 MTDs.

We used a variety of data sources to better understand the professional vision held by individual participants during the institute. Primary data included pre-institute surveys, institute-provided participant notebooks, and field notes of participants' small group discussions. Secondary data included field notes of whole group discussions, video recordings of the lab class, and data from a CPTM designed two-part follow-up to the institute. In the first part, two years after the conclusion of the institute, CPTM researchers invited Summer Institute attendees to complete an online survey to gain information about their current practice, impressions of the institute, and input for an upcoming institute reunion. Of the original 65 MTDs, 46 (11 of whom participated in this study) submitted responses to one or more of the online survey questions. Second, at the Summer Institute reunion, CPTM researchers conducted four 90-minute focus group interviews with 32 of the institute MTD attendees (3 of whom were study participants who completed the online survey and 1 of whom was a study participant who did not complete the online survey). The focus group interviews addressed the institute experience and paid specific attention to the lab class, the attendees' current practices, and the attendees' MKT.

There are limitations to what we can conclude from this data. The participants used their notebooks in a variety of ways, and some did not speak much during the institute sessions. Although the notebooks provided insights into aspects of the lab class and institute sessions the participants noted and analyzed, we cannot conclude the notebooks included *all* aspects of what participants noticed or analyzed. We do, however, claim that the notebooks provide us a sense of what was important to participants in that moment.

Table 1
Examples of Codes

Data	Observation Stance	Part of Instructional Triangle
"Discussing whether the answer 3/5 is 3/5 of 1 dozen or 3/5 of 3 dozen"	Describe	Mathematics
"[instructor] asked for others to share to get more of them to talk… [instructor] thinks that ss [students] get into a pattern of expectation on 1st day – if they speak on 1st day, they're more likely to continue sharing in subsequent lessons"	Interpret	Teacher – pedagogical decisions
"She has no concept of area… Think she was working from answer. Student is not bothered by unequal parts."	Evaluate	Student – mathematical claims or evidence

Four researchers analyzed the data, including one who collected data at the institute and one who participated in the design and implementation of the follow-up study. For the first level of analysis, we coded the primary data based on the observation stance of the participant's written or oral comments (see Table 1). The stance codes used were describing, interpreting (Wallach & Even, 2005), and evaluating (van Es & Sherin, 2002). These codes helped us identify when participants were taking an interpretative stance versus an evaluative stance, and consequently helped us distinguish between when an MTD viewed the lab class as a model class or an analysis class.

The participants were asked to focus on specific parts of the instructional triangle during their observations, so we also coded the data accordingly: *content* (the MKT being taught in the lab class), *teacher* (the lab class instructor), and *students* (the preservice teachers enrolled in the lab class). We found these three areas needed more specific descriptions. Under the code *students*, for example, it was useful to distinguish between participant comments on student solution strategies versus student affect. A pair of researchers coded each remaining notebook. After individual coding of each notebook, the pair compared codes, discussed and reconciled inconsistencies, and produced one final coded notebook file for each participant. Small group field notes and pre-institute surveys were then coded using the same scheme. The secondary data were not coded but used to confirm or disconfirm findings from the primary sources. The second phase of analysis entailed determining recurring themes in the data for each participant and across participants. We used content codes to examine patterns in what participants noticed and the stance codes to examine patterns in how participants documented what they noticed. Using the data we established themes. After establishing themes, we re-examined each participant's primary and secondary data to find confirming and disconfirming evidence of these themes. Finally, we composed an overall report for each participant.

Context for Results: Cookie Jar Problem

To illustrate our findings, we use a discussion of the participants' responses to the Cookie Jar Problem (CJP; see Figure 1), a mathematical task for which discussion spanned multiple lab class sessions. The mathematics that underlies the CJP aligns with the CCSSM, and the way the problem was implemented in the course encouraged students to engage in the Standards for Mathematical Practice (NGA & CCSSO, 2010). We chose to illustrate our findings with the CJP because it challenged the lab class students' content knowledge, providing the participants with a rich experience to observe and interpret. The MTDs solved the CJP in multiple ways and hypothesized student solutions and misconceptions prior to its introduction in the lab class session. We first present four student solutions and surrounding discussion that occurred during the two-day development of the problem.

