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Commentary

The Effect of Additional Math in High School on College Success

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Methods of causal inference are not widely used by education researchers, even though they can be extremely useful tools for eliminating selection bias and confounding factors in empirical studies. For example, researchers have established that taking additional math classes in high school is strongly correlated with success in college and higher earnings. More recent research seeks to show that taking additional math in high school actually causes success in college. Such analyses are difficult because researchers must draw meaning from naturally occurring data, rather than through experimentation. Researchers have employed a few different methods of causal inference with varying levels of success. Studies using the best methods suggest that taking additional courses in high school mathematics does, in fact, cause an increase in college enrollment and future wages. Education researchers should recognize the power of causal inference methods more widely in evaluating treatment effects.

This commentary explores the effect taking additional mathematics courses in high school has on college success. Knowing if additional mathematics has a positive impact on overall college and career success is important to help both government and privately funded educational institutions create policies around the teaching of secondary school mathematics. The question of curriculum effects is used as an illustrative example to show how methods of causal inference can be applied more broadly to help the education research community.

For years, mathematics has been one of the core subjects taught in school. Success in mathematics is often viewed as being critical to overall success in school. Does taking additional math in high school cause college success? Many studies have

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shown a strong correlation between additional math and overall educational success (Altonji, 1992; Gamoran & Hannigan, 2000; Levine & Zimmerman, 1995; Smith 1996). However, these results alone do not imply that additional mathematics causes overall educational success. This question is difficult to answer because a study must differentiate somehow between students whose circumstances make them more likely to both succeed in college and take more mathematics and students who become more prepared for college by taking more math. To account for these confounding factors, some researchers use methods of causal inference such as propensity score matching, instrumental variables, and regression discontinuity designs.

This paper takes a deeper look into why it is so difficult to discover if additional high school mathematics causes college success and examines the methods that have been used to overcome these difficulties. Taking additional math in high school is certainly correlated with college success. Additionally, studies by researchers using rigorous methods of causal inference consistently indicate that taking additional math in high school actually *causes* an increase in college enrollment and wages. Some studies also show that additional math increases college completion.

Methods and Related Work

The literature reviewed herein was found through keyword searches in Google Scholar and EBSCO Academic Search Premier using phrases such as *math success effects*, *math curriculum effects*, *effect of high school math success on later educational success*, and similar phrases. Literature from any discipline was considered if it was relevant to the topic and the authors used some method of causal inference to estimate the causal effect of additional math in high school on college success. More papers were found and screened in the references of papers found by keyword searches. In total, about 45 papers were screened, with eight being selected for detailed review. The key details of each of the included papers, including the discipline(s) of the authors, are shown in Table 1. Most of the literature that seeks to establish the causal relationship between additional math classes in high school and college success was published in journals and written by authors working in the fields of education and the economics of education.

There is a substantial body of research that addresses the relationship between high school curriculum and college outcomes without estimating causation (only looking at correlation). This work has been excluded, as it is not the focus of the present commentary. Those wishing to explore this body of research could start by reading Adelman (2006), Aughinbaugh (2012), and Stein, Kaufman, Sherman, and Hillen (2011) and following the references provided therein. Stein et al. (2011) conducted a literature review specifically to understand more about the universal algebra movement: Who is taking algebra by the eighth or ninth grade, and what are the outcomes for these students? They conclude that taking algebra by the eighth or ninth grade is strongly correlated with later high school and college success, but only for students who are prepared for it or who are given the extra support that they need to succeed. The present study takes a different approach, focusing on the power of methods of causal inference to estimate the causal effects of curricular policy.

The Inherent Problems

There is a multitude of problems that come with seeking to estimate the effect of additional high school mathematics on college success, which fall into two main categories: selection bias and confounding variables. All of the literature that seeks to show that additional high school math causes college success touch on some of these difficulties (Altonji, 1992; Attewell & Domina, 2008; Byun, Irvin, & Bell 2015; Dougherty, Goodman, Hill, Litke, & Page, 2017; Joensen & Neilsen, 2009; Kim, Kim, DesJardins, & McCall, 2015; Levine & Zimmerman, 1995; Rose & Betts, 2001). This section discusses how selection bias and confounding variables make it difficult to study the effect of additional math in high school on college success, and the next section will discuss how methods of causal inference can help overcome these difficulties.

Selection bias is a problem because the sample of students who choose to take additional math in high school is not random: More advanced students are more likely to take more math. One way to avoid student-level selection bias is to compare students from a school that requires more math in high school to a school that requires less. However, this would introduce a school-level selection bias (Allensworth et al., 2009; Altonji, 1992). Some researchers have combated this issue by creating a study design that eliminates student-level selection bias, and then adding extra controls for some of the school effects (Dougherty et al., 2017; Joensen & Neilsen, 2009; Kim et al., 2015; Rose & Betts, 2001).

