The Mathematics Identity Bicycle: A Metaphor for the Role of Agency in Mathematics Learner Identity Development

Stephen L. Caviness

In this conceptual paper, I trace the role of agency in mathematics learner identity development across several empirical studies to illuminate understanding of how researchers have conceptualized agency as it relates to identity in mathematics education research. Building on this line of research, I propose an adapted conceptual framework for examining the role of agency in mathematics learner identity development by attending to both micro and macro influences on mathematics learners. I argue that such a conceptual framework is needed to support novel research pursuing equity in mathematics education. Finally, I offer the metaphor of the mathematics identity bicycle to aid in understanding how this conceptual framework might be taken up by other researchers in the pursuit of equity in mathematics education.

Expressing individual agency may be understood as authoring one's own identity (Holland et al., 1998). This empowering notion of authoring one's identity has important implications for the pursuit of equity in mathematics education. By equity, I partly mean equitable access to and achievement in mathematics, or what Gutiérrez (2009) refers to as the dominant axis of equity. I also understand equity in the context of mathematics education to mean opportunities for all learners to develop intellect that supports criticality, experience joy, and learn not just about oneself but the identities of others (Muhammad, 2020; 2023). A key component of equity that I focus on in this paper is identity. A greater understanding of the role of agency, including factors that may encourage or hinder individual expression of agency, holds promising implications

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toward understanding mathematics learners' identities. More broadly, a greater understanding of agency and its role in mathematics learner identity development may lead toward a more equitable mathematics education for all learners.

I begin by defining mathematics identity and situating research involving agency within the broader scope of mathematics identity research. I then share a lineage of conceptual frameworks involving agency that researchers have built over time. Finally, I propose an adapted framework, along with the metaphor of the mathematics identity bicycle, based on this line of work conceptualizing agency in mathematics identity research. My purpose in doing so is to situate the role of agency in mathematics identity research and primarily to offer a conceptual frame that may be helpful for researchers, especially those whose work focuses on pursuing equity in mathematics education.

Mathematics Identity

Researchers conceptualize mathematics identity as it relates to both teachers of mathematics and learners of mathematics (Darragh & Radovic, 2020; Graven & Lerman, 2020). Researchers have also used multiple definitions for understanding and operationalizing mathematics identity (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019). Darragh and Radovic (2020) developed a definition for mathematics learner identity intended to be universally applicable across the different perspectives of identity that mathematics education researchers hold. They defined mathematics learner identity as "a socially produced way of being, as enacted and recognized in relation to doing mathematics" (Darragh & Radovic, 2020, p. 582). I adopt this definition for mathematics learner identity based on its general applicability across the field of mathematics education.

Theories Emphasizing Agency

In addition to defining mathematics identity in multiple ways, mathematics identity researchers have utilized many

different theoretical frameworks. Langer-Osuna and Esmonde (2017) categorized these theoretical perspectives into four categories, two of which emphasized the role of agency as well as its implications for researchers. Specifically, Langer-Osuna and Esmonde suggested that researchers who conceptualize mathematics identity with post-structural and positioning theories are most likely to attend to the role of agency in mathematics identity development. It follows that the literature discussed in this paper comes from mathematics identity research wherein researchers have adopted post-structural and positioning theories. Characterizations of these theories are clarified in Table 1 with examples of studies that have been guided by these approaches. Importantly, Langer-Osuna and Esmonde acknowledged that these categorizations overlap and that there are limitations to organizing a vast body of research into only four categories.

Table 1
Post-structural and Positioning Theories of Mathematics Identity

	Characteristics of Theory	Examples
Post-structural	Generally maintains a zoomed-out perspective to understand an individual's lived mathematical experiences in their broader context Considers the broad power structures which influence mathematics identity Expression of mathematics learner agency is made available or constrained from the top-down	Booker & Goldman (2016) Hodge (2008) Jung & Wickstrom (2023) Lange & Meaney (2011)
Positioning	Generally maintains a zoomed-in perspective to understand an individual's lived mathematical experiences in their immediate context Considers the situated and context dependent factors which influence mathematics identity Expression of mathematics learner agency is expressed in direct relation with others	Bishop (2012) Edelen et al. (2022) Gardee (2021) Smith (2022)