> There was a jar of cookies on the table. Kira was hungry because she hadn’t had breakfast, so she ate half the cookies. Then Steve came along and noticed the cookies. He thought they looked good, so he ate a third of what was left in the jar. Niki came by and decided to take a fourth of the remaining cookies with her to her next class. Then Kayla came dashing up and took a cookie to munch on. When Pam looked at the cookie jar, she saw that there were two cookies left. “How many cookies were in the jar to begin with?” she asked Kira.

Figure 1. The Cookie Jar Problem.

Stan sketched and explained his group’s solution using discrete objects while his group member Kate wrote corresponding number sentences (Figure 2). Working backward, Stan drew squares to represent cookies left and then those taken by each person, finishing his drawing with six squares to represent the one-half of the cookies first taken. He claimed that his final 12 represented the original number of cookies. This group also attempted an algebraic solution, but Stan was concerned because: “We proved it visually, we proved it by doing the math this way [the diagram], this way [the number sentences], and what we are hung up on is trying to prove it algebraically” (lab class video).

Three fourths of the student groups attempted an algebraic approach. The ensuing 30-minute discussion of this approach involved eight students actively sharing and critiquing ideas. Leading off this discussion, Melissa presented her group’s attempt to model the CJP with an equation (Figure 3a), but her group could not identify their error. Melissa described how each term of the original equation represented the number of cookies left after each person took a share. Nicole offered her solution (Figure 3b) as a comparison, “Instead of one third for what Niki took, I believe that it should be two thirds instead of one third because there’s two thirds of the cookies remaining. Because the one half is the half *remaining* times the two thirds *remaining*.”

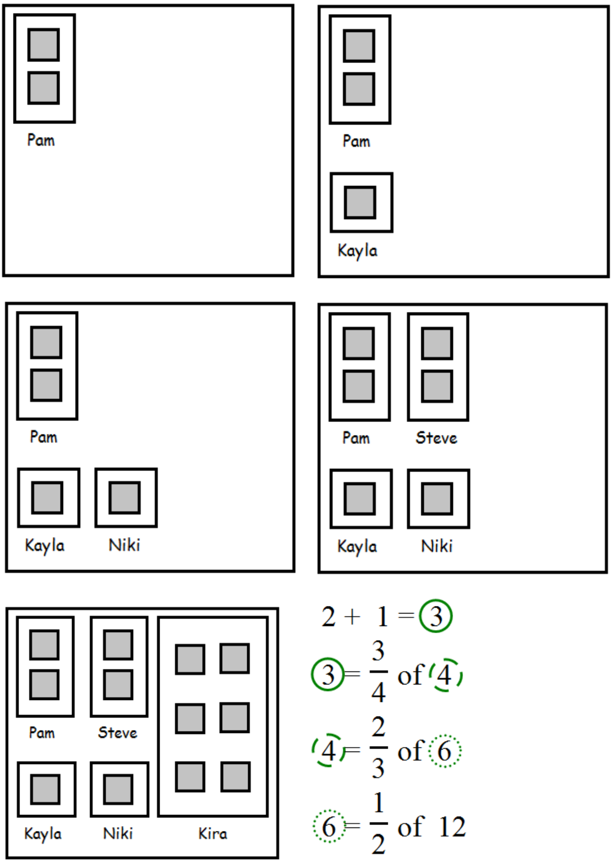

Figure 2. Stan's group's working backwards solution.

(a)	$x = \frac{1}{2}x + \left(\frac{1}{2}x\right)\frac{1}{3} + \left(\left(\frac{1}{2}x\right)\frac{1}{3}\right)\frac{1}{4} + 1 + 2$
(b)	$x = \frac{1}{2}x + \left(\frac{1}{2}x\right)\frac{2}{3} + \left(\left(\frac{1}{2}x\right)\frac{2}{3}\right)\frac{3}{4} + 1 + 2$

Figure 3. (a) Melissa's algebraic solution and (b) Nicole's algebraic solution.