When trying to deduce the effect of additional math, there are many confounding factors linked to student characteristics: When a student does well in college, it is difficult to tell if their success is due to the treatment (having taken additional math) or to other factors such as socioeconomic status, parental attitudes, or prior skill (Altonji, 1992; Byun et al., 2015). There are additional confounding factors that are not related to student characteristics, such as differences in instructor quality, impact of peers, and course requirements (Altonji, 1992; Schneider, 2009). Many of these confounding factors can be accounted for by using control variables, but control variables can only account for factors that can be measured. Because using control variables is insufficient, control variables should be coupled with methods that eliminate selection bias and confounding variables at the study design level, as discussed in the next section.

Mitigating the Problems Through Study Design

Clearly, many factors confound the data about the effects of additional high school math on college success. There are enough confounding factors that enumerating them all is impossible. Thus, it is critical to use methods that will control for the seen and unseen factors affecting the data. The most robust way to establish causation between variables is through randomized trial, which in theory eliminates both selection bias and confounding variables (Angrist & Pischke, 2015; Mosteller

& Boruch, 2002). To deduce the causal effect of taking additional math in high school on college success, a randomized trial would mean constructing an experimental design that involves randomly assigning students to take additional or more advanced math classes. It is important to note that such an experiment would eliminate selection bias and confounding variables related to student characteristics such as socioeconomic status and parental attitudes, but confounding due to other factors such as instructor quality, peer effects, and course requirements would remain unless controlled for in other ways. The political barriers to constructing such an experiment are very high, and some would argue that doing so is unethical (Mosteller & Boruch, 2002). Likely for these reasons, such an experiment has not been conducted. Instead, researchers must use natural experiments: naturally occurring situations where students were randomly or pseudo-randomly assigned to taking an additional or a higher-level math class (DiNardo, 2018). Methods used for analyzing data in such a way as to estimate a causal effect are known as methods of causal inference. Such methods are practiced by researchers in many fields including statistics, economics, and epidemiology. The reader can refer to Pearl (2009) for an introduction to methods of causal inference generally, or Angrist and Pischke (2015) for an overview of uses of causal inference in the social sciences in particular. Here we explore which methods of causal inference researchers have used to estimate the effect of additional math in high school on college success.

Matching

Matching techniques seek to obtain a sense of random assignment by matching each experimental subject who received treatment with another experimental subject of similar background who was not given treatment (Todd, 2018). One commonly used type of matching is Propensity Score Matching (PSM), where subjects are matched using a propensity score: the probability that they would receive treatment based on preexisting factors (Hahn, 2018). For example, suppose researchers want to study the effects of taking Algebra II in high school. Using PSM, to see the effect of having taken Algebra II, they would match each student who took Algebra II with a student who did not take Algebra II having similar socioeconomic standing, parental attitudes, past performance, and as many other relevant characteristics as possible. One of the drawbacks of matching techniques is that they only control for measurable variables, so they do not solve the fundamental problem of selection bias: Subjects make selections based on unobservable characteristics (Todd, 2018).

Attewell and Domina (2008) used propensity score matching to analyze data from a longitudinal study done by the U.S. Department of Education starting with eighth graders in 1988. They used parental education, income, employment status, and marital status along with student educational expectations and prior academic performance to create their propensity scores, then compared students with similar scores who took a slightly different curriculum in high school than the other, with one including higher level math. They found that students who took a more intense curriculum in high school math and English were more likely to attend college but were not necessarily more likely to graduate.

More recently, Byun et al. (2015) utilized PSM to analyze data from the Educational Longitudinal Study of 2002–2006. They matched students on a wide array of characteristics, including socioeconomic status, prior academic achievement, and parental educational expectation. They found the positive impact that taking advanced math classes has on college enrollment can mostly be explained by other factors such as family and school influences. They also found that taking advanced math classes has less of a positive impact for Black students than for White students. However, as previously mentioned, since PSM relies on matching based on observable characteristics, there is always some level of inaccuracy due to unobserved characteristics. There are other methods that control for both measured and unmeasured confounding factors.

Instrumental Variables

Instrumental variable estimation is another way of estimating a causal relationship between two variables when a randomized trial is not feasible. In order for an instrumental variable estimation to establish causation, there must be some variable, called an instrumental variable, that meets two conditions: (a) It is correlated with the explanatory variable, and (b) it is not correlated with the outcome variable except by virtue of its association with the explanatory variable (see Woolridge, 2013 for a proof that a variable fitting these requirements can be used to estimate causation). In estimating the effect of additional math in high school on college success, the explanatory variable is the math classes taken by a student in high school and the outcome variable is how well the student performs in college. Therefore, a valid instrument must be correlated with high school math curriculum but not with college success (except through its correlation with high school math courses).

