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# Facilitating Mathematical Reasoning Through Team-based Learning: Review and Discussion of Current Practice

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*Team-based learning (TBL) is a flipped classroom model, where small group discussions and peer learning play a central role. Some of its features, such as scalability to large classes and a high degree of structure, together with a well-documented success rate in other fields, could make TBL an attractive option for the mathematics educator wishing to transform their teaching. This article surveys available peer-reviewed literature to provide an overview of current use of TBL in mathematics, summarizes findings, and based on these, discusses TBL's potential to support mathematics learning. I pay particular attention to if and how TBL can be leveraged to shift students' focus from procedural towards conceptual learning and more creative forms of mathematical reasoning.*

As lifelong learning is becoming an increasingly central aspect of higher education, critical thinking and self-directed learning skills are often highlighted. In the OECD report "Education 2030", Schleicher (2018) said:

> In these times, we can no longer teach people for a lifetime. In these times, education needs to provide people with a reliable compass and the navigation tools to find their own way through an increasingly complex and volatile world. (p. 60)

Similarly, UNESCO (International Commission on the Futures of Education, 2021) said:

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> We should teach students (of all ages) to engage with knowledge creatively and critically, questioning its assumptions and interests (p.77).

Similar sentiments can be found in national education policies, such as the Swedish Higher Education Ordinance (Högskoleförordningen, 1993), which includes the ability to critically interpret and discuss information and phenomena as a degree requirement for a Bachelor's degree. Likewise, the Swedish government report Statens offentliga utredningar (2019) stressed that analytic ability, critical thinking, and independence are important in a world where future competence needs are partially unknown.

Thinking critically and making informed judgments relies on the ability to reason and evaluate the validity of others' reasoning. These are skills that students can practice in the mathematics classroom simply by doing what mathematicians do (e.g., determine the truth value of a conjecture by proof or counterexample), but undergraduate mathematics students are often not actively engaged in such tasks. Typical learning activities involve practice problems, but the exercises most found in calculus textbooks encourage procedural learning rather than reasoning (Lithner, 2008). Indeed, Selden, Selden, and Mason (1994) showed that mathematics students in general, even those with good grades, struggle and perform poorly on problems for which they possess the necessary content knowledge but have not been shown an explicit method.

Commenting on students' focus on procedural learning, Lithner (2011) asserted that "the main components of the traditional mathematics learning environment, teaching and textbook, at least reinforces but probably creates this behaviour among students" (p. 299). If, as Lithner proposed, the traditional teaching approach does not encourage students to develop reasoning skills, educators should look for other strategies that potentially do. A compelling case can be made for "active learning" strategies (Conference Board of the Mathematical Sciences, 2016; Freeman et al., 2014), but individual faculty trying to heed this call and transform the way they teach face the challenge of creating meaningful activities that help students reach the intended learning outcomes. Hence a ready-made and tested strategy can be an attractive way forward. Team-based learning (TBL) is one such “branded” strategy, and the purpose of this paper is to provide an overview of and highlight the use of TBL in mathematics education, describe findings from research literature, and discuss TBL’s potential to support mathematics learning and develop students’ mathematical reasoning.

Proponents of TBL claim that it supports quality learning while improving social and collaborative skills (Michaelsen et al., 2004). A study by Espey (2018) linked TBL to a self-reported improvement in critical thinking skills. Certain aspects of TBL, such as its scalability to large classes (Peters et al., 2020) and potential to emulate how mathematicians work (see Paterson and Sneddon, 2011, and later parts of this paper), could also make TBL an attractive option for mathematics educators. Although studies critically assessing the efficacy of TBL can be found in research literature reviews (e.g., Haidet et al., 2014; Liu & Beaujean, 2017), most of these are situated in a medical or health science context, and relatively few of the studies reviewed deal with TBL in science, technology, engineering, or mathematics. For example, the review by Haidet, Kubitz, and McCormack (2014), synthesized 40 articles of which 30 are medical/health science related and only one of which described mathematics education. However, looking at more recent publications suggests a current uptake of collaborative learning methods in mathematics education, and today, a number of TBL studies related to mathematics or neighboring fields can be found. After a brief description of TBL in the next section, I present a review of the current literature on the combination of TBL and mathematics.

## TBL Background, Principles, and Structure

### Background and Theory

TBL was first developed by Larry Michaelsen in 1979. He taught organizational behaviour at the University of Oklahoma and had to adapt his dialectic teaching method to a sudden growth in class size (Sibley & Ostafichuk, 2014). His adaptation, which he called Team-based learning, has since evolved into the current form practiced today and is described in this article. Hrynchak & Batty (2012) observed that TBL, if properly implemented, aligns closely with the main elements of constructivist learning theory. These elements, as set out by Kaufman (2003), advise that in constructivist teaching practice, educators should (1) be facilitators of learning rather than transmitters of knowledge; (2) provide opportunities to reveal inconsistencies between learners' current understanding and new experiences; (3) engage students in active learning, using relevant problems and group interaction; and (4) provide sufficient time for in-depth examination of new experiences. The following sections will reveal that the first and third elements are inherent in the TBL course design and that the second and fourth may be dependent on properly designed learning activities.

