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Scholarly Practice and Inquiry: Dynamic Interactions in an Elementary Mathematics Methods Course

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This paper presents research that exists at the crossroad of scholarly practice and scholarly inquiry. We outline the process in the design, enactment, and empirical examination of an elementary methods course activity, Exploring and Supporting Student Thinking (ESST), which engaged 18 elementary prospective teachers (PTs) in two sessions of one-on-one problem posing with 3rd grade students. Our results mirror outcomes from existing literature focused on student interviews and letter exchanges as well as reveal other potential PTs experiences from such interactions. We end by describing implications for future activity design and with a call for researchers to continue to contribute to scholarly inquiry in this area.

Research suggests that teachers who understand how students think about particular mathematical ideas will be better positioned to recognize, interpret, and support these ideas in their instruction (Brown & Borko, 1992; Fennema & Franke, 1992). Research on Cognitively Guided Instruction (CGI) has demonstrated teacher knowledge of student thinking, reasoning, and strategies can lead to gains in student achievement (Carpenter & Fennema, 1992; Carpenter et al., 2000). Ball and colleagues' work on mathematical knowledge for teaching identified knowledge of content and students as a crucial facet of pedagogical content knowledge necessary for teaching mathematics effectively (e.g. Hill et al., 2008).

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In light of these findings it has become increasingly important for mathematics teacher educators (MTEs) to assist prospective elementary mathematics teachers (PTs) in developing knowledge of children's thinking. Jacobs et al.'s (2010) work on professional noticing of children's mathematics has become a popular framework to explore the ways in which teachers attend to, interpret, and respond to students' mathematical thinking. Activities in mathematics methods courses often provide PTs opportunities to examine and interpret authentic (or instructor-generated) samples of student work depicting invented computational strategies or mathematical reasoning as a means to gain experience interpreting and responding to student thinking (e.g. Tyminski et al., 2014). We term these types of interactions between PTs and these artifacts as *static*, in that there is no student to interact with during the process of interpreting the work and, once interpreted, there is no opportunity to respond authentically to students and observe the result. We see value in static interactions for developing PTs' knowledge of students' mathematical thinking and include examples of them in our methods courses. Responding, however, is an important skill of noticing and static work does not offer authentic opportunities to respond to student thinking. Although noticing has been shown to be a complex skill to develop, research has demonstrated it is learnable over time (Jacobs et al., 2010; Tyminski et al., 2014). In order to scaffold the complexity involved in noticing, we wanted to begin with activities in which PTs first focus on one student's thinking. Jacobs and Empson (2015), asserted that one-on-one interviews between PTs and students can be valuable in developing PTs' ability to engage in professional noticing. Therefore, we sought to design and enact an activity in our methods course's early field experience that would foster PTs' understanding of how to elicit and support a student's mathematical thinking in a one-on-one situation. The activity would be *dynamic* in nature, allowing for a sustained exchange between the PTs and the student.

The research presented in this paper comes from a cycle of scholarly practice and scholarly inquiry (Lee & Mewborn, 2009). We outline the process of development in the design, enactment, and empirical examination of an elementary methods course activity[1] that we call Exploring and Supporting Student Thinking (ESST) and answer the question, "What are the experiences[2] of PTs within the ESST activity?"

Literature Review

Scholarly Inquiry and Practice

In methods course activity design and enactment, the authors seek to leverage the interplay between research and practice through the processes of scholarly inquiry and scholarly practice (Lee & Mewborn, 2009). Scholarly inquiry is the exploration of "issues and practices through systematic data collection and analysis that yields theoretically-grounded and empirically-based findings" (p. 3), while scholarly practices are "adapted from empirical studies of the teaching and learning of mathematics and the preparation of mathematics teachers" (Lee & Mewborn, 2009, p. 3). "Scholarly inquiry and practices are interrelated in that MTEs use empirical studies in mathematics education to build practices that are labeled scholarly. In addition, scholarly practices can inform directions for scholarly inquiry regarding PTs' mathematics teaching and learning" (Kastberg et al., 2017). To inform the design of the ESST Activity as an example of scholarly practice, we reviewed the literature on existing scholarly inquiry on dynamic interactions in a methods course, such as student interviews or letter writing exchanges. In our review of the literature we (a) present and synthesize knowledge on the potential impact of these activities on PTs' learning and (b) describe how the literature informed the design of the ESST activity through understanding the variation and commonalities of the activities described in other researchers' scholarly inquiry.

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[1] We define *activities* as situations MTEs provide for PTs.

[2] We define *experiences* as "ways in which the preservice teachers internalized those activities" (Mewborn, 1999, p. 31).

