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Preservice Teachers' Conceptualizations of Mathematical Tasks

Bima Sapkota

This study reports how 12 secondary mathematics preservice teachers (M-PSTs) described characteristics of mathematical tasks after participating in instructional activities, including reading, reflecting, and discussing task characteristics from two mathematics task frameworks and related book chapters. The findings demonstrated that after engaging in these activities, M-PSTs used formal, research-informed language in a way that highlighted student-related factors (e.g., student prior knowledge) and contextual factors (e.g., class time constraints), suggesting that the language and concepts offered through the task frameworks initiated M-PSTs' nuanced task descriptions. An emergent task framework, developed through the literature synthesis and data, as well as a series of instructional activities are proposed to enhance M-PSTs' skills to negotiate multiple task-related factors during task selections.

Providing preservice teachers (PSTs) with opportunities to engage in instructional activities related to mathematical tasks (hereafter "tasks") influences the ways they select and describe tasks (Anhalt & Cortez, 2016; Anhalt et al., 2006; Crespo & Sinclair, 2008; Lee et al., 2019; Norton & Kastberg, 2012). In particular, PSTs' task selection and description are influenced by instructional activities that involve reading about task characteristics, as well as reflecting on and discussing those characteristics. These instructional activities alter how PSTs consider mathematical and pedagogical factors in their task selection and description. For example, after engaging in related activities, PSTs emphasized the aesthetic value of tasks (Crespo & Sinclair, 2008) and selected problem-solving tasks as opposed to memorization tasks (Anhalt et al., 2006). As such, the growing body of research in the area of PSTs' task selection and description has suggested that PSTs often focus on particular task characteristics that are highlighted in readings, including

***Bima Sapkota** is a PhD candidate at Purdue University. Her primary research focuses on practice-based teacher education, and discipline-specific teaching and learning. She investigates how mathematics preservice teachers develop discipline-specific practices when they engage in practice-based instructional activities.*

task frameworks, and discussed during instructional activities in teacher education programs.

However, inquiry into how a series of instructional activities involving reading about, reflecting on, and discussing several task characteristics, from multiple mathematics task frameworks (hereafter "task frameworks") might influence secondary mathematics preservice teachers' (M-PSTs) task description has not yet been conducted. As mathematics teachers need to examine multiple factors (e.g., difficulty level of tasks, student age) during task selection, providing M-PSTs with only a single task framework might limit opportunities to examine multiple factors influencing their task selections. Thus, engaging M-PSTs with multiple task frameworks prepares them for future teaching. In this study, I investigated M-PSTs' descriptions of tasks following their engagement in instructional activities related to the following two task frameworks: (a) Stein et al.'s (2000) Task Analysis Guide and (b) Leinwand and Wiggins's (1991) Criteria for Performance Tasks. I report M-PSTs' complex and in-depth task descriptions as they considered a wide-range of task-related factors after engaging in these activities.

Literature Review

In this section, I synthesize extant literature that describes how (a) PSTs' conceptualizations of task characteristics enrich their teaching practices, and (b) task-related instructional activities influence PSTs' task selection and description.

Describing Task Characteristics: An Important Mathematics Teaching Practice

The National Council of Teachers of Mathematics (NCTM, 2014) identified task selection and implementation as a key mathematics teaching practice. They proposed that effective mathematics teachers use tasks to motivate students and to develop their mathematical reasoning. Here, "tasks" refer to mathematical problems and associated instructional activities that teachers select and/or construct to create a student learning environment (Stein et al., 2000). All tasks do not provide students with the same opportunities to engage in reasoning and problem solving (Henningsen & Stein, 1997); therefore, PSTs need the skills to analyze task characteristics to ensure purposeful selection of tasks that foster students' problem solving and reasoning in their future classes (NCTM, 2014). Task selection includes selecting tasks from the curriculum and "setting up" (e.g., modifying if needed) those tasks for implementation in the classroom. Teachers should consider several factors (e.g., difficulty level of tasks, students' grade levels) while selecting tasks. The type of task can affect the learning environment (e.g., students explore solutions vs. teachers provide solutions) and the mathematical knowledge students develop (e.g., procedural vs. conceptual knowledge) (Hiebert & Wearne, 1993; NCTM, 1991). By examining PSTs' task selections and descriptions, mathematics teacher educators can investigate the factors that PSTs highlight while selecting tasks (Crespo & Sinclair, 2008).

Influences of Instructional Activities on PSTs' Task Selection and Description

Previous research has suggested that different task-related instructional activities have a range of influences on PSTs' task selection and description. Those activities include constructing lesson plans and discussing the tasks included in the lesson plan with colleagues (Anhalt et al., 2006); reading about and reflecting on theoretical literature (Lee et al., 2019); categorizing tasks using a task framework and discussing task characteristics (Arbaugh & Brown, 2005), and discussing particular task features (e.g., the aesthetic value of tasks; Crespo & Sinclair, 2008). These activities, which included reading about, reflecting on, and discussing specific task characteristics, contributed to changes in how PSTs negotiated mathematical (e.g., selecting tasks to build on students' reasonings) and pedagogical (e.g., addressing students' grade levels) aspects associated with tasks (e.g., Crespo & Sinclair, 2008; Lee et al., 2019). Those alternations were influenced by the specific task characteristics highlighted in the activities.

Studies have indicated that PSTs began to prioritize student-related factors, such as students' prior knowledge, and contextual factors, such as time constraints, once they had analyzed and discussed task characteristics from theoretical literature and task frameworks. For example, Lee et al. (2019) found that M-PSTs focused on "mathematical ideas that students were supposed to inquire, and whether those were appropriate for the students' cognitive level tasks" (p. 982) once they engaged in "noticing activities." These noticing activities included analyzing textbook tasks using a framework, finding alternatives for the textbook tasks, and modifying textbook tasks based on mathematics teaching and learning theories. These findings highlight the significance of opportunities for M-PSTs to discuss and reflect on the characteristics of tasks. In related work, Anhalt and Cortez (2016) found that M-PSTs conceptualized how mathematical modeling tasks were connected with real-life situations as they progressed through modeling activities, including reading and discussing modeling task characteristics from the literature and creating and reflecting on modeling problems.