## Guiding Principles of TBL

The term "team-based learning" refers to a particular form of collaborative learning, in which small student groups repeat a specific cycle of learning activities. It is an implementation of the flipped classroom model, in which students are expected to acquire content knowledge before each class, typically from course literature or instructor-prepared teaching material, while classroom time is entirely dedicated to theoretical or applied exercises facilitated by the instructor.

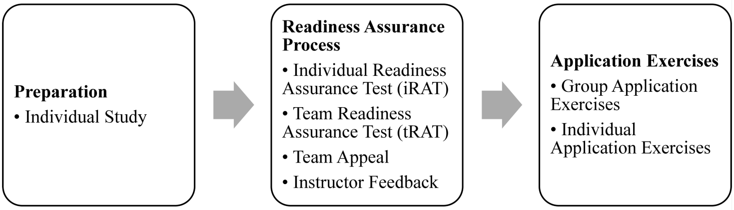
Michaelsen (Michaelsen et al., 2004, Ch. 2) claimed that if the course design adheres to certain principles, student groups will develop into self-managed learning teams over the duration of the course and that individual students will take responsibility for their initial exposure to course content preparing themselves for the in-class teamwork. These principles are (1) teams must be properly formed and managed, (2) students must be held accountable for individual and group work, (3) team assignments must promote both learning and team development, and (4) students must have frequent and timely performance feedback. The TBL framework, which is centered around these principles, provides a high degree of structure to the flipped classroom model, prescribing not only what to do in class but also how to organize the entire course.

## The TBL Course Structure

In TBL, a course is divided into a few themes or major topics that are presented in sequence, and Michaelsen recommends four to seven themes within a course (Michaelsen et al., 2004, Ch. 2). Each theme consists of a three-phase cycle, with the individual phases being (see Figure 1) preparation, readiness assurance process (RAP), and application exercises (AEs). This cycle is completed once for each theme.

**Figure 1**

*TBL Activity Cycle for a Course Theme.*



*Note.* Image by Nurul (2021).

### *Phase 1: Preparation*

In the first phase of a TBL cycle, which happens outside the classroom, students prepare for the subsequent in-class activities. The instructor's main role is to facilitate this process, e.g., by providing recorded lectures or a reading list covering the necessary course content.

### *Phase 2: Readiness Assurance Process (RAP)*

After students arrive in class, their understanding of fundamental concepts is assessed by a multiple-choice quiz executed in two steps. In the first step, referred to as the individual Readiness Assurance Test (iRAT), each student completes and submits the quiz, which is immediately followed by the second step, the team Readiness Assurance test (tRAT). For the tRAT, students take the same quiz again, but this time they collaborate in small teams, and each team has to come to an agreement on which answer to submit. Team discussions generated by this process are an important aspect of TBL, and for this reason the tRAT typically involves an immediate feedback and resubmission mechanism meant to encourage further debate and team cohesion. After submitting their tRAT, teams get immediate feedback on their performance and may re-evaluate and resubmit (with a score penalty) until they submit all answers correctly. If, for example, a dominant team member has pushed for an incorrect answer that the team has agreed on without understanding why, this mechanism may encourage previously passive team members to speak up. The tRAT may also involve an appeal process in which teams can argue that an answer marked as incorrect should be accepted.

Like the preparation phase, the instructor's role is to facilitate rather than teach. The exception is a short concluding lecture in which the instructor can address any remaining issues or unresolved questions. The immediate feedback and resubmission process described above is naturally implemented through an online submission platform or by using scratch cards (Immediate Feedback Assessment Technique cards) specifically designed for this purpose.

### *Phase 3: Application Exercises (AEs)*

In the last phase students build upon the knowledge they have gathered and processed in the previous two phases by working together (in the same teams) on more complex exercises. These exercises form the bulk of in-class activities in the course and should adhere to "the 4-Ss" (Sibley & Ostafichuk, 2014, Ch. 7), which are design guidelines for activities that encourage the whole team to work together. The 4-Ss include the following components: (1) a Significant Problem that is meaningful to the course and is rich enough to involve the whole team, preferably beyond the individual capability of any single team member; (2) the entire class works on the Same Problem at the same time, in order to generate greater investment before a class-wide discussion or debriefing; (3) teams should be able to express their solution to a problem with a Specific Choice, which forces team members to all come to agreement on a single, clearly defined answer in light of vague or conflicting information; and (4) responses from teams are revealed Simultaneously in order to encourage accountability, preventing other teams from modifying their answers based on answers revealed by another team.

### *Example: A Readiness Assurance Process from a Calculus Course*

By the end of a first-year undergraduate calculus course I teach, students are expected to know the limit definition of derivative, understand the idea of the derivative as representing a rate of change or the slope of a tangent line, and master procedures for calculating derivatives (i.e., differentiation rules). In this context, the procedural learning issues previously described manifest as students focusing on the mechanics of differentiation while sometimes neglecting definitions and conceptual understanding. In TBL however, the purpose of the Readiness Assurance Process is, as the name suggests, to solidify the fundamental knowledge needed for the next sequence of exercises. Subsequent application exercises or practice problems, e.g., optimization problems or theoretical investigations of the derivative concept, require students to have a firm grasp of the differentiability concept, and the following example addresses a common source of confusion. I will present the question with possible solutions followed by a likely scenario in a TBL class.