Altonji (1992) introduced the idea of using variation of curriculum across high schools (specifically, number of courses required in each subject) as an instrumental variable. To justify using this as an instrumental variable, Altonji assumed that the number of required courses in each subject is correlated with the number of courses taken by each student, but not correlated with any other unobserved student or school characteristics which would influence college and career success. By analyzing data from the National Longitudinal study of 1972, he found that taking one additional academic course in high school yields little effect on future success, but the effect of taking additional math is greater than the effect of taking more of any other subject. Since the natural experiment he leverages creates a sense of random assignment for students, selection bias and confounding variables based on student characteristics are minimized. By Altonji's own admission, however, the assumptions his study relied on are not realistic, failing to account for other factors such as school and peer effects. This highlights one of the primary weaknesses of the instrumental variables approach: The causation argument only holds true if the instrumental variable is correlated with the explanatory variable but otherwise not correlated with the outcome variable, and such an instrumental variable is often difficult to find.

Levine and Zimmerman (1995) used methods similar to Altonji's (1992), with the same instrumental variable, to analyze data from the National Longitudinal Survey of Youth (NLSY) and High School and Beyond (HS&B). In the NLSY, they found no effect for women but found that men who take more math classes are more likely to attend college but are no more likely to graduate. In the HS&B data, they found no effect for men but found that women who take more math in high school are more likely to attend and graduate from college. Across both data sets, they found that additional math in high school increased the wages for women who graduated from college.

Rose and Betts (2001) also used data from HS&B and instrumental variable methods similar to Altonji's (1992). They also improved on Altonji's method by adding more controls for school effects, helping the instrumental variables assumptions that he used become more convincing. They showed that taking higher level math classes in high school had a significant effect on college enrollment, college completion, and earnings 10 years after high school graduation. According to their results, taking higher level math in high school had a stronger positive effect on college success for women.

More recently, Joensen and Nielsen (2009) used instrumental variables techniques to estimate the effect of additional math on college success. However, the natural experiment that they took advantage of doesn't require such strong assumptions as Altonji's (1992). They studied high school students in Denmark. In the 1980s, there was a pilot program for a new curriculum instituted at some schools which gave students more flexibility in their course options. In the pilot program, some students took more advanced math than they would have without it. As their instrumental variable, Joensen and Neilson (2009) used whether or not a student's high school decided to enroll in a pilot program *after* the student was enrolled in the high school. They add additional controls for the preexisting qualifications of students. The fact that the schools self-selected to be a part of the pilot program introduces some bias, but they control for this using the average GPA of students at the school. They found that those who took more mathematics than they would have without the pilot curriculum received a higher education and had higher earnings later on.

Kim et al. (2015) analyzed data regarding students in Florida, following them from grades 7–12 and into college and the workplace. They used county unemployment levels during the ninth-grade year of the subjects of interest as their instrumental variable and controlled for variation in how much students learned in class by studying subjects who can be compared to one another on a standardized test. Thus, their study minimized the confounding factors related to student characteristics through the natural experiment and eliminated some of the instructor effects by adding the control based on the standardized test. They concluded that completing Algebra II in high school has a significant impact on a student deciding to attend college, but no impact on their completion of college. However, it is debatable if their instrumental variable meets the criteria for being a valid instrument. It seems unlikely that unemployment during a student's ninth grade year has no correlation with his or her decision to attend and perform well in college. They supported their claim by showing that the unemployment rate changes enough from year to year that it won't have an impact. However, if unemployment has an impact on what math classes a student chooses to take in ninth grade, as they claimed it does, it would be likely that it also impacts many other variables that would affect students' decisions to attend and perform well in college.

Regression Discontinuity Design

Using a regression discontinuity design (RD design) is another way of extracting randomness from a situation that does not seem random. Regression discontinuity design is achieved by comparing only subjects close to some arbitrary cutoff to obtain a sense of randomness (Lee & Lemieux, 2010). Consider that students are often assigned to take certain courses based on a placement test. Assigning students to take a different class based on a test is not random assignment. However, it could be considered random assignment for those students who scored close to the cutoff on the test. Suppose that students who scored 80% or higher on the placement test were placed into the higher-level math class. On average, the students who scored 79% and the students who scored 80% have nearly identical mathematical abilities. Thus, by only comparing students who scored close to the cutoff, researchers can obtain a sense of random assignment—this is the strength of using an RD design (Lee & Lemieux, 2010). In order for a regression discontinuity design to estimate causation, the subjects must not be able to precisely control the variable that decides which subjects receive treatment, and all other factors related to the outcome variable must be continuous with respect to the treatment variable. Compared to the assumptions necessary to establish causation under other methods of causal inference, these assumptions are much easier to satisfy and much easier to test (Lee & Lemieux, 2010). Therefore, causal effects that are estimated using an RD design are often more robust than those using other methods.