Mathematics identity researchers who adopt post-structural theories generally conceptualize a learner's ability to exercise agency as shaped by, and often constrained by, broad power structures. Researchers using this theoretical approach do not simply ask "'Is x true?' but rather 'What makes x possible?' and 'What are its effects?'" (Mendick et al., 2009, p. 72). Therefore, a post-structural approach to understanding agency does not only seek to understand whether learners express agency in mathematics, but also what broad structures of power are making such agency expressions possible and what the ramifications of these influences are. Examples of broad power structures might include administrative policies within a school or government mandates surrounding curriculum expectations. In this sense, the educator's role may be viewed as one cog in a broader machine of power structures that is influenced by authorities from the top-down.

In mathematics identity research guided by positioning theories, learner agency is generally considered within local, situated contexts rather than broader power structures as in post-structural theory. For example, Bishop (2012) described how a mathematics teacher might overtly position their students by consistently referring to them as mathematicians. A learner in this classroom context may then express agency by affiliating with, complying with, or rejecting the social positioning offered to them (Cobb et al., 2009). Students may reposition themselves but only as negotiated in a social context. Therefore, an individual's expression of agency may be constrained or supported by educators or, as Bishop (2012) showed, by other learners. With these characterizations in mind, in the following section I expand on the ways in which researchers have conceptualized agency in mathematics identity development.

Agency as Authorship

Agency may be understood as self-authorship of one's identity (Holland et al., 1998). Several researchers have literally referred to agency as authorship (e.g. Booker & Goldman, 2016; Goos & Bennison, 2019; Losano et al., 2018) whereas many

others have implied this notion through their descriptions of learners' choice (Gardee, 2021), participation (Gresalfi et al., 2009), or the extent to which learners can connect mathematics to their lived experiences (Jung & Wickstrom, 2023). In Holland et al.'s (1998) words, "the world must be answered—authorship is not a choice—but the form of the answer is not pre-determined" (p. 272). This suggests that the expression of agency is both necessitated and context dependent.

The theoretical perspective of agency as authorship has proven helpful for mathematics identity researchers in understanding agency. Still, much more work is needed to understand the ways that researchers have conceptualized the role of agency in mathematics identity development. Below, I elaborate further on several studies while highlighting conceptualizations of agency wherein the educator's and learners' roles concerning agency are closely intertwined.

Agency in Figured Worlds

One theoretical perspective that many mathematics identity researchers have used to guide their work is Holland et al.'s (1998) conceptualization of identity in figured worlds. A figured world is "a socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al., 1998, p. 52).

Boaler and Greeno (2000) drew on Holland et al.'s (1998) notion of figured worlds to conceptualize agency as self-authorship. In their study, Boaler and Greeno compared the identity development of secondary mathematics students in different AP Calculus classes across different schools. These AP Calculus classes were considered different figured worlds and were characterized by the pedagogical discourses that teachers employed: didactic, which is closely aligned with traditional or teacher-centered instruction, and discussion-based, which is well aligned with reform or student-centered instruction. Within this context, Boaler and Greeno (2000) understood agency as a mathematics learner's ability to author their own identity. This

ability is socially constructed and therefore influenced by an individual's positioning in a figured world.

Boaler and Greeno (2000) took up the question of "what different mathematics environments influence agency?". Specifically, they sought to understand the "possibilities and forms of authoring that are created in different mathematics environments" (p. 173). Their findings suggest that mathematics teachers have significant influence in the development of mathematics learners' identities through the *pedagogical choices* they make. In a series of interviews with students, Boaler and Greeno (2000) found that students who rejected mathematics or, in other words, did not develop a sense of belonging to mathematics, talked about feeling unable to express agency in the mathematics classroom. By this, students meant there were limited opportunities to express themselves or to be creative—opportunities that they experienced in other courses. Boaler and Greeno (2000) shared, "[t]o be a successful participant of a traditional [mathematics] classroom, students need to give up their choice and decision making, which is reflected in the students' comments about obedience and compliance" (p. 189). In their study, mathematics teachers who adopted didactic pedagogies corresponded with students feeling constrained in their expression of agency.