**Question:** Let $f(x) = x^{2/3}\sin(x^{1/3})$. What is $f'(0)$?

**Possible Solutions:** Procedural knowledge, i.e., differentiation rules, will tell students that

$$f'(x) = \frac{2}{3}x^{-1/3}\sin\left(x^{\frac{1}{3}}\right) + \frac{1}{3}\cos\left(x^{\frac{1}{3}}\right) \qquad \text{for} \qquad x \neq 0$$

but this expression is undefined for $x = 0$. Thus, it does not provide a direct answer to the question. On the other hand, the limit definition of derivative does:

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{h^{\frac{2}{3}} \sin\left(h^{\frac{1}{3}}\right) - 0}{h} = 1.$$

### *A Likely Scenario in a TBL Class.*

During iRAT (the individual quiz), some students give the wrong answer; a common misunderstanding being that when the differentiation rules fail us, the function is not differentiable. Some students find the correct answer but are unsure of why it is correct, and other students have correctly solved the problem using the definition or some other logically correct argument. Worth noting is that even in the last case, the student may still be unsure of why their correctly deduced answer is not provided by the differentiation rules. They may incorrectly think of the differentiation rules as giving a different answer instead of the differentiation rules as giving no (direct) answer.

During the tRAT, which directly follows the iRAT, before any feedback on the iRAT has been given, the students as a team answer the same question again, but now they have to agree on a common answer. The students are likely to have arrived at different answers individually or reached the same answer with different approaches. (One approach not mentioned above is applying the differentiation rules and then taking the limit of $f'(x)$ as $x \to 0$, which works, but it takes some further reasoning to understand why it works.) Ideally, this leads to a win-win situation for all, in which students with different strengths or different levels of understanding learn from each other. Research literature supports the idea that explaining one's own reasoning to peers and receiving explanations from peers is beneficial for mathematics learning (Webb, 1982; Pijls et al., 2007).

The above scenario, although based on certain assumptions, is not entirely hypothetical. During the fall of 2022, this question was part of a RAP in the aforementioned calculus class. Roughly half of the students gave the correct answer on the iRAT, and most teams were able to deduce the correct answer for their first tRAT attempt. At the concluding debrief, one team asked for further clarification on how the differentiation rules can seemingly contradict the definition, and another team asked about the validity of the alternative approach mentioned above. The previously mentioned issue with dominant team members is also real and has been documented by Lim (2022) and Nihalani et al. (2010), who both observed that once a team has identified a high performing "superstar" team member, the rest of the team might then defer to the superstar's authority without questioning or understanding, thus defeating the purpose of the tRAT. The instant feedback and resubmission mechanism of the tRAT is intended to counteract this behavior (Michaelsen & Sweet, 2008), and there is observational evidence that it does, to some degree, have the desired effect (Lim, 2022).

I have described the general principles of TBL and how TBL can be used to teach mathematics with certain objectives in mind. The review that follows next looks at examples from available research literature on how TBL is being used to teach mathematics. The findings from this review will then form the basis for the discussion section.

## Literature Search

This review examines peer-reviewed articles describing or investigating the use of TBL in a mathematics instructional context. To find available sources satisfying these criteria, I conducted two advanced searches of the Scopus database using the following queries:

- TITLE-ABS-KEY("team-based learning") AND ( LIMIT-TO ( SUBJAREA,"MATH" ) ) AND ( LIMIT-TO ( DOCTYPE,"ar" ) )
- TITLE-ABS-KEY("team-based learning" mathematics) AND ( LIMIT-TO ( DOCTYPE,"ar" ) )

The first search yielded 13 hits (as of July 2023) and returned articles in the database containing the phrase "team-based learning" in the title, abstract, or keywords, restricted to the "article" document type and mathematics subject area. The second search produced 18 results partially overlapping with the first search but with a different filtering approach. All search results were then manually filtered to remove articles not discussing TBL in a mathematics instructional context. Nine articles passed the manual filter, and an additional article, which satisfied the criteria but did not appear in the original searches, was found by following references from the original set of articles. Hence, the following 10 articles were selected for this review: Clair and Chihara (2012), Ford (2018), Lewis and Estis (2020), Lewis et al. (2021), Naughton et al. (2020), Nanes (2014), Paterson and Sneddon (2011), Peters et al. (2020), Sheryn and Ell (2014), and Vance (2021).

The selected articles were thoroughly read with particular attention given to the following guiding questions: (1) What were the authors' reasons for implementing TBL in mathematics education?, (2) Did the authors encounter any challenges particular to mathematics education in implementing TBL, and how were those dealt with?, (3) Did the authors report on student feedback?, and (4) Did the authors perform any quantitative analysis of the efficacy of TBL in mathematics education? A summary of findings is presented in the next section.