Dynamic Interactions[3]

Examples of dynamic interaction activities within the research literature include asynchronous activities such as letter writing exchanges (e.g. Crespo, 2000; 2003; Norton & Kastberg, 2012), as well as face-to-face activities such as interviews with learners (e.g. Ambrose, 2004; Jenkins, 2010), PTs' use of scripted interview protocols in interviews with learners (Moyer & Milewicz, 2002), and PTs' work with small groups of learners (e.g. Nicol, 1998). Each of these studies represents an example of scholarly inquiry into PTs dynamic interactions with student thinking. A brief summary of each activity, context, and the authors' findings are presented for each example.

In Ambrose (2004), elementary PTs worked in pairs to pose open-ended problem solving activities focused on whole number operations and fractions to children. The goal was to impact PTs' beliefs about teaching by leveraging current beliefs as caregivers, with the potential outcome of shifting PTs beliefs from teaching as explaining to understanding student thinking. Ambrose concluded that as a result of these open-ended problem solving activities, PTs developed new beliefs that were incorporated in existing belief structures.

Jenkins' (2010) intervention involved six middle grade PTs working in pairs in alternating roles to pose open-ended tasks focused on patterns and proportions to students. Jenkins searched for evidence of PTs' "interpretive listening skills and awareness of the different ways that middle school students make sense of mathematics" (p. 147). Jenkins reported "the structured interview process fosters an interpretive orientation to listening and initial awareness of the variety of ways that middle school students think about mathematics" (p. 147).

Moyer and Milewicz (2002) engaged 48 PTs in using scripted diagnostic interview protocols focused on rational number tasks to guide their interactions with children. The PTs conducted interviews with children throughout the semester.

[3] This section on dynamic interactions is built from Tyminski and his colleagues' prior work (Kastberg et al., 2017), which served as an example of the role of scholarly inquiry in developing scholarly practices.

The final interview was recorded, transcribed, analyzed, and reflected upon by the PTs and served as evidence of PTs' experiences and use of questioning. Analysis of the interviews revealed a beginning classification for the types of questioning: (a) "check listing," which involved asking the protocol's questions with little regard for student responses; (b) "instructing vs. assessing," in which PTs explained mathematics directly to the student with little regard for students' reasoning; and (c) "probing and follow up questions," characterized as PTs genuinely listening to student responses and generating follow-up questions to elicit further student thinking.

In Nicol's (1998) activity, 14 PTs were engaged in weekly interactions with small groups of 6th and 7th grade students (ages 12-13). The PTs solved problems involving multiplicative reasoning in class and then posed adapted or extended versions of these tasks to students. Nicol examined PTs' abilities to question, listen, and respond to students using prospective teachers' journal reflections as sources of evidence for these behaviors. Across the weekly implementations of the activity, PTs began to shift their approaches from those that focused on arriving at a correct answer toward an inquiry-based approach focused on eliciting and understanding student thinking.

Crespo (2000) examined the ways in which elementary PTs listened to the responses of the fourth-grade students in a series of six interactive letter exchanges. Reflective journals were used by PTs to explore their interactions with students. PTs' initial interpretations of student work focused on correctness and tended to contain conclusive claims about student understanding based upon small samples of student thinking. PTs' interpretations began to focus on what the student intended or meant in a solution by the fifth week of the course. Crespo (2003) used the same letter writing activity and data to explore PTs' problem posing behavior. Initially, PTs attempted to "make their problems less problematic and more attainable to their pupils" (p. 251). PTs' questions were worded to avoid student errors or confusion rather than to generate learning opportunities for students or for themselves as teachers. Problems included in the last three letters "were puzzle-like and open-ended, encouraged exploration, extended beyond topics of arithmetic, and required more than computational facility" (p. 257). These questions were posed to challenge or extend student thinking and often asked for multiple solutions and explanations.

This synthesis of examples of scholarly inquiry suggests PTs' dynamic interactions with student thinking can potentially: (a) develop PTs' knowledge of students' mathematics and strategies (Ambrose, 2004; Crespo, 2000), (b) encourage PTs to shift their focus in working with students from attaining correct answers to eliciting and understanding student thinking (Crespo, 2000; Jenkins, 2010; Moyer & Milewicz, 2002), and (c) develop PTs' emerging abilities to use student thinking in crafting responses and posing new problems (Crespo, 2003).

From the literature we also identified four context factors as potentially supportive in the design and enactment of such an activity. They are:

1. PTs should have opportunities to solve challenging mathematical problems prior to posing them to students.
2. All PTs should pose the same problems to students to give PTs common experiences to discuss.
3. PTs require opportunities to reflect on their experiences in a whole group setting as well as through individual, targeted reflection.
4. MTEs must consistently respond to PTs' reflections.

We used these findings on learning outcomes and context factors to design the ESST activity and to analyze PTs' experiences with it. We describe this process in detail below.