Students in the discussion-based mathematics classroom were afforded more opportunities to author their own identities. "In the discussion-based classrooms students were, quite simply, given more agency" (Boaler & Greeno, 2000, p. 189). The teachers in this study strongly influenced identity development through their control, or release of control, over how students might express agency. This control corresponded with their pedagogical choices. Teachers who stifled student agency and did not provide opportunities for voice and choice resulted in most students dis-identifying with mathematics. Teachers who created a classroom culture of discussion found that students were more likely to develop a positive mathematics identity.

There were, however, some students in this study whose identity development appeared to be an outlier from the majority of students in their class. These students were able to author a positive mathematics learner identity despite the didactic

pedagogical discourse employed by their teacher. To make sense of the few students who did not develop mathematics learner identities as expected, Boaler and Greeno (2000) circled back to the conception of agency that, despite the teacher's ability to limit or encourage self-authorship, there is still always *some* ability for self-authoring one's identity.

Building on these ideas, Cobb et al. (2009) presented a theoretical framework intended to help researchers operationalize mathematics identity. They then shared how this framework might be used in a sample analysis of data from a design-based research study. In an effort to directly expand on Boaler and Greeno's (2000) work, Cobb et al. (2009) first distinguished between what they referred to as *personal identity* and *normative identity*. Personal identity is an individual student's view of themselves in relation to mathematics, whereas normative identity refers to classroom obligations co-created in a classroom space that define what it means to be successful or competent in mathematics (Cobb et al., 2009). Students may exercise personal agency in a mathematics classroom as they comply with, cooperate with, or resist their classroom obligations (Cobb et al., 2009). In doing so, they contribute to the co-creation of normative identity by either shaping classroom obligations through resistance or allowing the continuation of the classroom status-quo for obligations by complying or identifying with those obligations

Within this context focused on normative identity specifically, Cobb et al. (2009) discussed how agency and authority are closely related. They defined authority as "'who's in charge' in terms of making mathematical contributions" (Cobb et al., 2009, p. 44). This authority has the effect of defining *whom* students are accountable to and *what* students are accountable for, which make up the classroom obligations that define the normative identity of a figured world, such as a classroom. Agency is conceptualized in Cobb et al.'s (2009) work in a few specific ways. Firstly, Cobb et al. (2009) make clear that they do not view agency as if it were on a continuum that one might have more or less of. Rather, their conceptualization of agency refers to "the ways in which students can legitimately exercise agency in particular

classrooms" (Cobb et al., 2009, p. 44). Secondly, they draw on Pickering's (1995) work to define two forms of agency: conceptual agency and disciplinary agency. Conceptual agency "involves choosing methods and developing meanings and relations between concepts and principles" whereas disciplinary agency "involves using established solution methods" (Cobb et al., 2009, pp. 44–45).

Drawing on data from a larger design-based research study, Cobb et al. (2009) exemplified how their theoretical framework might support analysis of empirical data. They compared two classrooms with characteristics similar to Boaler and Greeno's (2000) didactic versus discussion-based classrooms. The "Algebra" class closely resembled the didactic class whereas the "design-experiment" class closely resembled the discussion-based class in Boaler and Greeno's (2000) study. They found that the teacher of the Algebra class primarily offered students opportunities to exercise disciplinary agency through execution of mathematical procedures. However, students did not identify with mathematics because they felt their ability to exercise agency was limited to complying with or resisting the teacher's authority. They therefore did not affiliate with repetitively practicing procedures that appeared irrelevant to them. Students in the design-experiment class developed identities that affiliated with the classroom obligations, as they experienced authority as shared. Therefore, they did not feel limited in expressing either disciplinary or conceptual agency. Of note, no students in either class openly acted on their ability to resist classroom obligations, which is a limitation for understanding this specific expression of agency in mathematics learner identity development.