On the second day of work on the CJP, two geometric solutions were presented. Sharon represented the cookie jar in its original state with a rectangle (Figure 4a), which she partitioned into halves, one of the halves into thirds, and two of those thirds each in half. She concluded one of the four squares was the fourth eaten by Niki and that the three small squares remaining represented the one cookie eaten by Kayla and the two cookies left in the jar. She claimed that each small square represented a single cookie to determine 12 cookies were originally in the jar.

(a)

(b)

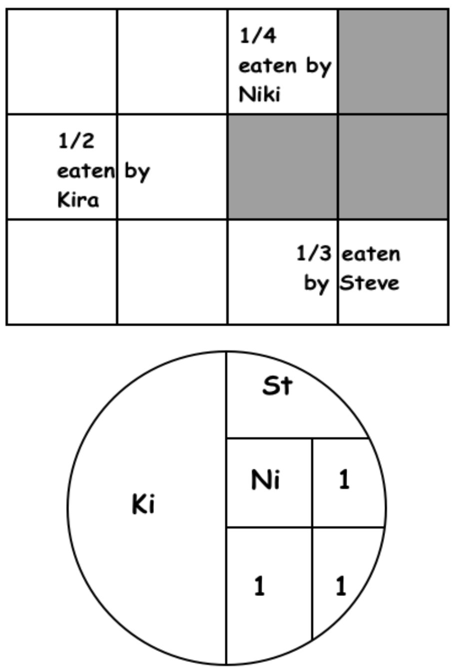

Figure 4. (a) Sharon's geometric solution and (b) Tara's geometric solution.

Tara presented the fourth solution (Figure 4b) using a circle to represent the original cookies in the jar. She then labeled one half of it to represent the half taken by Kira. Two horizontal segments in the right half "divide[d] this into three parts" with the top portion showing the cookies eaten by Steve. Tara then partitioned the remaining unlabeled portion of the circle into four parts with a vertical segment. She experienced some confusion about labeling these four new parts, and initially placed the numeral 1 in the lower right portion before erasing it to rethink her solution. Tara determined there were 12 cookies originally.

Results: Participants' Views of the Lab Class

Participants reacted to these four student solutions differently. For some, the algebraic solution had an elevated status, but for others, algebra was too sophisticated and masked misconceptions about the inherent mathematics. Participants who favored the geometric representations thought the solutions showed a conceptual understanding of unit fractions. Others became absorbed with the flaws in Tara's geometric representation. To illustrate our findings, we discuss how a subset of our participants—Fiona, Jon, Rachel, and Cathy (pseudonyms)—made sense of the lab class's work. We selected these four participants because they reflect the possible ways participants could react to the demonstration class, and specifically whether they viewed it as a model class, an analysis class, or a combination of the two (see Table 2).

Table 2
Background Information and Results for Participants

	Type of Institution	Highest Degree	Department	Results Pertaining to CJP
Fiona	University	PhD	Education	Viewed the demonstration class as a model class
Jon	University	PhD	Mathematics & Statistics	Viewed the demonstration class as an analysis class
Rachel	University	PhD	Mathematics	Initially viewed the demonstration class as a model class and then shifted towards an analysis class
Cathy	School System	ABD	Mathematics Education	Viewed the demonstration class as a model class and an analysis class, depending on the focus of her attention

Fiona: A Model Class

Fiona approached her observations of the lab class as a model class that should be emulated by MTDs but did not wholeheartedly agree with all the pedagogical moves she observed. Fiona was an experienced MTD who worked in a university education department with both preservice and inservice teachers. Fiona established her expectations for the lab class to serve as model class before her first observation, writing in her notebook: "This is a wonderful opportunity to watch master teachers in action with a diverse group of students. I want to hear the instruction and then listen to what the students heard. I want to follow their trains of thought." Early in her notebook, she compared the lab class to a "real" third-grade classroom and wondered how teachers could be trained to implement the instructor's techniques. After the first observation, she wrote that she intended to "share this [the instructor's work on fractions] with my teachers."