Dougherty et al. (2017) observed the results of a pilot program in the Wake County Public School System in North Carolina that placed many students on a higher math track than usual based on a placement test. Their RD design minimized student-level selection bias and confounding factors, and they added additional controls for teacher quality and peer effects. They found that taking a more rigorous math curriculum in middle school causes students to obtain higher composite scores on a college preparedness exam that tested math, science, English, and reading, with not all of the increase in their scores coming from the math portion. The students' intent to attend college also increased.

Because of the great strength of the RD design in estimating causal effects with few assumptions, more researchers should consider employing an RD design in examining the effect of high school math curriculum on college success. The existing study by Dougherty et al. (2017) used sound methods to estimate causation, but it unfortunately only measured intent and preparedness to attend college rather than examining actual college enrollment and completion data. A study that uses similar methods to estimate causation but follows the students all the way through college enrollment and completion would be a great addition to this body of work.

Discussion

When looking to answer questions of policy effectiveness, methods of causal inference have more power to eliminate selection bias and confounding factors than control variables alone. Propensity score matching is the easiest of these methods to apply but is the least rigorous as it only accounts for observed factors. Instrumental variables can eliminate confounding factors in theory, but it is difficult to apply in practice because the assumptions it requires are difficult to satisfy. RD designs also have the ability to eliminate all confounding factors, but it is easier to test that the assumptions are valid than with an instrumental variables approach. Given their explanatory power, such methods could be a great benefit to the education research community if applied to more scenarios.

The aggregated results of the surveyed literature are displayed in Table 1. Despite the limitations of each of the methods of causal inference utilized, there is a consensus across them all that taking additional math in high school increases college enrollment and wages. The effect on the completion of college is disputed. A few of the studies draw attention to a difference in curriculum effects for students of different gender, race, or socioeconomic status (Byun et al., 2015; Dougherty et al., 2017; Levine & Zimmerman, 1995; Rose & Betts, 2001). However, there is not enough consistency across studies to say with certainty which particular group receives less of a benefit from treatment.

In *The Math Myth,* political scientist Andrew Hacker (2016) argued that the only reason success in mathematics correlates so strongly with college success is that the current educational system mandates math for college entrance exams and general education requirements. Although he did not have evidence to support his claim, he did pose an interesting question that is worth investigating: Would success in high school mathematics lead to college success even if it was not used by colleges to decide who to admit? Attewell and Domina (2008) argued that since a more rigorous curriculum improved college success for all students, not just those already in higher curriculum brackets, the effects are due to an actual improvement of the students' abilities and not just because colleges look to accept students with certain math credentials. However, this argument seems tenuous and needs to be backed by further study. Answering the question of why success in high school mathematics is so strongly related to college success presents another opportunity for education researchers to utilize methods of causal inference.

Conclusion

It is important to understand the impact that mathematics education has on overall educational success so that local, state, and national leaders can make better policy and curriculum decisions. Though it has been long known that additional math in high school is correlated with college success, applying methods of causal inference to this problem gives additional insight, showing that taking additional math in high school actually *causes* increased college enrollment. Further research should also be done on related questions: How can we ensure that all students are reaping the benefits of a more rigorous high school math curriculum? Would a strong high school math background increase college success even without requiring it on entrance examinations? These questions, along with many other questions education researchers grapple with, would benefit from the use of methods of causal inference to eliminate selection bias and confounding factors.

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Table 1

Summary of Effects of Taking Additional Math in High School

Authors	Discipline	Methods	Treatment	Increases College Enrollment	Increases College Completion	Increases Wages	Different Effects by Groups
Altonji (1992)	Economics	Instrumental variables	Additional year of HS math	Yes	Yes	Yes	N/A
Attewell & Domina (2008)	Sociology	Propensity score matching	Taking more advanced curriculum (across 5 levels)	Yes	No	N/A	N/A
Byun, Irvin, & Bell (2015)	Education	Propensity score matching	Completing 1 class beyond Algebra II in HS	Yes	N/A	N/A	Yes
Dougherty, Goodman, Hill, Litke, & Page (2017)	Economics/ Education	Regression discontinuity design	Placement on a college-prep math track in MS	Yes[a]	N/A	N/A	Yes
Joensen & Nielsen (2009)	Economics	Instrumental variables	Additional high-level math class in HS	Yes	Yes	Yes	N/A
Kim, Kim, DesJardins, & McCall (2015)	Economics/ Education	Instrumental variables	Completing Algebra II in HS	Yes	No	N/A	N/A
Levine & Zimmerman (1995)	Economics	Instrumental variables	Additional year of HS math	Yes	Yes	Yes	Yes
Rose & Betts (2001)	Economics/ Education	Instrumental variables	Additional year of HS math	Yes	Yes	Yes	Yes

Note. HS = High School; MS = Middle School

[a]Intent and preparedness to attend college

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