Continuing to expand on this work, Gresalfi et al. (2009) reemphasized that agency is not a quantifiable trait but is the choices available to an individual, which are shaped by sociocultural factors. They shared,

It is important here to dispel the notion that people "have" or "lack" agency. In virtually any situation, even the most constrained, people are able to exercise agency; at the basic level, by complying or resisting. The ways that agency can

be exercised, and the consequences for doing so, are what change in a particular context. Said differently, an individual can always exercise agency, it is the nature of that exercise that differs from context to context. (Gresalfi et al., 2009, p. 53)

In this conceptualization, the role of an educator is hinted at as one who has particular influence in shaping the context that an individual student finds themselves in. For this reason, Gresalfi et al. (2009) sought to understand not just whether students act but "how they are given opportunities to act" (p. 53). Their findings continued to build evidence for the educator's role in influencing learners' agency by shaping the choices that learners might act on. Specifically, they noted that the educator may do this through the mathematical tasks that they provide to students and the degree of openness these tasks have.

Although they did not specifically draw on Holland et al.'s (1998) framework of identity in figured worlds, Lange and Meaney (2011) did conceptually build from Boaler and Greeno's (2000) and Cobb et al.'s (2009) work. Instead of studying learners' mathematics identities in a mathematics classroom, Lange and Meaney (2011) analyzed parent-child interactions, which involved a parent offering help with mathematics homework to their child. As a first notable finding, their study exemplified how parents may play the role of educator in much the same way a teacher might. Secondly, Lange and Meaney (2011) found that constraining learners' agency and positioning the learners as passive receivers of knowledge leads to "mathematical trauma" (p. 38). Lange and Meaney's (2011) contribution offers an important reminder that mathematics learners' identities are not shaped solely within the confines of a mathematics classroom.

Agency as a Mediator

In a continuation of Boaler and Greeno's (2000), Cobb et al.'s (2009), and Wenger's (1998) work, Gardee (2021) developed a theoretical framework building on this line of literature. Gardee (2021) incorporated the work of Marks and

O'Mahoney (2014) and Archer (2002) to offer an expanded conceptualization of mathematics identity. In this paper, I have drawn primarily on Marks and O'Mahoney's work that influenced Gardee's conceptual framework. Therefore, I first describe Marks and O'Mahoney's work before elaborating further on Gardee's (2021) framework.

Marks and O'Mahoney (2014) heavily emphasized agency in their conceptualization of mathematics identity. In their own words, "agency is central" (Marks and O'Mahoney, 2014, p. 72). Agency is literally central in their visualization of agency as a mediating role between social and personal identities. Like Cobb et al. (2009), Marks and O'Mahoney (2014) theorized identity both individually and collectively. Marks and O'Mahoney (2014) referred to personal identity as "the individual's own beliefs" and social identity as "the roles in social structures which humans imperfectly inhabit" (p. 85). They conceptualized agency as a mediator between these two identities to elaborate on how identities are shaped. Marks and O'Mahoney (2014) shared, "the transformative capacity of agency is a human potential, rather than an actuality, and the potential of that capacity to be actualized is constrained, not only by social structures but also the human's position in that social structure at their birth" (p. 73). In this sense, they hint at both the educator's role of influencing social structures within the classroom and the broader social structures beyond the classroom that may potentially constrain learner agency.

Gardee (2021) incorporated these ideas into her theoretical framework for understanding mathematics learner identity in the mathematics classroom (see Figure 1). This framework was then elaborated further by Gardee and Brodie (2022, 2023). Gardee drew on the sociocultural learning theories of Sfard and Prusak (2005) and Wenger (1998) to highlight the role that mathematics educators play when offering social identities to learners. Although such sociocultural theories informed Gardee's (2021) framework, she argued that the learning theories of Wenger (1998) and Sfard and Prusak (2005), which are heavily utilized in mathematics identity research (Darragh, 2016), are insufficient on their own for understanding mathematics as they underemphasize the role of agency. Therefore, she incorporated

Boaler and Greeno's (2000) and Cobb et al.'s (2009) notion of individual agency, which involves a learner's choice to affiliate with, comply with, or resist social identities offered to them. Notably, Gardee also added marginalization from mathematics communities as a choice for a learner to express individual agency, not just as a social positioning offered by others. Together, these manifestations of individual agency lead to both personal and social identity construction. In this sense, agency is a mediator as in Marks and O'Mahoney's (2014) theorization.