Methods

Participants and Context

This study examined the experiences of 18 junior-level PTs enrolled in the required elementary mathematics methods course for their university elementary teacher education certification program as they engaged in the ESST Activity. We view PTs' learning from a social constructivist perspective (Ernest, 1994); in which we attend to both the individual conctruction of ideas and the ways in which socal interaction affects this process. We also employ an inquiry approach to our methods course instruction (Dewey, 1938) in that we engage our prospective teachers in experiences that approximate the practice of teaching. The intent of the course is to provide PTs with structured opportunities to explore content and engage with resources designed to support PTs in learning to teach mathematics drawing from children's ideas. The design of the methods course was guided by three main ideas:

1. Children's mathematical ideas and understandings emerge from solving problems.
2. Teachers can use questioning to scaffold the development of children's mathematical understanding and sense making
3. *Standards*-based[4] curriculum materials can be useful learning tools for teachers and students (Drake et al., forthcoming).

The course is built around the work of teaching and consists of pedagogical strategies devoted to classroom practices such as: opening routines (Parrish, 2010; TERC, 2008; UCSMP, 2007), problem posing (Smith et al., 2008), making sense of and responding to student work (Jacobs et al., 2010), and facilitating whole class discussions (Smith & Stein, 2011). We use video and written work from a set of collaborative classroom teachers as representations of practice (Grossman et al., 2009), and, in addition to examples of *Standards*-based curriculum, develop our methods course activities around these artifacts. Prior to engaging in the ESST activity, the topics addressed within the methods course included standards documents (CCSSO, 2010; SC Department of Education, 2015), CGI problem types and student strategies (Carpenter et al., 1999), responding to students

[4] In using the term *Standards*-based curriculum, we are referring to the curriculum materials funded by the National Science Foundation and aligned with the NCTM Standards (1989; 2000), including: *Investigations in Data, Number and Space* (TERC, 2008); *Everyday Mathematics* (UCSMP, 2007); and *Math Trailblazers* (UIC, 2008).

through questioning (Jacobs & Ambrose, 2008), number choice and number choice progressions (Land, et al., 2014), and opening routines in the Grade 2–6 mathematics classroom (Drake et al., forthcoming). PTs had also completed four mathematics content courses that were designed for elementary mathematics teachers, including a course on problem solving. They were concurrently enrolled in a fifth content course addressing content for middle grades mathematics teachers.

Exploring and Supporting Student Thinking Activity

The design of the ESST Activity was developed as an example of scholarly practice informed by the literature described above on dynamic interactions. The ESST activity engaged PTs in solving, planning, and posing a series of five tasks. As the instructor of the methods course, the first author provided PTs with five tasks designed for use with 3rd grade students (Figure 1). As a class, the PTs and the instructor planned for the enactment of each task using an adaptation of the Thinking Through a Lesson Protocol (Smith et al., 2008; Figure 2). During week five of our course, PTs visited our partner elementary school where each PT was paired with a student from a third-grade class. During a half-hour session, PTs were asked to pose as many of the five problems as their student could work through, employing pre-planned scaffolds and extensions as they saw fit. PTs video recorded their sessions and posted them on the Edthena (2017) electronic video coaching platform. PTs were asked to reflect on their own video in terms of their student's solution path and their interactions with the student. They watched and provided feedback for three of their peers, also commenting on solution paths and interactions. The instructor (first author) also provided feedback on these foci. Following this process, PTs synthesized their reflection and feedback into a written plan of ways to improve their facilitation of each task. In week 7 of the course, the PTs returned to the partner school and enacted the same five tasks with a student from a different third grade class. As before, PTs recorded, uploaded, reflected, and commented on the videos of this session. The instructor provided feedback on these sessions as well. To complete the activity, PTs wrote a reflection paper summarizing their work and what they learned through their enactment and observations. The assignment instructions are provided below:

> The paper will summarize your learning as a result of all associates activities. It will consist of four sections, which mirror the four steps prior:
>
> 1. Day 1—Reflections on my student's thinking and my practice
> 2. Observations of other students' thinking and my colleagues' practice
> 3. My written plan of action
> 4. Day 2—Reflections on my student's thinking and my practice

We used their reflection papers as our data source to make sense of PTs' experience with the activity.

Figure 1
The Five Tasks to be Used with 3rd Grade Students

1. Candy Problem - Marcus has 523 pieces of candy. Josette has 275 pieces. How many more pieces of candy does Marcus have?
2. Sticker Problem - Beth has 398 stickers. Her friend gives her some more. Now she has 710 stickers. How many stickers did her friend give her?
3. Brownie Problem - Alan, Rose, Kevin and Benny share 37 brownies equally. How many brownies does each person get?
4. Coin Problem - Nadia has 5 coins in her pocket that total 51 cents. What coins can she have in her pocket?
5. Barnyard Problem – How many legs are in the barnyard? How do you know?