In her observations of the lab class, she focused on the teacher, as was assigned to her, and took a descriptive stance. Fiona also attended to the mathematics. In the prelesson discussion on day 1 of the CJP, she worked out four different solutions, indicating some investment in the mathematics of the problem. She composed a numbered list of the instructor's moves, yet her descriptions of the instructor's actions did not attend closely to the detail of those moves. For example, she wrote that the instructor circulated to the groups of students "asking probing questions" and that she "had students put a variety of methods on the board." Her notes were more detailed on the second day of the CJP, when institute leaders instructed her to attend to the mathematics. She took careful notes on the geometric representations of the problem (Figures 4 and 5) and considered what thinking might have led to the students' constructions and representations. Fiona hypothesized that Tara was perhaps working backward to show the validity of the answer determined using another method. This was the first appearance of an interpretive stance in her notes, but her reaction shifted to an evaluative stance for the final solution (Figure 5) when she noted the poor quality of the work presented, twice stating that Tara had "no concept of area.". In attending to the nuances of Tara's solution, Fiona extended her thinking to meaningful representations of the unit, commenting that Tara was "not bothered by unequal parts. Circle is more valued as unit" (Fiona's notebook). She expressed concern about the implications of Tara's understanding, asking in her notebook, "What if she showed this to children?"

There was no evidence that indicated Fiona considered the instructor's rationale for not addressing the incorrect diagram or that Fiona considered the potential opportunities for mathematics learning. Field notes showed Fiona's dissatisfaction with how the instructor had handled fractions in the CJP continued even after the lab class had moved to other problems. She led a discussion among her fellow MTDs on the issue of defining fraction, questioning whether a definition should be directly presented to students a priori to working with a concept or if it should be allowed to develop as students work on a concept. Fiona was invested in the lab class mostly as a model class, and though she disagreed with the way the master teacher responded to a student solution, she continued to view the class as an exemplary model of teaching.

Jon: An Analysis Class

Jon approached the lab class as an analysis class, and his stance remained mostly interpretive throughout the institute. Jon taught mathematics courses for preservice teachers and other mathematics courses in a university. Jon wanted to "ramp up" the mathematics he taught preservice elementary teachers (pre-institute survey) because at his university the mathematics was not deemed worthy of college credit in two of three content courses and were rejected from university core courses. The mathematics was a main point of focus for him and later he considered the instructor's work in bringing out key mathematical ideas. Both his interest in mathematical rigor and the internal conflict that he developed about what contistuted a worthwhile mathematical task influenced how he viewed the lab class.

The CJP, and, in particular, the conceptual understanding related to the algebraic solution, were a point of interest for Jon. He questioned whether the CJP was rigorous mathematics and focused on this question throughout the institute. When discussing how the lesson progressed, Jon noted, "By [the class] going into the algebra so deeply, it hindered the mathematical thinking of the problem. It distracted them [the students] from a deeper understanding of fractions" (field notes). As Jon returned to this issue throughout the institute, he took an interpretive stance and considered a variety of perspectives as he wrestled with the mathematics of the CJP and how those ideas were reflected in other problems used in the lab class. He related his concern about mathematical rigor to pedagogical decisions about when and how to introduce the idea of a fraction of a unit: explicitly or through discovery. In field notes, institute researchers described Jon's effort to understand how tasks were chosen and implemented: "He really struggled ... to articulate and understand his own thinking about the difficulty of tasks and how to draw out the nuances of a problem." This struggle continued in the class sessions following the CJP. Jon questioned, "What are the characteristics of a problem that prompt such [mathematically rich] discussion?" (field notes). Jon became intensely invested in the interplay between the teaching and the mathematics, wondering whether some problems were inherently good or if any problem could be good because of the teacher's pedagogical decisions regarding the development of the mathematics. At the week's end, he offered an extension to the CPJ: "Turn the cookie problem around and take the fractional parts away in the reverse order from the original problem. The result is exactly the same as before. Why?" (field notes). At the institute's conclusion, he intended to return to his university and encourage his fellow mathematics colleagues to "think about how deep mathematics is disguised as mathematics for elementary teachers" (field notes).