## Common Themes in Surveyed Articles

### Purpose or Motivation for Using TBL in Mathematics Education

Among the ten studies included in my analysis, the reasons given for implementing TBL differed but had a few overlapping themes, which I discuss below.

### *Promoting Deeper Understanding and Mathematical Thinking*

Paterson and Sneddon (2011) argued that mathematical thinking can be fostered through learning activities that encourage students to "think mathematically and behave like a mathematician" (p. 881) and described how TBL enabled them to design such activities. As an example of mathematician-like behavior that would emerge from the learning activities, they described how students would put forward and test different conjectures, thus creating a need for the students to convince themselves and others of the conjecture's validity. Ford (2018) stressed the need for a framework that emphasizes critical thinking and effort, rather than specific algorithms, and an explicit goal for the TBL implementation by Peters et al. (2020) was to help students attain a deeper understanding of calculus concepts.

### *Improving Student Learning Outcomes, Engagement, and Pass Rates*

Similar goals relating to student engagement, pass rates, and attrition rates were stated by both Nanes (2014) and Peters et al. (2020). A goal of Nanes (2014) was to "engage with weak students so that students who historically would have failed or withdrawn from mathematics courses can instead pass the class" (p. 1209). Peters et al. (2020) stated four goals, two of which were to "increase student engagement in the course" (p. 212) and "improve student success rates so more students remain STEM majors" (p. 212).

### *Scalability and Structure*

An often-claimed benefit of TBL is its scalability to large classes (Michaelsen et al., 1982, 2004), and both Nanes (2014) and Peters et al. (2020) mentioned large enrollment as one deciding factor when choosing to implement TBL. Another practical reason was given by Lewis, Clontz, and Estis (2021) who aimed to facilitate collaborative inquiry through Inquiry

Based Learning (IBL) but found it difficult to balance specific content goals with the open-ended inquiry model suggested by IBL. They also sought additional structure that would encourage collaboration and guarantee student preparedness for in-class activities and found that TBL can both integrate the principles of IBL and provide such structure. Structure was also a deciding factor for Vance (2021), who observed that the TBL course design promotes individual accountability towards the team, encourages teams to deal with conflict, and enables teams to improve their cooperative process.

## Subject Specific Challenges in Implementing TBL

Proponents of TBL have stressed that TBL is not a teaching technique but a holistic strategy, consisting of certain learning activities in a particular sequence, and have provided several recommendations on how these activities should be constructed and conducted (Michaelsen et al., 2004). On the other hand, different teaching circumstances or the educators' own teaching philosophy can contradict these recommendations. This section highlights cases from the surveyed articles in which the authors perceived contradictions between the general recommendations and their view of how mathematics should be taught.

All except one of the surveyed articles reported implementing a Readiness Assurance Process. There were diverging views regarding the purpose of the RAP, which affected the course structure. If, as Figure 1 suggests, each course theme should be assessed in a corresponding RAP, then Nanes (2014) and Peters et al. (2020) considered the recommended division of a course into four to seven themes as too few for mathematics learning but suggested bearing in mind that the material learned one day often relies on a working understanding of the material learned the day before. On the other hand, Lewis, Clontz, and Estis (2021) took the "tautological view" that the purpose of the RAP is to ensure student readiness for the following AE. By focusing the RAPs on the prerequisite skills and concepts needed for the subsequent AEs rather than attempting to cover the entire syllabus, they found that fewer RAPs were needed.

Regarding Application Exercises and the 4-Ss, Nanes (2014) faced difficulty designing class activities that represented a Significant Problem beyond the individual capability of a single student, and he found such complexity unnecessary. The 4-Ss are guidelines for designing activities that encourage the whole team to work together, and Nanes found that these goals could still be achieved by presenting mathematical challenges similar to test or homework problems. Several authors (Nanes, 2014; Peters et al., 2020; Paterson & Sneddon, 2011) also seem to agree that for mathematics assessments, where the solution process might be more important than the answer, the Specific Choice principle is hard to implement strictly. It is noted however that the principle does not rule out the possibility of a question having several correct answers or solution strategies. Paterson and Sneddon (2011) found that limiting teams to submit only one solution that occupies no more than a single page created situations where teams would discuss and argue to agree on the best solution to submit.

### *Student Experiences and Feedback*

Several authors (Clair &Chihara, 2012; Lewis & Estis, 2020; Lewis, et al., 2021; Vance, 2021) gathered student feedback on their TBL implementation, typically through post course surveys, obtaining moderate to strong agreement on questions pertaining to their perceived learning (e.g., "The use of TBL during class time was a valuable learning experience"). Such surveys can however be misleading (Deslauriers et al., 2019) and may be more reflective of students' acceptance of the TBL pedagogy rather than its efficacy.

In pre- and post-course interviews, Sheryn and Ell (2014) asked students about their attitudes towards group work and found that these were either consistently positive or changed from ambivalent to positive. Naughton et al. (2020), whose objective was to raise aspirations, conducted pre- and post-course surveys focused on confidence in mathematical ability, mathematical interest, and interest in pursuing university studies and found an increase in positive responses to 14 out of 15 survey questions. When students were asked to elaborate or comment on their answers, some common themes appeared across several studies.