Arbaugh and Brown (2005) found that after engaging with the levels of cognitive demand from the Task Analysis Guide (Stein et al., 2000) and performing task-sorting activities (i.e., using task frameworks to sort tasks), mathematics teachers were able to explain how their students would engage with mathematical tasks in their classrooms, suggesting that the teachers identified that different mathematical tasks create different learning opportunities for students. Before their engagement in those activities, teachers highlighted procedures required to solve tasks. This finding indicated teachers' attention shifted from task procedures to reasonings presented in the tasks after learning about and discussing task characteristics. Crespo and Sinclair (2008) found that elementary PSTs emphasized mathematical aesthetic values of tasks after discussing mathematical aesthetic criteria of tasks, suggesting that PSTs' task descriptions mirrored the task characteristics they had discussed.

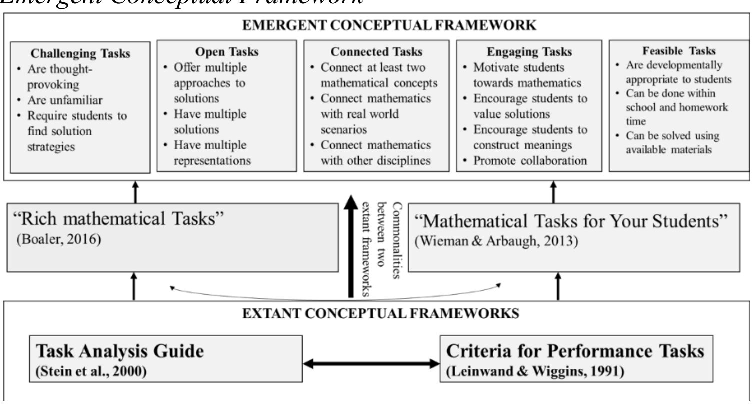
Studies have also shown that PSTs negotiated student-related factors through task-posing activities, which included designing/selecting tasks and analyzing students' responses. Crespo (2003), for example, found that after PSTs posed tasks for their students and received feedback through letter exchanges, they started selecting open-ended and cognitively demanding tasks. They began to consider tasks that fostered students' reasoning skills instead of trying to choose procedural tasks that would lead to students producing correct answers. Norton and Kastberg (2012) also found that secondary M-PSTs began examining student-related factors after they posed tasks and exchanged feedback with students through letter writing.

In summary, existing research has suggested that PSTs use the language and concepts from task-related literature to select tasks and to justify their selections after they reflect on and discuss specific task characteristics from a single task framework or from the literature highlighting a specific task characteristic. However, focusing on only one framework might limit PSTs' opportunities to broadly conceptualize factors that should be considered during task selection. Thus, there is a need for further research that investigates which factors PSTs consider once they have opportunities to learn and use task characteristics from instructional activities related to multiple task frameworks. To address this need, I designed a sequence of instructional activities that invited M-PSTs to read two distinct task frameworks and then reflect on and discuss task characteristics from both frameworks. As such, I provided M-PSTs with opportunities to engage with the following two task frameworks: Stein et al.'s (2000) Task Analysis Guide and Leinwand and Wiggins's (1991) Criteria for Performance Tasks. In addition, I encouraged them to use these frameworks to describe tasks. I extended current research related to developing M-PSTs' conceptions of a specific task characteristic to examining the ways in which M-PSTs consider and describe multiple task characteristics while justifying their task selections.

Conceptual Frameworks

In order to identify task characteristics in M-PSTs' task descriptions, I developed an emerging conceptual framework (see Figure 1). The framework initially drew upon four related sources: (a) Stein et al.'s (2000) "Task Analysis Guide;" (b) Leinwand and Wiggins's (1991) "Criteria for Performance Tasks;" (c) Boaler's (2016) "Rich Mathematical Tasks;" and (d) Wieman and Arbaugh's (2013) "Choosing Mathematical Tasks for Your Students." The framework descriptors were later revised using the data. Stein et al. (2000) proposed task categories based on the cognitive demands of tasks: memorization, procedures without connections, procedures with connections, and doing mathematics. Wieman and Arbaugh (2013) introduced eight criteria, such as essential vs. tangential, rich vs. superficial, that compared the characteristics of worthwhile tasks with those of "less worthwhile tasks. Boaler (2016) and Wieman and Arbaugh (2013) described task characteristics similar to those proposed by Stein et al. (2000) and Leinwand and Wiggins (1991).

Figure 1
Emergent Conceptual Framework



Note. The two sources on the bottom row are the task frameworks. The sources in the middle row are the book chapters that illustrate task characteristics presented by the frameworks. The boxes on the top row present task categories and their descriptors derived from all four sources. The horizontal arrows between boxes indicate that the two frameworks and the book chapters present some common task features. The vertical arrows indicated that the sources in upper-level boxes are derived from the lower-level boxes.

Given this, I identified the common task features discussed by all four author teams. For instance, I used the following features from the four sources to develop the descriptors for challenging tasks: (a) high-level cognitive demand tasks challenge students' thinking (Stein et al., 2000), (b) thought-provoking (Leinwand & Wiggins, 1991), (c) do not suggest pre-proposed problem-solving strategies (i.e., students find problem-solving strategies/unfamiliar; Boaler, 2016), and (d) foster student reasoning (Wieman & Arbaugh, 2013). Here, Stein et al.'s (2000) description of a task requiring complex non-algorithmic thinking, was represented by descriptor "thought-provoking" in the challenging tasks category. This process of synthesizing multiple sources to generate task descriptions arguably allows researchers to develop nuanced ways to describe task characteristics.