Jon paid attention to the mathematics of the institute and his attention included pedagogical moves that supported exploration of the mathematics. He also exemplified MTDs who developed an internal conflict during the CJP about some aspect of the lab class and consequently worked to reconcile that conflict. In

Jon’s case, the CJP stimulated a conflict about what counted as worthwhile mathematics tasks (those that relied on sophisticated symbolic mathematical skills or tasks that could be solved in a variety of more accessible ways). He was deeply invested in how enactment could influence the rigor of a seemingly trivial task to address important mathematical ideas.

Rachel: A Shift

An experienced MTD, Rachel worked with elementary preservice teachers who chose to study mathematics beyond their required coursework or as part of a senior thesis. At the outset, Rachel intended to notice preservice teacher understanding and collaboration as well as the instructor’s questioning techniques. Many of her stated interests included the interaction between students or between students and the instructor. Rachel focused primarily on the instructor and students, specifically attending to pedagogical issues. In the follow-up survey, she said she had expected the institute would “verify methods she was already using.” Initially, Rachel did not take an interpretative stance or evaluative stance, instead adopting a solely descriptive stance. Over time she developed a conflict related to the mathematical ideas within the problems used in the lab class and her stance shifted to an interpretative one.

In relation to the CJP lessons, Rachel paid close attention to the teacher’s actions, attempting almost to transcribe everything the instructor said and did. She noted the specific instructions given to the students to set up their work on the CJP, as illustrated with Rachel’s notes from her notebook (Figure 6). Rachel also noted which groups of students interacted with the instructor during work time and the exact questions the instructor posed during the whole group setting: “[The instructor] asked [students] if they can show the relationship between the algebra and the picture. She asked if someone who hadn’t been involved in the earlier explanation could share their thinking” (Rachel’s notebook). In the final pages of her notebook, Rachel noted that the instructor made her practice explicit throughout the institute; not only had the instructor clearly stated course goals, but she provided the rationale for those goals, linking the activities of the institute to the real work of teaching. Two years later, Rachel returned to it in the focus group interview and shared how she believed she is a much better teacher because she makes her teaching more explicit to students. Rachel demonstrated she not only valued the lab class as a model class to be emulated but also carried pieces of the instructor's practice into her own teaching.

> [The instructor] asked ss [students] what the problem is asking.
>
> She asked ss if there were any other thoughts.
>
> She asked ss to solve the problem then go the extra step to explain their work & how they know they're right.
>
> [She] told tchrs [teachers] it's good practice to allow ss time to read the problem & think about what it entails before they begin to solve the problem. (Rachel's notebook)

Figure 6. Rachel's notes about what the instructor asked the students.

As the institute progressed, Rachel demonstrated a view of the lab class as an analysis class. While engaged in another problem, Rachel reflected back on the CJP, noting, "This problem seems very simple compared to the cookie jar problem. I wonder if the purpose of this problem is to get students to think about fractions as a representation for division?" (Rachel's notebook). Rachel's critique went beyond considering the mathematics at hand to take into account the students' interpretation of that mathematics:

> It seems the more natural approach to this problem is to find the actual number of bagels. . . . In order to actually divide up these bagels, they would probably have to find that 3.5 of a dozen is 7.2 bagels. So it's not surprising that students would find these numbers, then work back toward the less natural solution of 3/5 dozen (which in a practical sense is not helpful). (Rachel's notebook)

The tone and nature of Rachel's comments changed with the introduction of the new problem, moving from descriptive to interpretive as her attention turned more generally to the concepts underlying the problems rather than the problems themselves. Her analysis was a result of her conflict with the mathematics involved in the problem. She questioned the purpose of the problem, the various student strategies to the problem and how student work connected to the overall purpose. In the final pages of her notebook, Rachel discussed how fractions are represented and the importance of choosing fractions that make sense for a given problem context. Her burgeoning interpretative analysis was a result of her conflict with the mathematical ideas involved in the problem.