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Figure 2
Prompts Used to Prepare Each of the Five Tasks

Preparing to use the task

1. Describe how will you present the task (orally, written, both)?
2. What tools (manipulatives, representations, technology) will students have access to?

Preparing to interact with student's thinking

1. What will you say and do to be sure the students understand the task you are asking them to engage in?
2. What will you do (tasks, questions, actions) to scaffold the task for students who are having trouble?
3. What are all the ways the task can be solved? (solve the problem yourself) Describe likely student errors or misunderstandings.
    a. For each strategy and/or error describe how you will respond. For example, "Some children might solve this problem very quickly by guessing, but be incorrect in their answer. I will ask these children to check their guess within the problem situation and see if the answer makes sense."
4. What extension will you use to further the student's mathematical thinking?

Data Analysis

The authors began the process of data analysis by identifying individual units of analysis within PTs' written reflections. A unit was defined as a collection of phrases or sentences addressing the same topic. When the topic of the writing changed, a new unit was identified. Next, the authors individually coded the units of analysis within the written reflections of the 18 PTs using the three *a priori* codes concerning potential learning outcomes within the literature review: *Knowledge of student thinking*, *Obtain correct answers or Support student thinking*, and *Respond using student thinking*. Through several readings of the data and discussion of our existing codes, the three main codes were refined and operationalized, generating our final coding scheme with descriptions and sub-codes (Table 1).

Beyond the codes developed from our synthesis of prior scholarly inquiry, we also employed open and emergent coding techniques (Strauss & Corbin 1998) in order to identify other themes within the units of data. Two new themes emerged from within the PTs' reflection on their experiences. The first, "PTs' tendency to label or evaluate students based on minimal evidence," was also a finding from Crespo (2000) in which she stated that PTs' initial interpretations of student work focused on correctness and tended to contain conclusive claims about student understanding based upon small samples of student thinking. The second theme that emerged from the data was "the importance of unpacking a task for students."

After units were identified within a written reflection, they were coded with both a main code and a sub-code if applicable. Some units did not receive a code as they did not address one of the five themes. Although each unit was assigned exactly one code, within an entire reflection it was common for different codes to appear multiple times or for reflections to contain codes that seem to be in opposition to each other. Inter-rater reliability for the coding was completed between the two authors demonstrating 74% agreement across 370 units coded within the 18 collected reflections. Disagreements were resolved through discussion. As we did not utilize a pre-post assessment model, we identified evidence of each code and do not make claims on whether or not the identified idea was newly developed knowledge. Rather, we examined frequencies of each code in order to understand the experiences of PTs in their interactions with students. Representative examples of each code follow in the results section.

Table 1
Coding Scheme

Code	Subcode
Knowledge of Student Thinking	Describing Student Solution Strategies
	Conceptions and Misconceptions
	Task Difficulty
Obtain Correct Answers or Support Student Thinking	Obtain Correct Answers—Aware
	Obtain Correct Answers—Unaware
	Elicit and Understand Student Thinking
	Reflection Focused on Student Thinking
Respond Using Student Thinking	Action
	Reflection
"Unpacking" a Task for Students	
Evaluating or Labeling Students	

Results

In this section, we present our findings related to our five main codes and their applicable sub-codes. Table 2 presents the number of units coded for each of the sub-codes. In order to describe our analysis process, the following sections present illustrative examples of each of the sub-codes, which were taken from the written reflections of our PTs. All PTs names and student names are pseudonyms.

Table 2
Coding Scheme and Number of Coded Units (N = 370)

Code	Subcode	Units	Total Units
Knowledge of Student Thinking	Describing Student Solution Strategies	14	48
	Conceptions and Misconceptions	27	
	Task Difficulty	7	
Obtain Correct Answers or Support Student Thinking	Obtain Correct Answers—Aware	13	120
	Obtain Correct Answers—Unaware	12	
	Elicit and Understand Student Thinking	42	
	Reflection Focused on Student Thinking	53	
Respond Using Student Thinking	Action	29	128
	Reflection	99	
"Unpacking" a Task for Students			58
Evaluating or Labeling Students			16
		Total	370

Code 1: Knowledge of Student Thinking

As anticipated based upon our literature search, our analysis of the data revealed a number of instances where PTs identified specific mathematics in their student's work and displayed evidence of knowledge of student thinking in their reflections. Forty-eight units were identified and coded as examples of knowledge of student thinking. To further distinguish among these reflections, sub-codes were developed and utilized to identify three specific categories of student thinking: anticipating possible student solution paths, conceptions and misconceptions, and task difficulty (See Table 2).

Describing Student Solution Strategies

The subcode "describing student solution strategies" was used when PTs identified insights on how a student might solve a problem. Susan provided the following evidence of developing such knowledge while interacting with her first student:

> Madison struggled with the coin problem trying to use five coins to make 51 cents. I thought it was very interesting while watching the videos that so many students started with putting out five dimes and a penny. My first reaction is always to make 50 cents with two quarters. Perhaps they all started this way because it would get them closer to five coins total.