The five task categories are not mutually exclusive. For example, open tasks are often engaging (i.e., interesting to students; Boaler, 2016). The levels of cognitive demand should be evaluated in the context of student factors such as prior knowledge because the feasibility of tasks influences the challenge of tasks (Stein et al., 2000). However, by attempting to retain the main theme of each source, I developed descriptors for the task categories to differentiate among them. The descriptors of each task category are unique yet closely related to each other. For example, thought-provoking tasks (a descriptor for challenging tasks) often require students to construct meaning (a descriptor for engaging tasks). Here, the two descriptors convey distinct meanings, but the same task can be characterized by both the descriptors. The task categories were not exhaustive representations of all possible types of tasks; instead, they were the most frequently discussed in all four sources. For instance, Leinwand and Wiggins (1991) proposed that worthwhile tasks use discipline-appropriate solution methods; I did not include this feature in the framework as the other three sources did not highlight this feature.

Methods

In this study, I investigated how a group of M-PSTs described task characteristics after engaging in a series of task-related instructional activities. The following question guided this study: After M-PSTs had opportunities to read about, reflect on, and discuss task characteristics from task frameworks and book chapters, how did they describe the characteristics of mathematical tasks? The instructional activities (see Table 1) included (a) reading about task characteristics from two task frameworks and two book chapters; (b) describing the characteristics of given tasks; (c) sorting tasks using given frameworks; (d) sharing individual reflections on task characteristics with the class; and (e) selecting and describing tasks.

Table 1
Descriptions of the Instructional Activities Related to Mathematical Tasks

Days	Activities	Descriptions of the Activities
Day 1	Reading task-related literature	M-PSTs read "Choosing Mathematical Tasks for Your Students" (Wieman & Arbaugh, 2013) and "Rich Mathematical Tasks" (Boaler, 2016).
	Reflecting on and discussing task characteristics	M-PSTs individually solved Martha's Carpeting Task and the Fencing Task (Stein et al., 2000). They explored task characteristics from these two different (procedural vs conceptual) yet related tasks (i.e., both were word problems included the concept of area and perimeter). M-PSTs individually completed an instructor-directed written reflection on the tasks (i.e., they described task characteristics based on the given prompts). M-PSTs shared their reflections with the class. M-PSTs were assigned in small groups to reflect on the readings based on the following questions: What are the main ideas discussed in the readings? What are your insights from the readings about the significance of task selections in mathematics teaching and learning? M-PSTs shared their responses about the tasks that they solved on Day 1 (in relation to the readings) in their groups and later with the class.

Day 2	Reading a task framework	M-PSTs read "Analyzing Mathematics Instructional Tasks" (Stein et al., 2000).
	Reflecting on and discussing task characteristics	M-PSTs shared their thoughts about the reading with the class in relation to Martha's Carpeting Task and the Fencing Task. Some guiding questions included: How does a particular task feature indicate levels of cognitive demand? How do you explain the student learning opportunities provided by the tasks? M-PSTs collaborated in the "Task Sorting Activity" and the follow-up discussion (Smith et al., 2004, Stein et al., 2000, p. 18)
Day 3	Reading, reflecting on, and discussing a task framework	M-PSTs individually read "Criteria for the Performance Tasks" (Wieman & Arbaugh, 2013), completed an instructor-directed written reflection, and shared the reflection with the class.
	Selecting and describing a task	Each M-PSTs individually selected a task and described task characteristics based on the following prompt: Find/construct a high-school task you deem worthwhile. Write a one-page (minimum) description of the task, providing the general rationale on why the task was appealing to you for your teaching. You can use two frameworks to justify your task selections.

Note. The shaded activities are the data sources (i.e., initial and final data) for the study.

The activities (a), (b), and (c) were inspired by Arbaugh and Brown's (2005) activities, in which PSTs sorted given tasks using Stein et al.'s (2000) task categories and subsequently discussed the task characteristics. The activity (d) was similar to Crespo and Sinclair's (2008) interventions, in which PSTs read and discussed task characteristics. The activity (e) was inspired by the activities in Crespo's (2003) and Norton and Kastberg's (2012) studies, in which PSTs posed tasks through letter-writing exchanges with students and reflected on the effectiveness of those tasks. Although my study did not include task-posing activities, it involved opportunities to reflect on the effectiveness of selected tasks by analyzing task characteristics.

Context and Participants

The participants were 12 M-PSTs—six males and six females—who were enrolled in a secondary mathematics methods course at a large midwestern university. All of the M-PSTs were in the same section, in which I was one of two course instructors. M-PSTs were required to take the methods course in two semesters prior to their student teaching. This course was M-PSTs' first mathematics education course in the program. Prior to this course, they completed several mathematics content and general education courses. As an instructor of the course, my role was to design and enact the instructional activities so that all students experienced the same curriculum. I facilitated the discussions by prompting with guiding and follow-up questions. M-PSTs' individual descriptions of selected tasks during Day 1 and Day 3 activities (i.e., beginning and end of the activities) were the data for the study (see shaded area in Table 1) to analyze the shifts in M-PSTs' task descriptions from the beginning to the end of the activities.

Data Analysis

The focus of the study was not to analyze how an individual M-PST selected and described task characteristics, but rather to gain insights into shifts in the group of M-PSTs' descriptions of tasks. Thus, following a single case study approach (Yin, 2017), the task descriptions generated by 12 M-PSTs were considered the unit of analysis even though data were collected from each M-PST. Despite the decision to focus on the whole group rather than individuals, all data sources were de-identified. I used both bottom-up (identifying codes from the data) and top-down (categorizing the codes using a theory or framework) interactive modes of analysis (Chi, 1997). Initially, I used the bottom-up approach because I aimed to incorporate all the task characteristics highlighted in M-PSTs' task descriptions. Using a descriptive coding method, I identified codes based on the perceived meaning of the text rather than the exact phrases used in M-PSTs' task descriptions (Saldaña, 2016). For instance, one M-PST described, "I chose [this task] because students could learn a concept instead of simply doing a calculation." I coded this response as "build students' conceptual understanding" (see Table 2). To capture all possible meanings from a single instance, I used simultaneous coding methods (Saldaña, 2016). For instance, one M-PST described, "This task requires students to reason through the process of how to go from having two unrelated pieces of information to solving one for a new piece of information." I coded the phrase "students have to reason through…to solve for a new …information" as "student reasoning," and the phrase "two unrelated pieces of information" as "connecting multiple areas of mathematics." The simultaneous coding method allowed me to challenge potential bias with the single and personal interpretation of M-PSTs' task descriptions. Next, I used the emergent conceptual framework (see Figure 1) to categorize codes (a top-down approach) because I aimed to report the shifts in M-PSTs' initial-to-final task descriptions in terms of the five task categories. Since I constructed the framework by identifying common, unifying concepts and descriptors from four extant sources, the framework served as an existing source for the categorization of codes.