Cathy: A Model and an Analysis Class

Cathy worked for a school district and had more than 15 years of teaching experience in a K-8 setting. When Cathy discussed what she wanted to learn from the first lab class, she listed, "how to ask students thoughtful questions to push their thinking [and] how to move the mathematics forward in a positive, non-threatening, [and] nuturing way" (Cathy's notebook). Later in the survey data, Cathy stated that she was interested in watching the instructor teach. Cathy took a descriptive stance in her notebook, often writing word-for-word accounts of student comments or solutions. For the CJP, she wrote, "Marsha chimed in that Elisha did like a subtraction problem. . . . [Elisha said] I don't know really. I tried to break it down. Maybe I did something wrong. I don't know" (Cathy's notebook). In her notebook she also wrote detailed notes about what other participants said during sessions.

Cathy took an evaluative stance when her attention turned to the teacher. She did not write down the instructor's comments or questions during the lab class. In a postlesson discussion she said that the instructor did a great job leading a discussion about explanations with students (field notes). Throughout the week, Cathy often made evaluative comments about the mathematical tasks chosen for the lab class or the instructor's pedagogical moves. For example, Cathy stated the instructor ignored Tara's incorrect circle (see Figure 5) and thought the instructor should have intervened (field notes).

Cathy's notebook and discussion comments suggested she viewed the class as a model when the attention was on the mathematics or the instructor's pedagogical moves. However, Cathy became invested in student participation. Cathy's stance became interpretative when her attention focused on student participation or class discourse patterns. During the prelesson and postlesson discussions, Cathy offered interpretive statements about why a student, Sharon, did not share her solution. Cathy focused on Sharon throughout the CJP and read Sharon's written work prior to the postlesson discussion, noting Sharon only engaged in the discussion after the diagram was displayed on the board (field notes). As the week progressed, Cathy showed her continued investment in students' participation and mathematical understanding by tracking specific students. Overall, Cathy believed the institute experience had a profound effect on her professional outlook, noting, "I had no clue that it would be such a defining moment in my educational life . . . and has had a powerful impact in my professional career" (online survey response).

Conclusion

The purpose of this study was to examine MTDs' professional vision by identifying the focus of their attention while observing a demonstration class and how they interpreted those observations. The CPTM Summer Institute provided a unique professional development experience enabling us to study how a diverse group of MTDs observed and interpreted a mathematics class for preservice teachers. One could speculate the diversity in expertise and work settings may have influenced the MTDs' observational stance with respect to classroom events. However, with our limited data, we do not have evidence that supports the idea that educational background or work setting predicted observation stance. The MTDs' stated goals for and interests in attending the institute, however, did align with what MTDs noticed during the lab class. For example, those who, from the outset, planned to focus on teacher questioning did seem to consistently maintain that focus throughout, with other aspects of the class potentially becoming salient as well.

As researchers we originally thought participants would either view the lab class as a model class to emulate or as analysis class to critique, but the evidence suggested otherwise. Participants did not fall neatly into one category with respect to how they viewed the lab class. For example, while there was evidence that Fiona observed the lab class as a model class, there was also evidence she took an interpretative stance for parts of the CJP. Rachel illustrated how an MTD could develop an internal conflict and shift her or his observational stance over time. In addition, Cathy exemplified how participants may take different observational stances depending on what they focused on during the observation.

The institute organizers designed the lab class to be viewed as a site for analysis and reflection, but our results reveal this design did not ensure participants would approach observations with an analytical lens. Although some participants did not necessarily react to the institute design as intended, our data suggest both purposes for demonstration lessons have value in professional development and provide opportunity for professional growth. When MTDs viewed the lab class as a model class they had the opportunity to think critically about their own practice by comparing it to another's, consider new teaching strategies, and consider novel aspects of teaching and learning mathematics. When MTDs viewed the lab class with an analysis view they had the opportunity to examine the dynamic nature of instruction and explore the interplay between instructional strategies, mathematics, and students. Within the analysis class view, MTDs have the potential to shift their thoughts about teaching and learning. Although both purposes for demonstration lessons provided professional development opportunities, we found two aspects of the institute experience, observer investment and observer conflict, supported an analysis stance.