Another PT, Addie, provided the following evidence from her reflection on her interaction with her day 1 student on the Sticker Problem. The student had used a different solution path than she had anticipated:

> She used the manipulatives that I brought in interesting ways that I would not have thought of. For the sticker problem, she figured out that counting up would be a good strategy. Instead of counting up like my other student did normally, she used the little circle counters and had them represent different number amounts. She did not get confused, but I think this was a risky method because it would have been very easy to get confused about what each one represented. She started with 398 and used 2 counters so that she would get to 400. Then, she used the counters to start counting by 10's from 400 to 710. After she got to 710, then she counted up all of the counters. When she got her answer, she added it back onto 398 and realized that she did not get 710, so she needed to add 20 more to get there. Eventually, she got to the correct answer but it was a confusing method.

Of the 48 units identified as knowledge of student thinking, 14 involved describing student solution strategies. Despite their work as a class to generate possible strategies and misconceptions that students may have, PTs still saw ideas and strategies from students that they had not anticipated. The experience of working in a dynamic setting with children, with opportunities to question and learn more about a student's thinking beyond their written work, afforded PTs opportunity to develop this knowledge base in a way which examining static samples of student work cannot.

Conceptions or Misconceptions

This code applied to excerpts where PTs identified conceptions or misconceptions of students' mathematical thinking in their reflection. There were 27 units within PTs' reflections identifying student conceptions or misconceptions. Georgia identified her student's misconception about regrouping when subtracting during the Candy Problem.

> I noticed that my student thought in a different way than I was used to and had some misconceptions about regrouping while doing the standard algorithm for subtraction. In the candy problem, she didn't seem to fully understand how to count up by ones. She was counting by 5's and passed the number she was "counting up to." Then, when she started to count by ones she started at 5 then jumped to 10, 11,12,13.

Addie also shared her reflection on her student's work on the Candy Problem. Her student used the standard algorithm to subtract.

> She set the subtraction up correctly, however, she did not regroup at all. In fact, she didn't even consider regrouping the numbers. If the number was bigger on the top than on the bottom, she subtracted the smaller number from the bigger number and used that number in her answer.

These samples were typical of the types of misconceptions PTs identified in their reflections.

Task Difficulty

We identified 7 examples of PTs recognizing, almost always in retrospect, the potential difficulty of a task for students. Anna's comment from her student's work on the Brownie Problem was typical of these responses: "This problem was the hardest for me as a teacher because although he comprehended that each person got 9 brownies, he didn't understand that the last brownie got divided into four pieces and became a fraction". Other examples of PTs acknowledging the difficulty of the task include Deron, who provided the following example: "Both the candy and sticker problem was [sic] a little challenging for him." Similarly, Abbie's reflection included the following example: "This problem turned out to be a lot harder for Amy." These examples illustrate that despite the preparation of anticipating solution strategies, PTs were still unaware if or why a problem might be difficult for students. It was not until they interacted with the student that they considered that a task may be difficult. Then, in that moment of interaction, they did not have sufficient time to consider what to do to address this issue.

Code 2: Obtain Correct Answers or Support Student Thinking

Our examination of the existing literature suggested that PTs work with students can support a shift in PTs' focus from helping students attain correct answers to eliciting and understanding student thinking. Given we could not examine a shift with only a single point of data, we examined which of these foci PTs tended to focus on as well as their awareness of their own tendencies. There were 120 units coded as examples of these two foci and, within these, we categorized PTs' experiences further using sub-codes. In instances where PTs focused on their student obtaining a correct answer, we differentiated between PTs who identified their focus and reflected on it and those who described such actions but did not seem to recognize it, even upon watching their video. For units coded as examples of PTs reflecting on their moves to support student thinking, we identified two sub units: PTs' reflections which described moves we viewed as supporting student thinking and PTs' reflections on their work identifying the goal of the activity. In preparation for their work with students, we discussed as a class how the goal of the activity was not to use direct instruction, but to practice eliciting, interpreting, and supporting student thinking.

Obtain Correct Answers: Aware

The code "obtain correct answers—aware" was used when PTs self-identified their tendency to focus on students arriving at a correct answer. Riley's reflection serves as a typical example of this sub-code. She wrote,

> For the second task, the student struggled in understanding how to approach the problem. I made the mistake of telling him that he should possibly add up. Because of the goal of this assignment, I should not have suggested a strategy to him. Then, I guided him too much through that strategy. Instead I needed to let him approach the problem on his own.