Figure 1

Gardee's Theoretical Framework for Mathematics Identity.
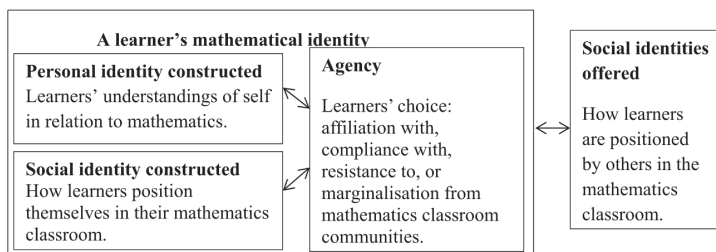
Reprinted from "Relationships between Teachers' Practices and Learners' Mathematical Identities," by Gardee, A., 2021, International Journal of Mathematical Education in Science and Technology, 52(3), p. 379.

In Gardee's (2021) framework, an educator may influence learners by offering them social positionings either in affiliation with the classroom community or marginalization from the classroom community. She observed two mathematics teachers and found that the means by which these teachers offered social positionings of affiliation or marginalization were through pedagogical approaches and social relationships. This was evident in the teachers' observable practices and the discourse they used.

Gardee (2021) also acknowledged that it is not just educators who position students, but that students may position other students. Gardee and Brodie (2023) specifically focused on this peer-to-peer influence in a later paper and showed how several momentary instances of social identities offered between

learners, or *micro identities*, developed into more enduring macro identities. Gardee and Brodie's (2023) study accounts for the way that a learner might express agency as resistance in one moment and as affiliation in the next (micro identities), though ultimately, learners trend toward one enduring pattern for how they express agency over time (macro identities).

Agency From a Zoomed Out Perspective

In the studies discussed above, researchers conceptualized mathematics identity and agency in ways that generally adhered to positioning theories. As a result, the analyses and conceptualizations of agency were "zoomed in" at the classroom level (Lerman, 2003). Cobb et al. (2009) referred to this level of zoom as the "classroom microculture" (p. 63). As shown, positioning theories are especially apt for this particular focus at the classroom level. Researchers whose work may be categorized as post-structural offer a different perspective by "pulling back" (Lerman, 2003) to capture the broader power structures that influence agency and mathematics identity development (Langer-Osuna & Esmonde, 2017). I offer some noteworthy examples of this macro perspective in this section.

Like other researchers, Booker and Goldman (2016) drew on Holland et al.'s (1998) conceptualization of agency as authorship, however they did not explicitly build on the line of work outlined in the sections above (e.g., Boaler and Greeno, 2000; Cobb et al., 2009). Although they conceived of agency as authorship, they also drew on Bandura (2001) to highlight the role of *collective agency* rather than solely focusing on individual agency. In this sense, the focus of their study zoomed out to a lens that included parents, students, teachers, and researchers at the level of a school community. They emphasized agency as being transformative and having the effect of systemic repair, particularly for parents to reclaim what they called *epistemic authority*: "exercising the right or the power to know" mathematics (Booker & Goldman, 2016, p. 223).

Given their wider lens and a participatory design research methodology, Booker and Goldman's (2016) analysis did not

focus solely on the educator's influence on mathematics learner identity development. Instead, their focus was on the collective learning of participants who engaged in mathematics problem solving workshops for families and school members. They found that participants in the study experienced an increase in individual and collective agency, but specifically at a small scale within their community of practice which included parents, teachers, and researchers. Interestingly, as agency increased along with increased epistemic authority, participants' awareness of the systemic and institutional barriers also increased. This resulted in a feeling of powerlessness due to feeling a lack of agency in the face of systemic power structures that they could not control. Booker and Goldman's (2016) findings exemplify the complex intersection of micro and macro level influences on agency in mathematics learner identity development as well as the need for researchers to attend to these different levels of "zoom."