Table 2

Codes and Categories Identified from M-PSTs' Descriptions of Task Characteristics

Task Categories	Codes Emerged from the Initial and Final Data
Challenging Tasks	Require students' independent and higher-level thinking/critical thinking
	"Students will not be able to figure out this task from memory but will need to test out hypotheses"
	Build students' conceptual understanding
	"I think that this activity could deepen students' conceptual understanding about what a factor is"
	Are unfamiliar to students/No formal pre-proposed task-solving strategies
	"It is unusual and likely different from what high school students have seen about circles before."
	Require students to communicate solutions
	"This activity includes cognitive efforts because students must explain what they are doing."
	Promote students' reasoning and problem-solving abilities

	"Students must reason through constructing an equation as they are not given the formula."
	Require students to find patterns
	"They have to do some nonalgorithmic thinking in order to find patterns."
Open Tasks	Can be solved using multiple task solving strategies
	"I think students have a variety of ways to get to that answer."
	Have multiple answers/solutions
	"This problem has multiple solutions that students can talk in the class."
	Have multiple representations
	"Students are using different representations (symbolic with the equations and graphical with graph)"
Connected Tasks	Connect various mathematical concepts/areas
	"I can talk about the relationship between surface area, volume, perimeter and things of this nature."
	Connect with advanced mathematical Concepts/raising other possibilities
	"Is there a relation between the surface area and the volume? This would be a good question to pose."
	Offer varied cognitive demands of tasks
	"First part is in low-level demand as we easily re-create the figure but the task itself is a high-level."
	Connect mathematics with other disciplines and real-life situation
	"The task has a geographical component; so, it connects math with geography"
	Connect students' current knowledge with their prior knowledge
	"It should only use concepts that they have already learned, but still challenge them."
Engaging Tasks	Excite/Interest students
	"More importantly, though it had an AHA moments and those bring people closer to math."
	Perceived as valuable by students
	"Students who do not like math may not see a purpose in completing this activity."
	Promote collaboration
	"The task is active because students could work on the activities in pairs."
	Require students to be decision makers
	"Students make big decisions when constructing the initial cube, which give them ownership"
	Require students to construct meaning
	"This task is very interesting as students construct meanings, which will give them a joy."

Feasible Tasks	Are doable in school and for homework "This task is safe and appropriate for students to work on during class." Are appropriate for students' grade level "A high school student should be at the level where he or she can recognize this pattern." Are based on students' prior knowledge "Students find this task interesting because they already know about linear equation." Are technologically accessible "I am worried of technological part. It requires internet, and I know how finicky technology can be." Can be solved using available materials "It is a simple task. All is needed is paper and pencil to complete this task called Ice Cream Scoop."

Note. The shaded codes emerged from both the initial and final data sets, the unshaded codes were found in the final data set exclusively. The text in the quotation marks are examples of codes from the data.

Although the process of categorizing the codes was a top-down approach, I also revised the task descriptors of the framework based on these codes (i.e., a bottom-up approach). For example, the codes "are technologically accessible" and "can be solved using available materials" were not initially represented through the descriptors of feasible tasks. I categorized these under "feasible tasks" because these represented the characteristics of feasible tasks (i.e., easily accessible to students). Afterward, I added the following descriptor of feasible tasks in the conceptual framework: can be solved using available resources (see Figure 1).

To identify M-PSTs' shifts, I compared the codes and categories that emerged from M-PSTs' initial-to-final descriptions. "Feasible tasks" was the new category that emerged only from M-PSTs' final descriptions. Further, there were three new descriptors of "engaging tasks" that emerged only from the final data. Besides comparing codes and categories, I also compared the ways in which the codes emerged from M-PSTs' task descriptions. Using simultaneous coding methods, I identified multiple codes from the same instances of M-PSTs' task descriptions and assessed how those multiple codes correlated with multiple task categories. For example, I identified the following two codes from a sentence of a M-PST's task descriptions: "require multiple task solving strategies" and "promote collaboration." These codes aligned with the descriptors of open and engaging tasks, respectively. I found several similar instances wherein M-PSTs described open tasks as engaging tasks in the same sentence; thus, I reported that M-PSTs' task descriptions were complex as they explored the connections between multiple task categories. To identify the shifts in M-PSTs' use of frameworks, I initially recorded how often M-PSTs used the framework language in their descriptions. Then, I identified themes on how M-PSTs' task descriptions changed when they used the framework language. In the finding section, I did not report how often M-PSTs used formal language; instead, I reported the identified themes from M-PSTs' use of language in their task descriptions.

For inter-rater reliability, a second coder (a PhD student in mathematics education) coded 15% of randomly selected data. She independently coded the data after I provided her with a list of my identified codes. I instructed her to create new codes if she thought a different code would more accurately reflect the data. She did not create any new codes and we achieved 80% agreement between our codes. We discussed and resolved any differences.