Georgia provided the following example of being aware of helping her student find the correct answer:

> Starting off on the candy problem, Jesse knew that it was subtraction but he started off writing the standard algorithm as the smaller number minus the bigger number. He was able to realize his mistake, but only after he asked me if it was right and I told him it wasn't.

An excerpt from Marisa's reflection also provides an illustrative example, "She had much more trouble with the Brownie Problem and the Coin Problem. I may have prompted her more through these two problems, but she eventually got to the right answer." Of the 120 units within Code 2, 13 received this code.

Obtain Correct Answers: Unaware

The code "obtain correct answers—unaware" was assigned when PTs seemed unaware of their decision or tendency to focus on the student getting the correct answer. Lawson described her work with her second day student.

> For my second one on one session at [redacted], I worked with a different student in a different class. My new student's name was Caleb. Caleb worked through the first problem with ease. I had him check the first answer in order to ensure that he had the correct answer. Caleb moved on to the next problem and again solved it with no issue. Caleb reached the brownie division problem and immediately drew an array. He became confused when the amount of brownies in each circle did not mach. I attempted to show him how to break up the last brownie into 4 pieces and redistribute giving each person 1/4 of a brownie. Unfortunately Caleb could not quite grasp this concept and after attempting a few times I decided that it was best to move forward to the coin problem. This problem again posed some problems for Caleb. There were times that Caleb made mistakes and seemed to struggle with this problem. He definitely struggled with the fact that he could only use 5 coins. Later on he made a simple mistake of confusing the amount his coins were worth. Eventually I was able to guide Caleb to the final and correct answer.

When his student reached a correct answer, Lawson moved on to the next task without any follow up or questions regarding his thinking. When Caleb had difficulty with a task, Lawson's focus was on ensuring he reached the correct answer. He did not reflect on this approach as something problematic or counter to the goals of the activity.

Marisa described her work on the first day with her student. Her reflection suggests that she may have thought that demonstrating correct answers was the goal of the activity. She wrote,

> I observed through working with [redacted] very many things. First, she has a great conceptual understanding of subtraction. She understands the concept of borrowing and counting up from the smaller number to the larger number to get the answer. She had much more trouble with the Brownie Problem and the Coin Problem. I may have prompted her more through these two problems, but she eventually got to the right answer.

There were 12 units identified in which PTs seemed unaware of their actions focusing on students obtaining the correct answer.

Action Focused on Eliciting and Understanding Student Thinking

The code "action focused on eliciting and understanding student thinking" was assigned when PTs' considered their intended teacing moves to help them better understand the student's thinking. For instance, Heather discussed the following steps she took when interacting with her student:

> For example, I created a road map to follow for each problem depending on the child's responses: whether she chose a successful solution path but could not explain the procedure, struggled and needed scaffolding, or completed the problem successfully and explained her thinking. The questions were not random, but rather flowed with her responses.

Kim's reflection also described her actions to focus on eliciting student thinking. "When Maggie was answering the problems, I would ask her throughout each one what her thinking was, what ideas she was using and how she was sure that her answers to the problems were correct." Marisa described her interactions with her student, which focused on supporting her understanding. She wrote,

> She would solve each problem and I would ask her to explain how she solved each problem. When explaining how she solved each problem, she would either catch herself saying something different than she wrote down, or I would remind her that she told me something different than she wrote down. I asked Kayla to explain how she solved each problem, not to just check if she was correct, but to understand her thought process in solving the problems.

There were 42 of 120 instances of Code 2 where PTs focused on eliciting or understanding student thinking.

Reflection Focused on Student Thinking

This code was assigned in instances where PTs reflected on the interaction with their student and reminded themselves to keep their focus on eliciting student's thinking. As one example, Leah wrote,

> To determine if Hayley's actions were due to my lack of setting up the problem, I altered my initial unpacking a problem method by expanding my question types to ask who, what, when, where, why, and how in later videos. This was, and still is, an improvement I need to continue to work on so I am able to better explore and support student thinking, assess what strategies a child knows, and determine a child's overall cognitive ability.

Margaret assessed her session as follows,

> I need to be careful of validating the student's answers and I need to encourage the student to check for herself whether it is right or it makes sense. In most cases when she got the answer correct I would say it was correct and congratulate her but it was suggested that I should encourage the student to figure out if it is correct or not.

Heather also commented, "Throughout the coin problem, I did not provide enough wait time and found myself explaining too much instead of letting her explore for a longer period of time."

Anna's submission contained the following reflection, "I have a tendency to just say 'good job that's the right answer' when a student solves a problem instead of letting them support their claim and prove to me that it's right". There were 53 of 120 instances where PTs stated that the goal was to focus on student thinking.