Other researchers offered conceptualizations of agency that align with post-structural theories of mathematics identity. For example, Jung and Wickstrom (2023) referred to "critical mathematical agency" (p. 5), which is when learners make connections between mathematics and contextualize it in their own lives. When learners express critical mathematical agency, they are active participants in creating knowledge collaboratively, with both peers and educators, rather than consumers taking in knowledge solely from a teacher. In addition to this, Grant et al. (2015) emphasized the importance of a post-structural perspective, though they did not adopt it in their particular study:

> Many critical researchers have acknowledged that mathematics classrooms and mathematical tasks are not neutral or without power dynamics, equitable access, and opportunity for engaging… While power dynamics and equity are not prominent in this study, we recognize that these dynamics influence agency and accountability and the importance of being mindful of such within both research and practice. Otherwise, there is no commitment to social justice and the status quo continues. (p. 95)

Herein lies not just the promise of, but also the importance of, considering the role of agency in mathematics learner identity development from a post-structural perspective.

Toward a Revised Conceptual Framework

The literature I have presented in this paper highlights both the ways in which researchers have conceptualized agency in mathematics learner identity development and the findings from those studies surrounding the influential role of educators on learners' agency. I shared researchers who conceptualized agency in mathematics learner identity development that adopted post-structual and positioning theories in their work. Generally, these researchers also conceptualized agency as self-authorship (Holland et al., 1998). A distinction was found between researchers' focus, or level of "zoom" (Lerman, 2003), which corresponded with their adoption of post-structural or positioning theories.

Agency is not quantifiable nor is it a personality trait one has—instead, agency is the choice that an individual has, which is shaped by sociocultural factors (Cobb et al., 2009; Gresalfi et al., 2009). The role of the educator is to do the shaping of these sociocultural factors. In a mathematics classroom, educators may influence learner agency through their pedagogical choices and the degree of authority they share with students (Boaler & Greeno, 2000; Cobb et al., 2009). And they may do this specifically through the tasks that they select for students to work on (Gresalfi et al., 2009). The social identities of affiliation with or marginalization from the classroom community are expressed through both pedagogical actions and social discourses with learners (Gardee, 2021). With this said, I argue that a focus on the educator's role alone is insufficient for understanding the many influences that may constrain or enable learner agency.

Balancing Agency

In my view, the danger in overemphasizing the role of agency in mathematics learner identity development is that the

responsibility, or more aptly stated, blame, for disidentifying with mathematics may be placed on students rather than the social, societal, and institutional influences that shaped them. It is for this reason that others have argued against the grit and/or growth-mindset movement (e.g., Gorski, 2016)—clearly, learners cannot simply author their way out of systemic racism, for example. At the same time, to underemphasize the role of agency in a learner's mathematics identity development diminishes the real power that individuals and collective groups of individuals have for effecting change. Not only this, but to belittle or to dismiss a learner's ability to express agency leaves those learners who do not feel a sense of belonging to mathematics hopeless. Therefore, I call for a balanced conceptual emphasis on agency when studying mathematics learner identities.

Assuming an appropriate balance of emphasizing agency in mathematics learner identity development, I have shown the importance of attending to agency in mathematics education research through the literature outlined above. I began this paper appealing to a focus on agency as potentially helpful for moving toward equitable mathematics education for all learners. To this end, I presented through the literature above that researchers experience certain limitations based on the extent to which they have conceptually zoomed in or zoomed out. Although many researchers in the studies above zoomed in closely, few zoomed out far enough to examine or acknowledge the broader power structures that influenced learners' agency. Surely there is nothing wrong inherently with researchers selecting a specific level to zoom in to, as evidenced in the knowledge that was gained from these studies, but it is important to maintain a macro level awareness of factors influencing agency. There is a need for a conceptualization well-suited for attending to both the micro and macro perspectives of agency in mathematics learner identity development. Therefore, in Figure 2, I offer a reimagined version of Gardee's (2021) theoretical framework to intentionally include macro perspectives and also make slight revisions that I feel more appropriately conceptualize agency as a mediator between personal and social identities.