### *Exposure to Different Ways of Thinking*

Lewis and Estis (2020); Lewis, Clontz, and Estis (2021); Naughton et al. (2020); and Sheryn and Ell (2014), all reported that students found value in experiencing multiple perspectives, e.g., their teammates' and the course instructor's, on the same problem.

### *Learning From Explaining to Others*

Lewis and Estis (2020); Lewis, Clontz, and Estis (2021); and Sheryn and Ell (2014) found that their students perceived themselves as learning from explaining their own reasoning to their teammates. Sheryn and Ell (2014) wrote that "five [out of ten] students mentioned that being the 'teacher' and explaining their understanding to others was extremely beneficial" (p. 873).

### *Engagement in the Course*

In several studies (Clair & Chihara, 2012; Lewis & Estis, 2020; Lewis et al., 2021; Sheryn & Ell, 2014), students reported being more engaged than they would have been in a traditional setting. According to Sheryn and Ell (2014) "the regular discussions motivated these students to be prepared and to be involved, and they were less likely to be passive learners" (p. 873).

### *Reduced Communication Apprehension and Math Anxiety*

Lewis and Estis (2020) administered the "personal report of communication apprehension questionnaire" (PRCA-24) before and after the course, and observed a significant decrease in communication apprehension among female (but not male) students on all sub-scales of the PRCA-24. Lewis, Clontz, and Estis (2021) reported reduced anxiety in students as observed by both instructors and students and noted that students recognize

TBL as a safe environment for productive struggle. Similarly, Naughton et al. (2020), based on their survey responses, suggested that a TBL-based approach can help students overcome mathematics anxiety issues.

### *Negative Impressions*

Although the selected studies almost exclusively reported positive feedback from students, there were also some negative comments. Lewis and Estis (2020) described some dissatisfaction with the flipped model, with students saying, “I don’t like having to teach myself what’s going on” (p. 174). They also reported concerns that teams were sometimes engaged in nonconstructive rather than constructive struggle. Sheryn and Ell (2014) described student concerns regarding the contributions from and personalities of other group members.

## Quantitative Effects on Performance

Three of the selected studies (Lewis & Estis, 2020; Nanes, 2014; Peters et al., 2020) involved a quantitative analysis of learning outcomes, comparing TBL versions with non-TBL versions of the same course.

Lewis and Estis (2020) and Nanes (2014) compared results from different versions of the same course (in both cases, linear algebra), either taught with a TBL approach or with a traditional lecture-based approach. Each study revealed a significant difference in favor of TBL. In both cases however, TBL sections were compared with a historical control of previous non-TBL iterations of the same course, which raises the issue of whether results from different semesters can be meaningfully compared. Partial justification is given by Lewis and Estis (2020), who argued that the standards-based grading (Elsinger and Lewis, 2020) they employ is more robust than “one-time assessments” such as final exams.

A more rigorous comparison is provided by Peters et al. (2020) who compared TBL and non-TBL sections of the same course, a large enrollment calculus course. In their study, comparisons were drawn between sections taught the same semester, assessed with the same exams, and graded with a uniform grading policy. Furthermore, they also calculated each student's normalized gain from pre- and post-course versions of the Calculus Concept Inventory (CCI), a test designed to assess understanding of fundamental calculus concepts (see Epstein, 2013, for a description of the CCI and definition of normalized gain). Data from two consecutive years was analyzed, corresponding to the first and second time TBL was introduced in the course, and for the second year a statistically significant difference in favor of TBL was observed, for both exam and CCI gain scores.

# Discussion

## Summary of Reviewed Articles

The overall picture painted by the selected articles is that mathematics educators choose to implement TBL hoping to promote deeper understanding and improve learning outcomes. Some reasons to choose TBL over other active learning strategies include the highly structured nature of TBL and its scalability to large classes, which are common for undergraduate mathematics courses. Some authors argued that the mathematics context calls for minor modifications to the recommended TBL course structure, but no major obstacles in adapting TBL to mathematics education were reported. Students find value in being exposed to different ways of thinking and learning by explaining their own understanding to others. Quantitative measurements on test performance support the claim that mathematics learning outcomes (at least as measured by these tests) can be improved by TBL. A birds-eye view of the selected studies also prompts some further comments on the crafting of team assignments and team composition, both being central aspects of TBL.

### *Crafting Appropriate Team Assignments is Important but may Take Time to Learn*

The quantitative data on learning outcomes as measured by Lewis and Estis (2020), Nanes (2014), and Peters et al. (2020) all showed an improvement from the first time TBL is introduced to later TBL iterations of the same course. A reasonable explanation would be that TBL instruction comes with a learning curve and that it takes one or several attempts for educators to become proficient with the new strategy. Transitioning from a lecture-based model towards TBL means crafting team assignments that will form the bulk of student in-class activities. According to the third TBL principle, these assignments should promote both learning and team development, but my review reveals that the construction of such exercises is one of the main challenges faced by mathematics educators converting their course to the TBL format. Comments from students, praising TBL as a safe environment for productive struggle but also criticizing the unproductive struggle sometimes occurring when the team gets stuck, lend further support to the idea that appropriately challenging exercises are essential to the TBL framework.