Code 3: Respond Using Student Thinking

We posited our PTs would have the opportunity to develop their abilities in using student thinking to craft responses and pose new problems based on their interactions with students. Prior to engaging in the ESST activity, PTs had read and discussed the Jacobs and Ambrose (2008) article, "Making the Most of Story Problems," which outlines a set of responding moves teachers can make to "support a child's thinking before a correct answer is given" (p. 261) or to "extend a child's thinking after a correct answer is given" (p. 263). There were 128 instances of PTs describing evidence coded as "respond using student thinking." These 128 units were further differentiated as either "action" or "reflection" using sub-codes. Units were coded as action if a PTs' response to a student demonstrated evidence of using student thinking. Units were coded as reflection if PTs retrospectively offered examples of how they might respond if faced with a similar situation in the future.

Action

Lawson's written reflection provided an example of using student thinking in his response:

> He seemed to be struggling conceptualizing the problem in his head so I attempted to help him by offering smaller numbers for him to work with. I had him solve the problem at hand using single digit numbers. Landon was able to solve the problem using single digit numbers. Next I had him return to the original problem and use the base 10 blocks to help him work through the problem.

Lawson discusses his responses during the interaction with the student and his goal of responding to the student's lack of strategy for the given task. He knows posing a simpler version of a problem can allow students to find a strategy they can apply to the larger numbers in the original task. Susan provided the following example of an action she took while responding to student thinking: "After she quickly solved the original problem, the extension I chose for Madison was still too easy for her. I decided to try asking her to write a number sentence representing the story." We identified 29 instances of PTs taking action during their session by responding to student thinking.

Reconsideration

Georgia's reflection on her interaction with a student caused her to reconsider her teaching moves. She wrote

> I could have done a better job at asking him to explain his process and how he knew that he needed to subtract. I just assumed that he knew how to do the problem, but it would have been helpful to understand his thinking and what words in the problem let him know that he needed to use subtraction. I also, could have had him do an extension problem with harder numbers to be able to observe his thinking with more difficult numbers.

Georgia's reconsideration of when it might be appropriate to pose an extension problem based on the interaction she had with the student is one example of how PTs reconsidered their interactions as a result of reflecting on the activity. Landon also reconsidered his actions with his student,

> I should have worded my scaffolding questions to him differently, and that maybe I was confusing him to impacting [sic] his mistakes. Instead of saying 'take away the smaller number form the bigger number', which is why he put the smaller number on top of the bigger number in his solution path, I could have said 'you have the bigger number and you want to take away the smaller number from it.' This

> could have prompted him to complete his solution path without any confusion.

Landon reflected on his interaction with the student and how he could have responded differently to help the student better understand the problem. There were 99 instances where PTs focused on a reflection, each of which posited alternative choices that could have been made in the moment or perhaps considered in the future.

Code 4: The Importance of "Unpacking" a Task for Students

One theme that emerged from our reading of the data was PTs' attention to the importance of "unpacking" a task for students. Prior to their work on the ESST activity, PTs had experiences in our course pertaining to problem posing, which asked them to consider how to (a) select a problem type and number choice, (b) choose a problem context, and (c) unpack a problem for students. The following excerpt from our course reading informed our class definition of unpacking a problem,

> After the problem has been developed, it is time to present it to your students. The key during this phase of problem posing is to ensure that all students understand the problem well enough to get started. You want to do this without telling students how to solve the problem or even leading them towards a particular solution strategy. (Drake et al., forthcoming, p. 59).

PTs watched three video examples of teachers introducing problems to their students and each were discussed in terms of what PTs noticed about what the teachers did and did not do while introducing the problem to their students.

Upon watching their video PTs often reflected on their interactions with students and identified the role of unpacking the problem for their student before allowing the student to explore. In one session, Haley worked with two students together because of an unbalanced number of PTs and students. She reflected on how she unpacked a problem for her students,

> Unpacking the problem is one thing that I could have done significantly better - making sure the students understood what the story was about and then the method used to solve it. I worked with two students and found that one was on a much higher math level than the other student causing me to work and speak toward the upper level student in the beginning and the lower student as we began working. When I saw that the higher student understood the problem I was quick to move on without checking the lower students understanding. Although I moved on from unpacking the story quickly I think that both could have benefitted from a more in-depth explanation.

While we note Haley viewed one point of unpacking a problem to ensure students understood "the method used to solve it," she also focused on the idea of making sure students understood what the problem was asking. Eloise provided another reflection on the importance of unpacking a problem, "I want to work on 'unpacking the problem' more. I think it's beneficial for students because it really sets them up correctly for the problem and gives them the most help to complete the problem." Leah also arrived at this conclusion concerning the importance of unpacking a problem, "To determine if Hayley's actions were due to my lack of setting up the problem, I altered my initial unpacking a problem method by expanding my question types to ask who, what, when, where, why, and how". There were 58 instances of PT's commenting on unpacking problems for students.