Figure 2

A conceptual framework for understanding agency in mathematics learner identity development with both micro and macro perspectives.
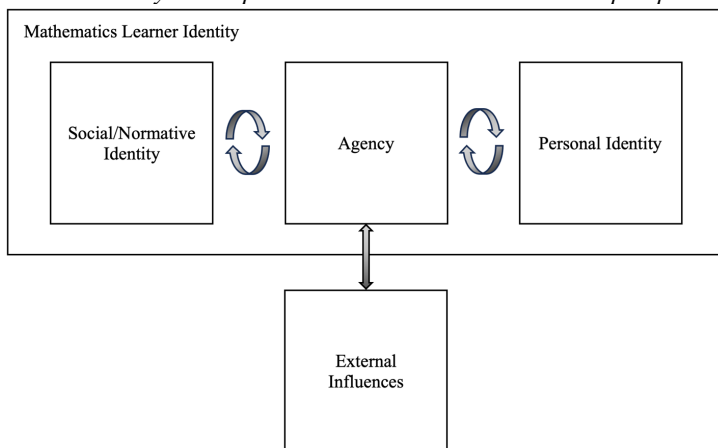
Adapted from "Relationships between Teachers' Practices and Learners' Mathematical Identities," by Gardee, A., 2021, International Journal of Mathematical Education in Science and Technology, 52(3), p. 379.

One important difference in this adapted framework is that "external influences" is intentionally a general phrasing to allow for attending to both micro and macro influences. External influences may very well be the social identities that a mathematics teacher offers a learner as conceptualized by Gardee (2021), but it may also be the role of the parent as shown in Lange and Meaney's (2011) study, other educators, or the broader institutional structures as shown in Booker and Goldman's (2016) study. I have used the terminology mathematics *educator* rather than mathematics *teacher* throughout this paper to acknowledge these broader educators who influence mathematics learners' identity. With this in mind, I call for more researchers studying mathematics identity to zoom out and consider the influence of mathematics educators beyond the mathematics classroom. The framing of external influences that I have proposed is conceptually broad enough to support this call.

Another distinction in the adapted mathematics learner identity framework is that the external influences arrow directly influences agency, rather than the entire box of mathematics identity as a whole. This is because I am conceptualizing agency as a mediator between external influences and the ways in which identity is developed. Like Gardee (2021), the arrow connecting external influences and agency is bidirectional. Conceptually, this is because learners simultaneously shape and are shaped by the world around them through expressions of agency. In addition to this, I view the bidirectional arrow as symbolic of the need to maintain balance between overemphasizing and underemphasizing the role of agency in mathematics learner identity development. Within reason, learners do have the capacity to shape their external surroundings. A learner's expression of agency might very well influence a teacher's pedagogical choices based on their affiliation with, compliance with, or resistance to identities offered to them. A learner might even influence some of the broader structural decisions that affect them—perhaps by voicing their thoughts at a school board meeting. However, a balanced perspective also acknowledges the external influences on learner agency that are largely beyond a learner's control. To reiterate the danger of overemphasizing learner agency, external influences may constrain a learner's agency and generally do not allow for the learner to simply reauthor the surroundings they have found themselves in. As Booker and Goldman (2016) noted, such broader systemic influences are difficult to change and slow to change at best.

Lastly, a visual change in the adapted mathematics learner identity framework centers agency as a mediator between personal and social identities that, together, make up a learner's mathematics identity. This is in agreement with Marks and O'Mahoney's (2014) description of agency as "central" to identity and the point through which "personal identity and social identity are reproduced and transformed" (p. 72).

The Mathematics Identity Bicycle

My hope is that this conceptual framework might be taken up in mathematics identity research in a way that allows for researchers to understand agency from both micro and macro level perspectives. Therefore, I would like to offer a metaphor for understanding the role of agency in mathematics learner identity development as I currently understand it, based on the literature I have presented and based on the framework I have just proposed. As with all metaphors, it has its limitations. With this in mind, I offer the metaphor of mathematics learner identity being a bicycle and a visualization of this metaphor shown in Figure 3.