Findings

Here I report how the group of M-PSTs described task characteristics at the beginning and at the end of their participation in the instructional activities. I use the terms "initial" and "final" responses/descriptions to refer to the responses recorded on Day 1 and responses from the conclusion of the instructional activities, respectively. First, I describe the overall shifts in M-PSTs' initial to final task descriptions. I then compare the factors that M-PSTs highlighted in their initial and final task descriptions under each task category. In addition, I explain the nuances in how M-PSTs described the relationships between different task categories.

M-PSTs' initial and final task descriptions varied in two key ways. First, M-PSTs produced a broader set of descriptors to describe open, connected, and engaging tasks in their final descriptions; that is, there were more codes associated with these three categories from the final responses than from the initial responses. Second, feasible tasks emerged as a unique category in the final data set. In their final task descriptions, M-PSTs highlighted student-related factors (as seen in Table 2: students' grade level and their prior knowledge), time-related factors (doable in school and for homework), and accessibility (technologically accessible, can be solved using available materials). The M-PSTs' use of language in their task descriptions shifted from informal to formal and research-informed language. The use of formal and research-informed language seemed to have assisted M-PSTs in offering a broad set of task descriptors, and such a broad set of descriptions was produced through M-PSTs' considerations of multiple task-related factors, such as cognitive demands of tasks, accessibility to students,. Below, I present the details of these changes by category.

M-PSTs' Descriptions of Challenging Tasks

Challenging tasks foster students' critical thinking as they do not suggest a path for problem-solving and require students to explore and communicate problem-solving strategies (see Figure 1). A notable difference between M-PSTs' initial and final descriptions was a shift to include student-related factors in their descriptions of challenging tasks. In their initial responses, M-PSTs described challenging tasks in terms of the length of tasks and suggested that tasks asking for short solutions do not require as much cognitive effort as those calling for longer solutions. For example, one M-PST mentioned, "In Martha's Carpeting Task, I didn't really make any judgment, I just did it. It was mental math." Similarly, another M-PST stated, "The Martha's Carpeting task is a single step rudimentary problem while the Fencing task is significantly more difficult and required more thoughts." Here, the M-PSTs assumed that the short task, which did not require writing, also did not require much thinking. As such, they did not consider the short task to be challenging. In their final descriptions, M-PSTs began negotiating students' prior knowledge and/or grade level while justifying their selections of challenging tasks. For example, M-PSTs wrote:

> There will always be students in my classroom that struggle with the material. While a "Doing Mathematics" task is the *ideal for most students, the students who struggle may require* [emphasis added] a little less open-ended.

> The task seemed both *complex enough for students to give it some thought and easy enough to students who are not comfortable* [emphasis added] with proofs in geometry.

> *The students who have a very strong understanding of the materials covered in class will have no problem with this task and the students who struggle* [emphasis added] will also be able to complete this task without too much issue as everything is in their own words.

In these responses, the M-PSTs recognized that students' prior knowledge (e.g., "students who are not comfortable with proofs," "students who struggle") needed to be considered when determining challenging tasks. M-PSTs' final descriptions of challenging tasks included the characteristics of feasible tasks (i.e., students' prior knowledge). Unlike their initial evaluation of a task either as easy or difficult (i.e., requiring long solutions or short solutions), these quotes show M-PSTs' complex task descriptions related to how a task can be more or less challenging depending on students' prior knowledge. This finding suggests a shift in M-PSTs' attention from task length to student-related factors while describing challenging tasks.

In both initial and final responses, the M-PSTs highlighted the characteristics of challenging tasks as their reasons for task choices; they asserted that tasks should invite students to reason, problem-solve, and build conceptual understanding. For instance, in their initial description, one M-PST argued that tasks building students' conceptual understanding are better than procedural tasks because procedural tasks require students only to calculate: "Students could learn a concept instead of simply doing a calculation." In the final response, another M-PST stated, "the beauty of the circle and the uniqueness of a constant radius isn't made apparent by those procedural tasks, which is why the task is so appealing to me. This task also presents concepts like the Reuleaux curve." Here, the M-PST justified their task selection by stating that the task offered students opportunities to engage in other/multiple concepts related to the circle. Furthermore, in their initial and final responses, M-PSTs highlighted that a task should require a considerable amount of cognitive effort from students. They did not use the phrase "cognitive effort" in the initial responses; instead, they used informal phrases such as "more thoughts" and "less straightforward:"

> "[Students] are actually required to do some *thinking on their own* [emphasis added]."
>
> "The first task we just had to plug into a formula, but the second task requires *more thought* [emphasis added] from the students."
>
> "The first task was more straight forward and only asked if I could find area when given l and w. The second task was less straight forward as we had to try and find l and w."

They used the phrase "less straight forward" to explain that the task did not suggest formal pre-proposed task-solving strategies. This description was somewhat vague because M-PSTs used informal language and could not unpack the characteristics of challenging tasks such as what it meant to be less straightforward. In the third quote above, the M-PST mentioned that the "task was less straight forward …we had to …try and find l and w" without explaining what they meant by trying to find l and w. This could refer to the process of testing a hypothesis or inductively identifying several values of l and w.

M-PSTs continued to mention students' independent thinking in their final descriptions, but they began using the language from the task frameworks, indicating that M-PSTs' task descriptions shifted from initial informal language to the use of formal and research-informed language at the end of the activities. Their use of formal language allowed them to offer complex and in-depth descriptions by unpacking several task characteristics and exploring the connections between those characteristics. M-PSTs explained that high-level cognitive demand tasks considerably challenge students who are solving them. For instance, M-PSTs used language and concepts from Stein et al.'s (2000) framework to describe how a task was challenging for students:

> I consider this task as challenging. They have to do *nonalgorithmic thinking* [emphasis added] to solve the task by *using their reasoning* [emphasis added].

> This task is *procedures with connection* [emphasis added]. While ideally this task would be doing mathematical tasks, I cannot say that it is because to take the step from *procedures with connection into doing mathematical tasks* [emphasis added] it is important to require students devise their solution strategies.