Code 5: Evaluating or Labeling Students Based on Minimal Evidence

A second theme that emerged from our reading of the data was PTs' tendency to evaluate or label students after a minimal time of working with the student. There were several instances where preservice teachers broadly evaluated a student based on minimum experience. For example, Anna wrote, "I would classify Landon as an above average math student who understands most concepts but gets through problems by going through the motions and performing the standard algorithm." Anna had minimal experience working with Landon, but was quick to classify him as an above average math student based on little observation time. Margaret evaluated her student after working through a few problems: "I think one of the reasons for this was the fact that my student was very smart and she knew how to do all of the problems, and she for the most part solved them all correctly on the first try." Eloise's response compared the ability of her two students,

> On my first day [at the school], I had a boy who was very good at math…On my second day, I had a girl who was not as good at math. She knew her math facts pretty well and kept telling me she had a 99% in math. I think this is interesting because she struggled going through some problems, but apparently does really well in math.

There were 16 instances where preservice teachers worked with their student on a series of problem and then labeled the student's mathematical ability from this interaction. This was a finding from Crespo's (2000) work as well.

Discussion and Implications

In the examination of our findings, in light of the synthesis of prior scholarly inquiry on dynamic interactions, we see confirmatory evidence that supports the relationship between: (a) the identified facets of the activity which were embedded within the design of the ESST activity and (b) the hypothesized outcomes gleaned from across the prior studies. In and of itself, this connection informs mathematics teacher educators (MTEs) of one approach in activity design of dynamic interactions that can engender particular learning outcomes for PTs. We can draw several conclusions about the design of the activity and its potential to foster the three outcomes suggested by the literature: *knowledge of student thinking*, *obtain correct answers/support student thinking*, and *respond using student thinking*. As presently constituted the evidence suggests the ESST activity afforded PTs the opportunity to demonstrate and develop general knowledge of students' thinking (e.g., A third-grade student may not think to make fractions in the sharing problem with the leftover object), which was suggested by the literature (Ambrose, 2004; Crespo, 2000). The ESST activity did not however afford PTs the opportunity to provide specific interpretations of students' mathematical abilities. (e.g., In a sharing situation where there are less objects than people, students tend to partition each object so that each of the n people get 1/n of each object to be shared). The activity did seem to provide the opportunity for PTs to consider and perhaps reconsider their role of listening to and supporting student thinking, echoing similar findings by Crespo (2000), Jenkins (2010), and Moyer and Milewicz (2002). Similar, to Crespo's findings (2003), our PTs' demonstrated evidence of their abilities to leverage student thinking in crafting responses to students as well as when posing subsequent problems.

Beyond evidence for the three themes the literature suggested could result from such an activity, we also identified two additional themes: *the importance of unpacking* and *labeling students based on minimal evidence*. We see these themes as areas we need to pay explicit attention to in our course prior to our PTs work with students. Unpacking, or "launching," tasks is an important practice for novice teachers to develop. Unpacking can involve activating prior knowledge, ensuring student understanding of the task, establishing expectations, or removing barriers to engagement without lowering cognitive demand (Van de Walle et al., 2016). The importance of effective launching is well known and the participants in the study recognized how crucial it was to supporting student success. However, continued explicit attention to what constitutes an effective launch is something to be addressed within the context of this activity in methods courses. Research demonstrates teachers often struggle to provide support that provides access without removing opportunities for mathematical thinking from students (Gonzalez & Eli, 2017). One idea we see as important for further, future examination is if a teacher's launch of a task indicates if her/his underlying goal is to elicit and understand student thinking or places an emphasis on arriving at a correct answer. Based on the ways in which our prospective teachers contextualized their experience within the effectiveness of their launch of the problem suggests launching may be a fruitful area of future study.

PTs' reflections after their second session still focused on ways they could improve when working with students the next time. This suggests that, at the conclusion of the experience, there was a mismatch between enacted practice and desired practice that could provoke further pedagogical knowledge development. Thus, we propose two changes to future enactments of the activity which we believe could improve it. First, would be to extend the activity beyond two sessions. Second, we believe it would be useful to have PTs interact with a variety of age levels and age appropriate tasks in these subsequent sessions. Obviously, more experience working with different age students would afford PTs opportunity to continue to develop their skills in eliciting and supporting students' thinking in a variety of contexts. Expanding the types of tasks posed would afford PTs the chance to develop knowledge of student thinking in a variety of different content areas and grade levels.

To develop the existing, but limited, knowledge base on dynamic interactions requires MTEs to engage in the cycle of scholarly practice and scholarly inquiry. A concerted effort by MTE researchers to conduct and share confirmatory studies from a variety of contexts, as well as original investigations into these types of activities, will allow MTEs to leverage this knowledge in their own methods and field-based courses. The ultimate result of these efforts should be PTs who are committed to the idea that teaching mathematics is about listening to and understanding a student's current conceptions and responding in a manner which supports their development.

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