### *Team Composition Matters*

The first TBL principle, which states teams must be properly formed and managed, may deserve more attention than the reviewed studies provide. TBL literature (Michaelsen et al., 2004; Sibley & Ostafichuk, 2014) recommends teams be instructor-assigned, diverse, not too small (five to seven students are recommended), and fixed teams. Teams should be heterogeneous enough in terms of skills, opinions, and experience to handle complex tasks, which is hard to achieve if the teams are too small or if students are allowed to self-select their teams. The recommendation to keep the teams fixed throughout the course allows time to achieve group cohesiveness. In the mathematics small group learning context, Webb (1982, 1991), confirmed that group composition does indeed impact learning. By studying mixed-ability groups with high-ability, medium-ability, and low-ability students, Webb found that the low- and high-ability students benefited by forming teacher-student pairs, while medium-ability students were disadvantaged, and instead did better in homogeneous-ability groups.

## What About Mathematical Reasoning?

In my introduction I discussed critical thinking in undergraduate education and argued that in a mathematics context this translates to shifting focus from procedural knowledge towards more creative forms of mathematical reasoning. I then asked whether TBL can have a positive impact in this regard, and findings from the reviewed articles regarding test scores, flexibility in problem solving, and attempts to foster mathematician-like behavior do shed some light on this question.

### *Quantitative Studies and Conceptual Learning*

Quantitative studies involving test results are limited by test design and what the tests actually measure. If a test mostly involves procedural tasks, the results will tell how well students have mastered these procedures but not much else. For this reason, quantitative studies based on final exam scores without further detail of what the exam covers will not be considered in this section. Concept inventories on the other hand, are often deliberately designed to test conceptual understanding and diagnose known misconceptions. Peters et al. (2020) used the Calculus Concept Inventory (Epstein, 2013) to assess students' change in conceptual understanding, with results in favor of TBL over traditional lecture-based teaching. Broadening the view from TBL to active learning pedagogy in general, there is empirical support for the idea that active learning is particularly well suited to support conceptual learning. For example, Freeman et al. (2014) conducted a large meta-study that included 225 individual studies on undergraduate STEM courses and found that on average, performance on examinations and concept inventories increased by 0.47 standard deviations under active learning, compared to traditional lecturing. This positive effect was particularly pronounced on concept inventory performance (0.88 SD compared to 0.44 SD for instructor written course exams), which the article explained by pointing out that active learning has a greater impact on the higher-level cognitive skills that concept inventories are designed to assess.

### *TBL and Flexibility in Problem Solving*

Lewis and Estis (2020) mentioned similar concerns regarding procedural versus conceptual learning and approached this by looking at flexibility, which they defined as the ability of students to use multiple problem-solving strategies and identify the best strategy for a given problem. To investigate whether TBL increases students' flexibility in problem solving, they compared solutions to a particular assignment where students were asked to compute the determinant of a matrix. The problem could be completely solved by an entirely algorithmic approach (such as cofactor expansion) but a hybrid approach (e.g., performing a few well-chosen row operations before the expansion) could simplify the problem and make it less computationally intense. They noted that students from a TBL class were far more likely to apply a hybrid approach while students from a lecture-based class, despite having been taught the same methods, were more likely to favor an algorithmic approach. Lewis and Estis hypothesized that the peer learning experience provided by TBL fosters flexibility by exposing students to their teammates' different approaches to the same problem.

### *Encouraging Students to Behave Like Mathematicians*

Sibley and Ostafichuk (2014) explained TBL (in particular the Specific Choice principle) with a courtroom jury analogy: A jury sifts through evidence and reasons, guided by a set of rules (laws), to eventually produce a simple binary decision, guilty or not guilty. They then claimed that TBL sets up a similar dynamic when student teams are asked to arrive at a simple decision after interpreting complex information. Mathematicians' work also follows this model. A mathematical conjecture may be presented, and the task becomes to determine, by reasoning, whether the conjecture is true or false. Such problems, of suitable difficulty, can be posed in a TBL class and team members could contribute to the discussion by suggesting more or less complete arguments or by questioning and poking holes in their peers' reasoning. If TBL generates this kind of dynamic process in the mathematics classroom, then it essentially mimics the way mathematicians work and helps develop students' mathematical reasoning. It appears that Paterson and Sneddon (2011) looked at TBL from this perspective as they described how TBL prompted the creation of tasks that encourage students to think and behave like mathematicians.