Here, the M-PSTs were able to offer clear and complex task descriptions by unpacking several characteristics of challenging tasks using formal language such as nonalgorithmic thinking, need to use reasoning, and procedures with connections. For example, in the second response, the M-PST asserted they did not consider the task as the most challenging (i.e., doing mathematics) because it did not require students to formulate problem-solving strategies.

M-PSTs' Descriptions of Open Tasks

An open task can have multiple problem-solving approaches, multiple representations of the problem/solution, and/or multiple answers (Figure 1). Some open tasks have only one answer but contain multiple solution approaches, while others have only one effective solution approach with multiple representations. Only two responses in the initial data set described open tasks:

> It allows for *multiple different perspectives* [emphasis added] for how to solve the problem.

> I choose the [Fencing] task because it *gives them choices* [emphasis added] to make.

The first response clearly indicates multiple ways to solve the problem, but the second response is less clear. The M-PST mentioned "more choices," but did not clarify what they meant by "more choices." More choices could refer to multiple task-solving strategies or multiple representations. In their final responses, the M-PSTs described open tasks using three descriptors (multiple answers, multiple representations, and multiple solving strategies) and evaluated multiple factors associated with open tasks. For instance, one M-PST stated, "while the task seems open-ended there is only one answer, once found the students could move on." In this response, the M-PST mentioned the task has some characteristics of open tasks but is missing other features because it has only one answer. Another M-PST responded, "the second portion of the task is a maximized equation…students have a variety of ways to get to that answer…and is open to the students to ask deeper questions on the consistency of the answer." Here, the M-PST clearly mentioned the task has multiple task-solving strategies. Another M-PST mentioned, "This task is open. Students can represent the difference of cubes formula in figure, but it can also be solved other ways." In this response, the M-PSTs considered multiple facets associated with the openness of tasks, including multiple representations, and multiple ways to solve a problem. These responses suggested that framework language (i.e., multiple representations) allowed M-PSTs to provide more nuanced and clear descriptions of task by unpacking several task descriptors.

The other significant factor that appeared in the final description is that M-PSTs began interpreting open tasks as engaging tasks, mentioning that open tasks create productive classroom discussion as students can share various task-solving strategies and answers:

> "It is pretty *open-ended* [emphasis added], and students could figure out their results in different ways. I think it would also *lead to good class discussion.* [emphasis added]"

> "Students can *interact and work with others* [emphasis added] to solve, it is *up to students on the routes they take and ideas they implement* [emphasis added] to come to the solution."

In these responses, M-PSTs suggested that exchanging different problem-solving strategies creates a collaborative learning environment. As such, M-PSTs acknowledged that their choice of tasks influences the classroom environments that they create. For example, selecting open tasks would lead to a productive classroom discussion. This indicated that M-PSTs considered student-related factors such as student engagement while describing open tasks.

M-PSTs' Descriptions of Connected Tasks

Connected tasks provide students with opportunities to make connections between (a) multiple mathematical concepts (e.g., area and perimeter) or multiple areas of mathematics; (b) different disciplines; or (c) mathematics and real-world scenarios. M-PSTs stated only the first type of connections in their initial descriptions. For instance, one M-PST stated the connections between two concepts (area and perimeter), writing "I would choose Fencing Task because students have to use knowledge about area and perimeter to establish connections between them." The M-PSTs also described how the tasks could build students' competency in multiple areas of mathematics. For example, one M-PST shared, "while the task is based in geometry, I thought this would be a good, 'in-between' task; you could try to recreate the rectangle geometrically and show the sides algebraically." Here, the M-PST asserted that the task was good because it included both geometry and algebra.

In their final responses, M-PSTs continued to highlight the above-mentioned factors while also differentiating between tasks that were based on students' prior knowledge and those that were not. They argued that a task is connected if it connects students' current knowledge with their prior knowledge and experiences. As such, they asserted that it is necessary to assess students' prior knowledge to decide if a task is a connected task.

For example, one M-PST stated, "students who are having difficulty in the class can treat it as a 'Procedures without connections' task while a student who has a stronger grasp on the conceptual sides will be able to make connection, so for them it will be a 'Procedures with connections' task." Another M-PST stated connected tasks help students to explore connections between their prior knowledge of linear function and the concept of quadratic functions: "this task helps students to explore how their prior knowledge on linear function is related to quadratic function."

In the final task descriptions, M-PSTs also stated the utilitarian values of mathematics. They described tasks as a means of connecting mathematics with real-world situations and other disciplines: "the task has a geographical component to it. So, this task may be useful to students who change the 'background' from math to geography." Here, M-PSTs highlighted students' learning experiences in different disciplines, such as mathematics and geography, in a way that recognized connected tasks can be a useful means for students to explore other disciplines. Another M-PST mentioned: "this task is valuable because it allows students who have not yet mastered the skill of graphing a polynomial to continue to work on their skills, ultimately making it easier for them to graph a quadratic or higher degree polynomial." Here, the M-PST valued a task because it helps those students who are still learning about graphical representations of polynomials because the task provides an opportunity for them to learn about higher degree polynomials. The M-PST described that connected tasks help students use their prior knowledge to learn advanced mathematical ideas.

M-PSTs' Descriptions of Engaging Tasks

Engaging tasks motivate students toward mathematics because they are interesting and exciting. M-PSTs, in both their initial and final responses, mentioned that tasks should be interesting and exciting to students. One factor that seemed to drive M-PSTs' initial and final task descriptions was the extent to which a task evoked students' curiosity, showing that they highlighted student-related factors in task selection. One M-PST, for example, responded, "I think students will find this task very interesting." Another M-PST responded, "I see this task as a good way to pique students' curiosity about all the formulas and equations they are learning." In the final activity, M-PSTs also began considering student collaboration as a determining factor for their task selection. They assumed that all tasks do not generate productive classroom discussions; thus, they chose tasks that could create opportunities for student collaboration. In addition, M-PSTs described open tasks as likely to be engaging for students because students get to exchange their answers and strategies: "It is very engaging as it also allows for a variety of methods for students to explain themselves and [supports] *collaborative work between students* [emphasis added]."