Figure 3
The Mathematics Identity Bicycle
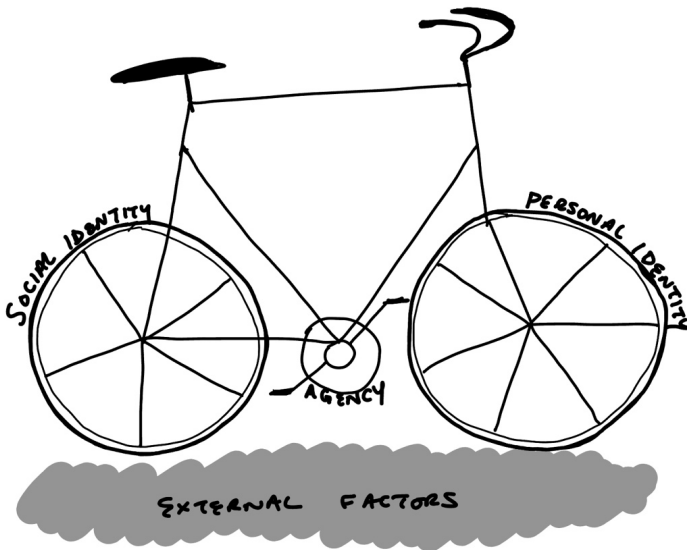
Firstly, the metaphor of the bicycle was chosen because mathematics identity is "something we do" (Darragh, 2016, p. 29) and biking is appropriately active. If mathematics identity is the bicycle, then I conceive of a mathematics learner metaphorically riding the bicycle as enacting their mathematics identity. Secondly, social identity and personal identity are both

labeled as the wheels, in no particular order, to represent that these identities are both always in motion, always fluid, and always connected to one another. For example, the way a learner might position themselves socially in a classroom is interrelated with their personal beliefs about mathematics and how they may view themselves. The front wheel's motion impacts the back wheel's and vice versa. Thirdly, in this metaphor, agency is conceptualized as the pedals of the bike. The pedals are the source of energy that make everything on a bicycle go, stop, or stall. Similarly, agency is conceptualized as affiliating with, complying with, or resisting external factors. This particular bicycle has pedals that, when pedaled in reverse, engage the brakes. Therefore, the mathematics identity bicycle will move forward when pedaled clockwise (affiliation), will 'coast' if held in place (compliance) allowing for the wheels to freely rotate, or will actively stop if pedaled counterclockwise (resistance). When riding the bicycle, a learner is always in some state of pedaling. Different states of pedaling the bicycle may be active, as with pedaling forward and braking, or passive, as with coasting. These states of pedaling align with the generally active nature of expressing agency through affiliation with or resistance to classroom norms and the passive nature of complying. In this sense, a learner who is riding the bicycle is *always* pedaling, and is therefore *always* expressing agency, consistent with Holland et al.'s (1998) conceptualization.

Finally and perhaps most importantly, the mathematics identity bicycle cannot be removed from the external factors or setting it is in, just as a real bicycle must have some context for where it is placed. These external elements directly affect the learner's ability to pedal, or in other words their ability to express agency. And these factors that may foster or hinder agency are generally outside of an individual's control. External factors might be a well-paved bicycle lane that is flat. Such conditions might allow for active pedaling and in no way constrain a learner's agency, represented by affiliation. Perhaps the external factors are a downhill grade in which a learner will likely coast with little effort, represented by compliance. Perhaps the downhill is so steep that the learner frequently brakes to prevent picking up too much speed, or possibly, a

roadblock might cause the learner to come to a complete halt, both representing resistance. Perhaps the grade is so steeply uphill that a learner simply cannot climb it without other supports. In this sense, a macro perspective of agency may be employed to understand broader systemic issues affecting learner agency, and researchers might ask what can be done to regrade the hill, a significant undertaking. Other external factors may be temporarily influential, like the weather—how a learner is feeling on a particular day in class. Or external factors may be more enduring, for example seemingly never-ending construction on one's commute, like mine, that require the learner to bike a longer route than necessary or precariously dodge the construction zone. It should be noted too that different learners have different routes. This truth requires attending to the unique ways in which learners' agency may or may not be constrained by external factors. It also raises the question "if learners have different routes, then what is the destination?". If the destination is school, then it should not be assumed that every learner arrives to school with an equally smooth bike ride. This allows for a reimagined view of what *counts* in mathematics. And in these ways, the bicycle metaphor for mathematics learner identity is helpful for understanding the complex ways in which agency and identity are interrelated. My hope is that this conceptual framing may be helpful for mathematics identity researchers in general, and especially those seeking to attend to external factors affecting learners with the goal of creating equitable and meaningful mathematics experiences for all students.

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