A limitation with the last argument is that it is mostly hypothetical. It describes an ideal situation where students develop their mathematical reasoning by engaging in constructive discussions, applying reasoning and logic to collaboratively work out the answer to a mathematical problem. The question remains about how common such a situation really is. Teams sometimes employ less ideal decision strategies such as deciding by majority vote (Lim, 2022) or relying on a trusted team member without questioning (Lim, 2022; Nihalani, 2010), neither of which involves any significant discussion within the team. A fundamental claim of TBL is that as long as the general principles are adhered to, "teachers create a context that promotes the quantity and quality of interaction required to transform groups into highly effective learning teams" (Michaelsen & Sweet, 2008, p. 12), but only a few TBL studies appear to challenge this claim. Looking at review articles and meta-studies (Haidet et al., 2014; Liu and Beaujean, 2017), there is a general consensus that TBL "works" (exam results improve, students are engaged and satisfied), but much less is known about the underlying mechanisms and how it works, in particular what kind of interactions really take place during team discussions, how course design affects these interactions, and how this affects student learning. A better understanding of such questions could help educators achieve more consistent success with TBL and other active learning strategies. Steps to address the first two questions were taken by Lim (2022), who categorized the main types of interactions taking place in observed tRAT sessions, and identified some situational factors associated with the different types. Based on Weregif's (2020) dialogic theory of learning, Lim classified team interactions as disputational, cumulative, exploratory, or playful. This classification does not take into account whether the interaction is founded in mathematical reasoning or procedural thinking, but the theoretical framework provided by Lithner (2008) does. Hence, analyzing team interactions through Lithner's lens could be an interesting continuation of Lim's work.

## Limitations

This review covered only peer-reviewed journal papers and ignores results presented in conference proceedings. Besides the quality aspect, practical reasons have factored into this decision, such as the time required to scan a larger body of text, and the fact that proceedings papers are often less readily available. Also, although this was not a selection criterion, the surveyed articles focused heavily on post-secondary education, and the findings of this review may not be generalizable to other levels.

## Conclusion

TBL has a documented success rate in mathematics teaching, particularly at the post-secondary level, and there is at least some evidence that TBL can be effectively leveraged to support the development of creative mathematical reasoning skills. However, student teams do not always adopt interaction patterns and decision strategies that are conducive for learning, and educators should strive to better understand what motivates students in this regard.

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## References

Clair, K., & Chihara, L. (2012). Team-based learning in a statistical literacy class. *Journal of Statistics Education*, *20*(1), 1–20. https://doi.org/10.1080/10691898.2012.11889633

Conference Board of the Mathematical Sciences. (2016). *Active learning in post-secondary mathematics education*. https://www.cbmsweb.org/2016/07/active-learning-in-post-secondary-mathematics-education

Deslauriers, L., McCarty, L. S., Miller, K., Callaghan, K., & Kestin, G. (2019). Measuring actual learning versus feeling of learning in response to being actively engaged in the classroom. *Proceedings of the National Academy of Sciences*, *116*(39), 19251–19257. https://doi.org/10.1073/pnas.1821936116

Elsinger, J., & Lewis, D. (2020). Applying a standards-based grading framework across lower level mathematics courses. *PRIMUS*, *30*(8–10), 885–907. https://doi.org/10.1080/10511970.2019.1674430

Epstein, J. (2013). The Calculus Concept Inventory—Measurement of the effect of teaching methodology in mathematics. *Notices of the AMS*, *60*(8), 1018–1026. https://doi.org/10.1090/noti1033

Espey, M. (2018). Enhancing critical thinking using team-based learning. *Higher Education Research & Development*, *37*(1), 15–29. https://doi.org/10.1080/07294360.2017.1344196

Ford, J. (2018). Blending team-based learning with standards-based grading in a calculus classroom. *Mathematics Teaching-Research Journal*, *10*(3-4), 52–70.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences*, *111*(23), 8410–8415. https://doi.org/10.1073/pnas.1319030111

Haidet, P., Kubitz, K., & McCormack, W. T. (2014). Analysis of the team-based learning literature: TBL comes of age. *Journal on Excellence in College Teaching*, *25*(3/4), 303–333.

Hrynchak, P., & Batty, H. (2012). The educational theory basis of team-based learning. *Medical teacher*, *34*(10), 796–801. https://doi.org/10.3109/0142159X.2012.687120

Högskoleförordningen. (1993). Sfs 1993:100. https://www.riksdagen.se/sv/dokument-och-lagar/dokument/svensk-forfattningssamling/hogskoleforordning-1993100_sfs-1993-100

International Commission on the Futures of Education. (2021). Reimagining our futures together: A new social contract for education. UNESCO. https://doi.org/10.54675/ASRB4722

Kaufman, D. M. (2003). Abc of learning and teaching in medicine: Applying educational theory in practice. *BMJ*, *326*(7382), 213–216. https://doi.org/10.1136/bmj.326.7382.213

Lewis, D., & Estis, J. (2020). Improving mathematics content mastery and enhancing flexible problem solving through team-based inquiry learning. *Teaching and Learning Inquiry*, *8*(2), 165–183. https://doi.org/10.20343/teachlearninqu.8.2.11

Lewis, D., Clontz, S., & Estis, J. (2021). Team-based inquiry learning. *PRIMUS*, *31*(2), 223–238. https://doi.org/10.1080/10511970.2019.1666440