In addition to mentioning that a task needs to be engaging for students, M-PSTs' final responses also elaborated on why a task might be interesting to some students and not to others: "this activity highlights useful qualities of a circle, making this task engaging (versus just plugging in numbers, which is fun for some people, is uninteresting to others)." They also argued that tasks that connect mathematics with "real life" are more interesting to students. For instance, one M-PST responded: "[This task] is relatable to most students as they have played pool at least once in their life. This might not be interesting for others who have never seen pool." In some cases, M-PSTs' description of engaging tasks did not come on its own; rather it included descriptions of other task categories. For example, one M-PST mentioned that "this task is interesting to students as they have to reason that there are 6 sides so they must have to account for each side of the cube." Here, the M-PST mentioned that a task that invites students to reason (challenging tasks) is also an engaging task. Another M-PST mentioned, "the most appealing part of this task to me was that it was unusual and different from what high school students have seen about circles before." Here, the M-PST claimed that a task that is unfamiliar (i.e., challenging) is likely to be an engaging task (i.e., appealing). Further, M-PSTs stated that whether or not a task is interesting to students may depend on the students' grade level: "I chose this task because it could be interesting for *high school* *students* [emphasis added]." This finding indicated that M-PSTs considered students' grade-level in their final task descriptions as a way to evaluate engaging tasks.

M-PSTs' Descriptions of Feasible Tasks

Feasibility of tasks refers to what extent a task is accessible to students of a particular age-group with the available time and resources. Feasible tasks was the category that emerged only from the final data set (see Table 2), suggesting that M-PSTs did not consider feasibility in their initial task description. In the final descriptions, M-PSTs' conceptions of feasible tasks emerged in two ways. First, they considered feasibility as a determining factor for their task selection. For example, below, M-PSTs described time constraints, accessibility of resources, students' age/grade level, and students' prior knowledge of tasks as reasons for their task selections:

> [I chose this task as it] is feasible; it can be done within *the constraints and time limits* of a classroom.
>
> The Encyclopedia Mathematica will be a web-based assignment *allowing students to access it anywhere including the classroom.* [emphasis added]
>
> The task seems *feasible for an early geometry level* [emphasis added], but *it is also good for an algebra class that has been exposed to proofs*. [emphasis added]

Second, as reported in the above subsections, M-PST considered the feasibility of tasks a crucial aspect of determining whether or not a task is connected, engaging, and challenging. One M-PST, for example, responded, "I think that this task is feasible because it requires the students to think and be engaged, but it isn't too out of reach. A high school student should recognize this pattern based on the questions that are asked." In this response, the M-PST interpreted the task to be engaging because it was feasible (isn't too far out of reach). In addition, M-PSTs began attending to the features of connected, engaging, and challenging tasks to discuss feasibility. One M-PST mentioned, "This lesson is very feasible as it is conceptual, students don't have to do much computation, which allows them to have in-depth discussions about circles and also gives the teacher flexibility as to the timing of the lesson." The M-PST illustrated how a task that focuses on conceptual understanding is feasible, suggesting that the M-PST sees a challenging task as feasible.

In summary, M-PSTs' final task descriptions were in-depth, complex, and nuanced in two ways. First, M-PSTs described tasks in a way that covered multiple task categories simultaneously, such as open tasks can be engaging. Second, M-PSTs emphasized pedagogical factors (e.g., students' grade level) in their final task descriptions while continuing to highlight mathematical factors (e.g., cognitively demanding). The same codes were present in the initial and final data sets in the category of challenging tasks; however, there was a new category (feasible tasks) in the final activity. This is because M-PSTs described challenging tasks using the same descriptors in the initial and final descriptions, but at the end they also incorporated descriptions about feasible tasks. Specifically, M-PSTs included student-related factors (e.g., students' age) and contextual factors (e.g., doable in class time) in their final descriptions, indicating that their attention to task characteristics shifted from an emphasis on cognitive demand to student accessibility. Furthermore, M-PSTs attended to the features of connected, engaging, and challenging tasks while describing feasible tasks.

Discussion

In this study, I investigated how M-PSTs described task characteristics once they had opportunities to read about, reflect on, and discuss two task frameworks. The findings reported here suggest that after these opportunities, M-PSTs began describing task characteristics using formal and research-informed language and emphasizing student-related factors and contextual factors. Those factors included students' age, grade levels, prior knowledge and experiences, and interests, and the available time and resources required to solve tasks. I argue that M-PSTs' engagement with two task frameworks enabled them to consider these factors and thus be able to provide detailed and nuanced task descriptions. For example, during the class discussions, M-PSTs mentioned that after becoming familiar with multiple frameworks, they began to consider how a particular type of task such as challenging tasks could be relevant for only some students and in certain contexts.

The broad set of descriptors in M-PSTs' final task descriptions shows their consideration of multiple factors associated with a task. In the initial activity, M-PSTs were only exposed to two tasks (Fencing and Martha's Carpeting Tasks) and they considered limited factors to describe the two tasks in their initial descriptions. In contrast, they considered multiple factors to describe a task at the end of the activities. For example, M-PSTs considered whether a task could be appropriate for students of certain grades or whether it could be connected with other mathematics topics in their final descriptions. Arguably, M-PSTs' abilities to consider multiple task-related factors were not affected by the number of tasks they examined, but were associated with their exposure to task factors from multiple frameworks. The emergence of a broad set of descriptors from M-PSTs' final task descriptions not only indicated an increased number of descriptors they were considering, it also suggested that M-PSTs were able to offer complex and in-depth descriptions of mathematical tasks.