The observer's investment in some aspect of the demonstration class seemed to influence her or his view of the class. Often MTDs became intensely invested in some idea and would follow the development of it throughout one or more lab class sessions. In these cases of investment, MTDs were more likely to interpret and analyze the lab class experience, rather than describe or evaluate. Three factors seemed to influence the development of MTDs' investments. First, MTDs came to the institute with their own professional goals, and these goals influenced the lens with which they viewed the lab class. Second, the content of the institute (MKT) aligned with MTDs' teaching responsibilities which supported their making connections to their own practice. Third, the MTDs had sufficient time to become invested in some aspect of the lab class. To foster participant investment, these three factors should be an intricate part of planning and facilitation of the demonstration lesson cycle of prediscussion, observation, and postdiscussion. Each component of the demonstration lesson cycle needs to have a clear focus, and MTDs need to see the connection between the focus and their teaching practice.

A conflict can encourage observers to take on an interpretative stance. These conflicts can arise from a variety of sources, such as the teacher, students, content, or some combination. Differences in MTD's ideas about mathematics and teaching mathematics surfaced throughout the week and initiated important, and, at times, intense discussions. One participant at the institute described the importance of this conflict during the focus group discussion: "[At the institute] I got the most out of tense conversations where there's some kind of opposition or polemic going on. . . . Those are the things that I remember and they cause me to think past the conversations." To encourage productive conflicts, professional development organizers should create a safe environment for open discussions and encourage participants to share diverse perspectives about observations.

Though policy and curricular changes have occurred since the design of this professional development, the work of preparing and supporting mathematics teachers remains largely unchanged. The release of the Common Core State Standards in Mathematics (NGA & CCSSO, 2010) was both an enormous change in mathematics education and the continuation of decades of work in the field. As the first attempt at adopting a (mostly) common national set of curriculum standards backed by Race to the Top funds, CCSSM represents a major push towards large scale instructional change. However, the CCSSM is also a product of the work that preceded it: The mathematics practices were informed by both the process standards of *The Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) and the strands of mathematical proficiency of the National Research Council report, *Adding It Up,* (2001). *The Mathematical Education of Teachers II* report (College Board of Mathematical Sciences, 2012) suggests that while much has changed since the first *Mathematical Education of Teachers* report (College Board of Mathematical Sciences, 2001), there are many themes in mathematics teacher education that are still relevant. For example, though teachers need to be proficient with school mathematics, it is not sufficient to know school mathematics for oneself. It is still important to develop a teacher's MKT (Ball, Thames, & Phelps, 2008) which is different than mathematical knowledge needed for other professions. For example, teachers need to be able to understand and connect multiple mathematical representations, make sense of students' mathematical thinking, and facilitate precise mathematical communication, all key features of the professional development experience in this study. Because much of teacher education and professional development continues to encourage teachers to shift to pedagogical strategies that are not solely teacher-directed, confront and revise beliefs about what it means to know and do mathematics, and develop MKT, the work of MTDs have maintained some commonalities over time.

A similar experience as the summer institute is not a professional development opportunity readily available to most MTDs due to the enormity and uniqueness of the experience. However, demonstration lessons can be utilized as a professional development tool in a local setting and on a smaller scale. For instance, groups of MTDs can work together to observe each other's classrooms and go through the cycle of prediscussion, observation, and postdiscussion. The findings in this study can also be extended to other professional development settings for MTDs, such as conferences and workshops. First, participants come to conference sessions with their interests and observational stances. Conference presenters can consider ways to connect to those interests. Conference presenters can safely bring out a variety of viewpoints in their sessions and provide opportunities for MTDs to grapple with those viewpoints. The different viewpoints can potentially create investments and conflict.

This study provides insights into how MTDs engage in a professional development experience with demonstration lessons and the professional vision of MTDs. Professional vision is not just a mental process but is greatly affected by "competently constructed relevant settings of a complex of situated practices" (Goodwin, 1994, p. 31). As such, professional development experiences influence participants' vision and vice versa. Professional development leaders should consider MTDs' vision and how they can tailor professional development to accommodate different lenses, enabling all MTDs the opportunity to "see" and respond in productive ways.

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