Lim, F. S. (2022). *Dialogues in team-based learning's team readiness assurance tests: are teams ready to dialogue?* [Unpublished doctoral dissertation]. National Institute of Education, Nanyang Technological University. https://hdl.handle.net/10497/24059

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, *67*(3), 255–276. https://doi.org/10.1007/s10649-007-9104-2

Lithner, J. (2011). University mathematics students’ learning difficulties. *Education Inquiry*, *2*(2), 289–303. https://doi.org/10.3402/edui.v2i2.21981

Liu, S.-N. C., & Beaujean, A. A. (2017). The effectiveness of team-based learning on academic outcomes: A meta-analysis. *Scholarship of Teaching and Learning in Psychology*, *3*(1), 1–14. https://doi.org/10.1037/stl0000075

Michaelsen, L., Knight, A., & Dee Fink, L. (2004). *Team-based learning: A transformative use of small groups in college teaching*. Stylus.

Michaelsen, L., & Sweet, M. (2008). The essential elements of team-based learning. *New Directions for Teaching and Learning*, 7–27. https://doi.org/10.1002/tl.330

Michaelsen, L., Watson, W.E., Cragin, J.P., & Dee Fink, L. (1982). Team learning: A potential solution to the problems of large classes. *Journal of Management Education*, *7*, 13-22.

Nanes, K. M. (2014). A modified approach to team-based learning in linear algebra courses. *International Journal of Mathematical Education in*

*Science and Technology*, *45*(8), 1208–1219. https://doi.org/10.1080/0020739X.2014.920558

Naughton, L., Butler, R., Parkes, A., Wilson, P., & Gascoyne, A. (2020). Raising aspirations using elements of team-based learning in mathematics: A pilot study. *International Journal of Mathematical Education in Science and Technology*, *52*(10), 1491–1507. https://doi.org/10.1080/0020739X.2020.1770880

Nihalani, P. K., Wilson, H. E., Thomas, G., & Robinson, D. H. (2010). What determines high-and low-performing groups? The superstar effect. *Journal of Advanced Academics*, *21*(3), 500-529. https://doi.org/10.1177/1932202X1002100306

Nurul Anis Huda Mohd Yem (2021). *Different question formats and quality of team discussion in team-based learning* [Final year project]. Nanyang Technological University. https://hdl.handle.net/10356/148495

Paterson, J., & Sneddon, J. (2011). Conversations about curriculum change: Mathematical thinking and team-based learning in a discrete mathematics course. *International Journal of Mathematical Education in Science and Technology*, *42*(7), 879–889. https://doi.org/10.1080/0020739X.2011.613487

Peters, T., Johnston, E., Bolles, H., Ogilvie, C., Knaub, A., & Holme, T. (2020). Benefits to students of team-based learning in large enrollment calculus. *PRIMUS*, *30*(2), 211–229. https://doi.org/10.1080/10511970.2018.1542417

Pijls, M., Dekker, R., & Hout-Wolters, B. V. (2007). Reconstruction of a collaborative mathematical learning process. *Educational Studies in Mathematics*, *65*(3), 309–329. https://doi.org/10.1007/s10649-006-9051-3

Schleicher, A. (2018). Educating learners for their future, not our past. *ECNU Review of Education*, *1*(1), 58–75. https://doi.org/10.30926/ecnuroe2018010104

Selden, A., Selden, J., & Mason, A. (1994). Even good calculus students can't solve nonroutine problems. In J. J. Kaput & E. Dubinsky (Eds.), *Research Issues in Undergraduate Mathematics Learning* (pp. 19–26). The Mathematical Association of America. https://doi.org/10.13140/RG.2.1.4303.7208

Sheryn, L., & Ell, F. (2014). Teaching undergraduate mathematics in interactive groups: How does it fit with students' learning? *International Journal of Mathematical Education in Science and Technology*, *45*(6), 863–878. https://doi.org/10.1080/0020739X.2014.884647

Sibley, J., & Ostafichuk, P. (2014). *Getting started with Team Based Learning*. Stylus.

Statens offentliga utredningar. (2019). *En långsiktig, samordnad och dialogbaserad styrning av högskolan*, SOU 2019:6 https://www.regeringen.se/rattsliga-dokument/statens-offentliga-utredningar/2019/02/sou-20196

Vance, E. (2021). Using team-based learning to teach data science. *Journal of Statistics and Data Science Education*, *29*(3), 277–296. https://doi.org/10.1080/26939169.2021.1971587

Webb, N. M. (1982). Peer interaction and learning in cooperative small groups. *Journal of Educational Psychology*, *74*(5), 642–655. https://doi.org/10.1037/0022-0663.74.5.642

Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education*, *22*(5), 366–389. https://doi.org/10.2307/749186

Wegerif, R. (2020). Orientations and group rules: A framework for researching educational dialogue. In R. Kershner, S. Hennessey, R. Wegerif, & A. Ahmed (Eds.), *Research methods for educational dialogue* (pp. 27-42). Bloomsbury Academic. https://doi.org/10.5040/9781350060111.ch-003