The use of formal language from the task frameworks allowed M-PSTs to produce a broad set of task descriptors and to provide more nuanced explanations for their task-related decisions. In the final activity, M-PSTs not only selected challenging tasks but also analyzed why those tasks were challenging for only some students, depending on students' prior knowledge and grade levels. Norton and Kastberg (2012), who found that some PSTs used the language drawn from course readings to justify task selection, cautioned that PSTs' use of formal language may suggest their attempt to follow the language used in the literature rather than their conceptualizations of the language. Similar to the PSTs in Norton and Kastberg's study, M-PSTs in my study used the language from the frameworks to describe tasks. However, I argue that M-PSTs' final task descriptions went beyond an attempt to follow the language from the frameworks. Instead, they used framework language to unpack their conceptions of multiple task-related factors and to explore the connection between the factors associated with multiple task categories. For example, M-PSTs described the characteristics of engaging and open tasks simultaneously. This indicated that they explored the relationship between multiple task categories, an explanation that would not have occurred if they were simply following the framework language.

In addition to engaging with the frameworks, the M-PSTs also engage in "task-sorting activities" and the follow-up discussions which possibly influenced the findings as these types of activities have been found to have an impact on PSTs descriptions and selection of tasks. For example, Stein et al.'s (2000) described how task-sorting activities raise teachers' awareness of the cognitive demand of tasks and encourage them to align their tasks with the goal of student learning. Arbaugh and Brown (2005) found that teachers began to anticipate students' classroom work while selecting tasks once they learned the levels of cognitive demand and performed task-sorting activities. During the class discussion that occurred during the study, M-PSTs shared different reasonings to justify their task-sorting. For example, some M-PSTs identified a task as high-level cognitive demand, arguing that students had to reason to solve that task. Others found the same task to be low-level cognitive demand, anticipating that students would memorize the task as they might not have developed the necessary reasoning yet. M-PSTs mentioned that these discussion sessions assisted them in noticing why they should consider students' grade-levels/age while selecting tasks. Thus, the discussion sessions and task-sorting activities provoked M-PSTs to consider student factors in their task descriptions.

M-PSTs' consideration of student-related and contextual factors to evaluate challenging tasks suggested that they appreciated the importance of cognitively demanding tasks (the concept from Stein et al.'s [2000] framework). Meanwhile, they realized the extent to which a task is challenging depends on contextual and student factors (a concept from Leinwand and

Wiggins's [1991] framework). These findings align with those of Lee et al. (2019) as well as Anhalt and Cortez (2016). Lee et al. (2019) found M-PSTs attended to student-related factors after reading and reflecting on related theories. Similarly, Anhalt and Cortez (2016) found M-PSTs' conceptions of modeling tasks were enhanced after reading and analyzing literature related to modeling problems.

Crespo and Sinclair (2008), who found that PSTs emphasized the aesthetic value of tasks in their task descriptions after discussing this construct in the class, provide support for the argument that M-PSTs in this study captured the essence of meanings from the discussion and reflected them in their task descriptions. The findings from my study also align with Anhalt et al.'s (2006) findings: M-PSTs included cognitively demanding tasks in their lesson plans once they reflected on and discussed selected tasks with their colleagues. In my study, after discussing the cognitive demands of tasks, M-PSTs provided justifications for why higher demand tasks were required for their students. While Crespo and Sinclair (2008) and Anhalt et al.'s (2006) studies suggested that PSTs considered specific factors (i.e., aesthetic values, cognitive demands) that were explicitly addressed in the activities, they did not suggest how PSTs would negotiate multiple factors from multiple readings while selecting tasks. In my study, M-PSTs were encouraged to use two task frameworks while describing tasks. The data allowed me to describe how M-PSTs negotiated multiple task-related factors from the task frameworks while describing tasks. Thus, the findings from this study extend the previous findings that M-PSTs' task selection improves after? discussion focused on a single framework or specific task characteristics. This study offers nuances into how M-PSTs negotiate several task-related factors and how they explore connections between several task categories when they engage with multiple task frameworks.

Conclusion and Implications

The instructional activities and data collection for this study were limited to a two-week period. Thus, the findings do not explain how long M-PSTs will retain the conceptions of tasks that they developed during these activities. However, the M-PSTs in this study did have important opportunities to continue to purposefully use these conceptions as they wrote lesson plans during the second methods course and student teaching semester. In their lesson plans, M-PSTs had to select tasks from available curricular resources and justify how those tasks were appropriate for their target students. I recommend future researchers implement these instructional activities across two methods courses (over a period of two semesters). Such extended periods of activity and study might provide opportunities for researchers to study which factors M-PSTs would continue to highlight in their task descriptions in the long term. It would be appropriate to investigate how M-PSTs' negotiation of different factors would shift while describing tasks after a long series of similar activities. This study focused on how a group of 12 M-PSTs described tasks; therefore, the findings do not provide detailed evidence on the progress of an individual M-PST's task descriptions. Thus, future studies should conduct individual interviews and record classroom discussions to explore how these instructional activities influence the change for an individual M-PST.

In this study, I investigated how M-PSTs described the characteristics of mathematical tasks as an initial attempt to understand M-PSTs' conceptions of mathematical tasks through their task descriptions. I acknowledge that a study on how M-PSTs would use task frameworks while making task selection decisions during their teaching is required to further understand M-PSTs' conceptualizations of tasks. In subsequent studies, I will investigate how M-PSTs enhance their conceptualization of tasks when they select, sequence, and implement tasks during their peer teaching.

To conclude, M-PSTs' conceptions of task characteristics were enhanced when they were afforded opportunities to engage in reading, discussing, and reflecting on multiple task frameworks. Reflection and discussion of task characteristics from multiple frameworks possibly assisted M-PSTs to consider a wide range of factors in their task selection and descriptions. Therefore, these findings are significant for mathematics teacher educators; they can and should use these instructional activities in their methods courses. These activities can potentially help M-PSTs to be better prepared to handle multiple task-related factors, including contextual factors, during their future teaching. In addition, teacher educators could use the emergent framework that I developed to enhance M-PSTs' conceptions of tasks. Mathematics education researchers could consider investigating how the framework assists in developing M-PSTs' task selection skills.

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